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## CALCULATION AND EXPERIMENTAL STUDY OF THE RETRACTING FORCE FOR MAGNETIC SPRINGS OF TWO TYPES

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*Designs for magnetic springs of two types have been proposed, and the methods of calculation of their retracting forces have been developed. Formulas are obtained for the retracting force in the main section of spring force characteristics. Experimental data are in good agreement with the results of theoretical calculations. The force characteristics of the proposed magnetic spring constructions can be varied for a specific application. The derived formulas are verified experimentally. Ways to change the force characteristics of magnetic springs according to specific requirements are demonstrated.*

*Keywords:* magnetic spring, permanent magnet, magnetic circuit, retracting force, residual magnetic induction, demagnetizing factor, coercive force.

### 1. Introduction

There are a lot of magnetic spring types. The majority of them have a power-law dependence of the retracting force on the displacement. However, the largest practical interest is attracted by magnetic springs, whose force weakly depends on the displacement [1, 2].

In two last years, we studied in detail two types of magnetic springs with an almost constant retracting force (the corresponding variation varies within the limits of 15%) over the working stroke of the spring. We consider the springs of those types to be the most promising for the majority of specific applications.

The first type of magnetic springs can be conditionally called “permanent magnet–magnetic circuit” (Fig. 1). In this type of springs, the interaction between a permanent magnet (as a rule, with a large energy product) and a magnetic circuit fabricated from a magnetically soft material is used. The second type of magnetic springs can be conditionally called “two permanent magnets” (Fig. 2). It consists of a tubular permanent magnet with the axial magnetization, with another cylindrical or ring magnet inside, the magnetization of which is antiparallel to that of the external magnet. This design allows the force characteristic shape to be varied by applying of a soft magnetic disk on the non-working end of the spring (see Section 5).

In this work, the calculation technique is developed, and the corresponding formulas are obtained for the determination of the retracting force magnitude in the main section of force characteristics, the models for magnetic springs of both types are developed, and the methods to vary the shape of a force characteristic at its initial and finite sections was proposed. All obtained results are verified experimentally on an automated installation for mechanical tests R-5.

### 2. Physical Model and Derivation of the Retracting Force Formula for a Magnetic Spring of the First Type

In Fig. 1, the schematic diagram of a magnetic spring of the first type (“permanent magnet–magnetic circuit”) is shown. The permanent magnet can be a cylinder with diametral magnetization or a rectangular prism with magnetization directed perpendicularly to a displacement.

The cross-section of the magnet oriented perpendicularly to a spring displacement direction can also have the shape of ellipse, rhombus, or trapeze: in all cases, the spring will work, but with a weaker retracting force, because the demagnetizing factor is smaller in those cases. The shape of the magnetic circuit must satisfy the following requirement: it must contain a cavity with the transverse cross-section that corresponds to that of the permanent magnet (it is desirable that the magnet should enter it with a sliding fit).

For the spring to be the most efficient, the magnetic circuit should transmit the total magnetic flux created by the permanent magnet. The smaller the gap between the magnet poles and the magnetic circuit, the lower is the magnetic flux scattering, and the stronger is the retracting force of the magnetic spring. The distance between the lateral sides (not poles) of the permanent magnet and the magnetic circuit can be arbitrary; it practically does not affect the force characteristics of magnetic springs of this type.

Earlier, we showed [3] that the retracting force can be calculated by the formula:

$$F = N^2(B_r^2/2\mu_0)S, \quad (1)$$

where  $N$  is the demagnetizing factor dependent only on the geometric shape of the magnet,  $B_r$  the residual magnetic induction of the material that the permanent magnet is fabricated from, and  $S$  the area of a transverse cross-section of the permanent magnet. Formula (1) was obtained, by using the virtual displacement method. At retracting the magnet by the distance  $dx$ , the work

$$dA = Fdx \quad (2)$$

is executed, where  $F$  is the retracting force. On the other hand, before the permanent magnet was introduced into the magnetic circuit, the demagnetizing field [4]

$$B = -NB_r \quad (3)$$

existed inside the magnet owing to the existence of magnet poles. In this expression,  $N$  is the demagnetizing factor, and  $B_r$  the residual induction of a magnetically hard material. After the magnet is introduced into the magnetic circuit, this field almost vanishes, so that the demagnetizing factor can also be conditionally adopted to equal zero. Therefore, in our approximation, the whole energy of the demagnetizing field can be considered to be spent on the work associated with a displacement of the permanent magnet. The magnetic field energy density equals

$$w = B^2/2\mu_0 = N^2B_r^2/2\mu_0, \quad (4)$$

where  $\mu_0 = 4\pi \times 10^{-7}$  H/m is the universal magnetic constant.

If the magnet shifts by the distance  $dx$ , the magnetic field energy  $W$  changes by

$$dW = wdV, \quad (5)$$

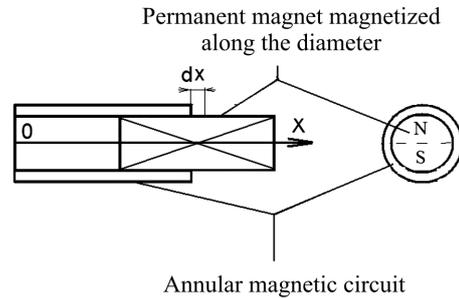


Fig. 1. Schematic diagram of the magnetic spring with a cylindrical magnet with diametral magnetization

where  $w$  is the density of the magnetic field energy, and  $V$  the volume, in which the field has changed. In our case,

$$dV = Sdx, \quad (6)$$

where  $S$  is the area of a magnet cross-section perpendicular to the cylinder axis. Substituting Eqs. (4) and (6) into Eq. (5), we obtain

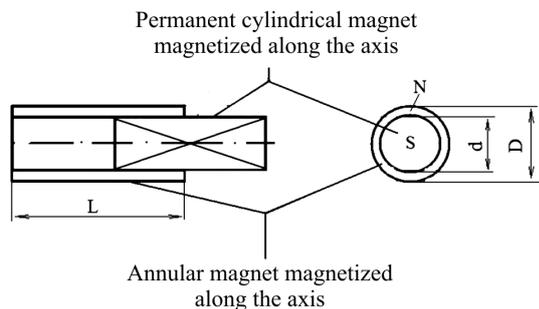
$$dW = N^2(B_r^2/2\mu_0)Sdx. \quad (7)$$

Equating Eqs. (2) and (7), i.e. putting  $dA = dW$ , and reducing the result by  $dx$ , we obtain the simple expression (1) for the force  $F$  of the permanent magnet retraction into a magnetic circuit.

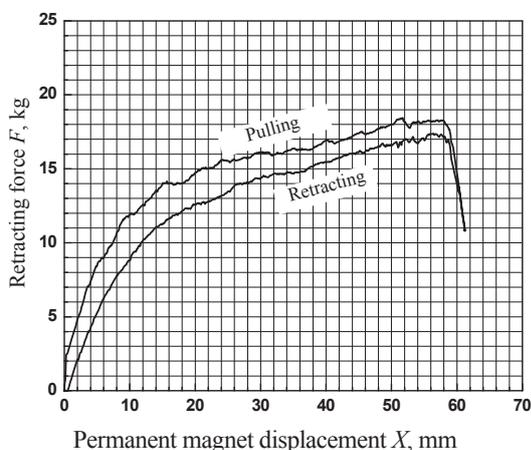
Formula (1) for the retracting force is in good agreement with experimental data. A typical experimental force characteristic of such a spring is depicted in Fig. 3. The difference between the force characteristics obtained at the retraction and the pulling is connected with the friction force between the permanent magnet and the magnetic circuit: the friction force is added to the retracting force at the pulling (upper curve), and subtracted at the retraction (lower curve). This design of a magnetic spring was protected by a patent for utility model [5].

We also experimentally proved the strong influence of the demagnetizing factor  $N$  on the retracting force [3] for magnetic springs with permanent magnets in the form of rectangular parallelepipeds characterized by the identical area of transverse cross-sections, but different distances between the poles. In accordance with formula (1), the forces turned out different by a factor of 1.7.

The advantages of this magnetic spring type include an easily controllable stroke length. The stroke



**Fig. 2.** Schematic diagram of a magnetic spring “two permanent magnets”:  $D$  and  $d$  are the external diameters of the ring and cylindrical, respectively, magnets;  $L$  the length of the external magnet



**Fig. 3.** Dependence of the retracting force on the displacement of the magnet in the tubular magnetic circuit for the cylindrical magnet 30 mm in diameter fabricated from Nd-Fe-B alloy, with diametral magnetization and the residual induction  $B_r = 1.25$  T

length of a spring depends on the length of an applied magnet. For a larger stroke, a longer magnet should be used. In this case, the retracting force does not change. As a shortcoming of this design, we can consider a strong force of attraction of the permanent magnet poles to the magnetic circuit. It can amount to 30–40% of the retracting force if the magnet is badly centered in the magnetic circuit. Both magnet poles should be equidistant from the magnetic circuit in order to reduce the friction force, which can result in an appreciable hysteresis in the spring force characteristic (Fig. 3).

In our experiments, we used ground magnets, which enter with a sliding fit into the magnetic circuit

fabricated from E12 electrotechnical steel. In order to reduce the friction force, the thin layer of a non-magnetic material with low friction coefficient can be used. This layer can also serve as an anticorrosion protection for the permanent magnet from humid air and help to mount the magnet poles at the same distance from the magnetic circuit.

### 3. Physical Model and Derivation of the Retracting Force Formula for a Magnetic Spring of the Second Type

In our opinion, another design of a magnetic spring, which can be called “two permanent magnets”, has wider capabilities. In the construction of this type, an external ring magnet and an internal cylindrical magnet are used. As the latter, a magnet with more complicated shape, but with cylindrical symmetry, e.g., a truncated cone, can be used. The magnetizations of both magnets are axial and antiparallel. The schematic diagram of such a spring is depicted in Fig. 2.

The internal magnet with a more complicated shape can be useful if the modification of the force characteristic of a magnetic spring is desirable. This construction can include a thin non-magnetic ring between the magnets to provide required changes in the force characteristic of the spring. The ring diminishes a little the retracting force in the main section, because the transverse cross-section area of the magnet becomes smaller. The corresponding decrease can be easily estimated, by using the formula derived below.

The retracting force can be varied by decreasing the area of the internal magnet pole. One can either diminish the internal magnet diameter or make an axial aperture in the latter, which turns out very useful if the magnet is attached to the control rod. The matter is that permanent magnets of the Nd-Fe-B system are characterized by the heated steel hardness, being at the same time very fragile, which makes it almost impossible to create any carving on them.

Let us consider a detailed derivation of the formula describing the retracting force for the design of this type. Earlier, it was shown [6] that the retracting force is well described in this case by the formula

$$F = B_r HS / \mu_0, \tag{8}$$

where  $S$  is the area of the transverse cross-section of the internal magnet,  $B_r$  the residual magnetic induction of a material, which the permanent magnet is

fabricated from,  $\mu_0$  the universal magnetic constant, and  $H$  the field in the external ring magnet, which is determined by the formula [4]

$$H = B_r[(1 + D^2/L^2)^{-0.5} - (1 + d^2/L^2)^{-0.5}]. \quad (9)$$

Here,  $D$  and  $d$  are the external and internal, respectively, diameters of the tubular magnet, and  $L$  its length.

If the residual inductions of both permanent magnets are identical, we obtain from Eqs. (8) and (9) that

$$F = B_r^2[(1 + D^2/L^2)^{-0.5} - (1 + d^2/L^2)^{-0.5}]S/\mu_0. \quad (10)$$

In the case concerned,

$$S = \pi d^2/4. \quad (11)$$

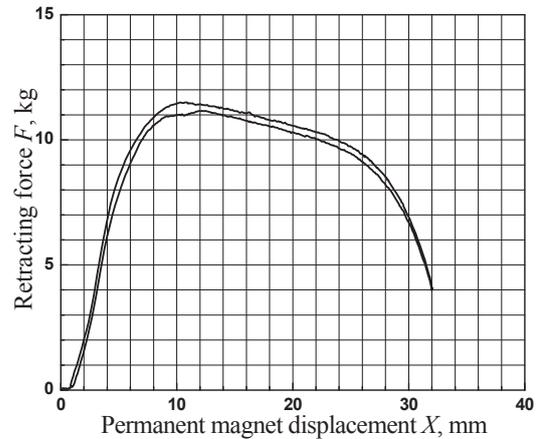
Substituting Eq. (11) into Eq. (10), we have

$$F = B_r^2[(1 + D^2/L^2)^{-0.5} - (1 + d^2/L^2)^{-0.5}]\pi d^2/4\mu_0. \quad (12)$$

Hence, we obtained the formula for the dependence of the retracting force magnitude on the geometric dimensions of applied magnets for a magnetic spring of the second type.

A comparison of magnetic springs of both types with identical diameters demonstrates that the second spring provides a retracting force, which is 40–100% stronger. Another advantage of this magnetic spring consists in an insignificant attraction between the internal and external magnets (no substantial hysteresis in the force characteristic resulting from the friction force was detected experimentally). A typical experimental force characteristic of a magnetic spring of the second type with  $D = 40$  mm and  $L = 35$  mm is shown in Fig. 4. This design of a magnetic spring also obtained the patent of Ukraine for utility model [7].

Perhaps, the only shortcoming of this construction is the increase of the spring stroke length, which is not so simple as in the first case. The matter is that if the stroke length grows, but the external diameter of the spring remains constant, the retracting force considerably decreases in accordance with formula (12). Now, let us consider its advantages.



**Fig. 4.** Dependence of the retracting force  $F$  on the core displacement  $X$  for a magnetic cylindrical spring with the following parameters: the external diameter of the tubular magnet equals 40 mm, the internal diameter of the tubular magnet 20 mm, the magnet length 30 mm, the internal magnet length 35 mm, and the internal magnet diameter 19.7 mm

1. As was said above, a magnetic spring of the second type is 40–100% more powerful provided the same diameter of the external magnet.

2. It can be optimized to a specific application depending on which parameter is a key one in this application. For instance, the diameters of magnets can be calculated proceeding from the stroke length of the spring and the required operating force.

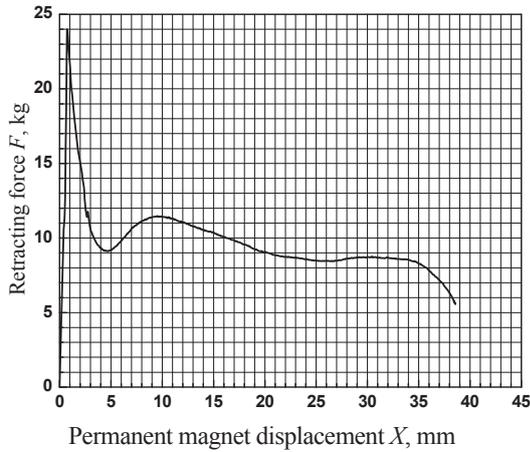
3. It is easily to provide a latch effort, which can be 2 to 4 times as large as the spring operating force, and the operating parameters of the spring in the main section will not be changed at that.

4. The section, where the spring force increases (in Fig. 4, these displacements are less than 10 mm) can be almost completely excluded.

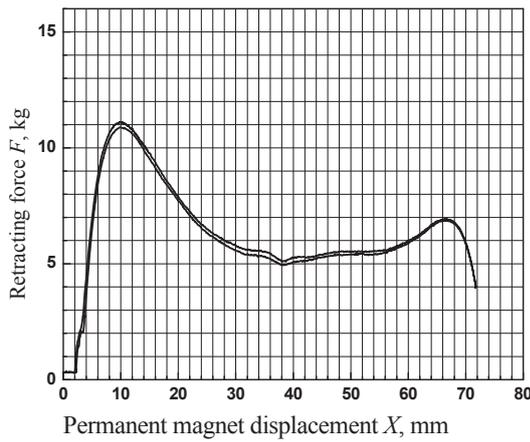
5. The friction force between the moving magnets is insignificant, and the exact centering of magnets, which is required in the case of a magnetic spring of the first type, is not necessary.

#### 4. Magnetic Spring of the Second Type with a Soft Magnetic Disk on Non-Working End of the Spring

The construction of the second type can be easily used, e.g., for the development of a door closer with prolonged service life. From this point of view, it satisfies all requirements put to those units: a retracting force of about 100–150 N in the main section and a re-



**Fig. 5.** Force characteristic of a spring of the second type with the end magnetic circuit: the external diameter of a tubular magnet equals 40 mm, the internal diameter of a tubular magnet 28 mm, the internal magnet length 35 mm, and the internal magnet diameter 25 mm



**Fig. 6.** Force characteristic of a magnetic spring with the double length: the tubular magnet length equals 70 mm, the external diameter 40 mm, and the internal diameter 25 mm

quired latch effort for the complete door closing. The latch effort is provided by attaching a cylindrical soft magnetic disk to the inactive end of the spring. The typical force characteristic of such a magnetic spring with a soft magnetic disk on the non-working end of the spring is depicted in Fig. 5. To make the latch effort stronger, the right end of the internal magnet can also be connected with a soft magnetic disk.

Any magnitude of the latch effort can be obtained within the interval from zero to the maximum by in-

roducing a non-magnetic interlayer with a required thickness between the soft magnetic disk and the internal magnet. This means that the curve in Fig. 5 can begin from any point within the interval from zero to 25 kg on the ordinate axis. The latch effort can be controlled by varying the thickness of the end magnetic circuit.

This design was successfully used by us in the return valve applied at the oil well washing. The valve is actuated when the pressure increases to 40 atm and is closed when the pressure drops below 10 atm. It is very difficult, if possible at all, to provide such a force characteristic with the help of ordinary mechanical springs.

### 5. Influence of Magnet Ends on the Force Characteristic

In the magnet spring construction of the second type, a large role in the formation of a force characteristic is played by the magnet ends, with their influence increasing with the ratio between the spring length and the spring diameter. A typical force characteristic of the spring usually contains two maxima (see Fig. 6). The first maximum is always larger than the second, because it is formed by both magnet poles. The “plateau” in the exhibited force characteristic (the section from 30 to 60 mm) can be calculated with a sufficient accuracy by formula (12), and the maxima are a consequence of the magnet end action. We experimentally proved this statement using a spring with the same diameter, but a double length. As a result, the corresponding maxima became sharper, and the “plateau” diminishes to an effort of 5.5 kg (Fig. 6).

If a smoother force characteristic of the spring is required, those maxima can be “cut off” by reducing the area of end cross-sections. Substituting the central cylindrical magnet by a combined one consisting of two truncated cones with a common base, it is possible to obtain a long enough spring with almost constant retracting force.

### 6. Conclusions

1. Designs of magnetic springs of two types—“magnet–magnetic circuit” and “two permanent magnets”—are proposed. Each of them is protected by the patent of Ukraine [5, 7].

2. Formulas for the dependence of the retracting force in the main section of a displacement are derived. They are confirmed by experiments.

3. On the basis of the models proposed, the parameters of magnetic springs can be calculated, e.g., for the required spring stroke length and effort.

4. On the basis of the obtained formulas, the suitability of the proposed constructions for their specific applications can be estimated.

5. The optimization of a magnetic spring like “Which geometrical dimensions of the magnetic spring are required to provide the maximum retracting force” or “Which type of magnetic springs with the same retracting force requires the minimum of a magnetic substance at its fabrication?” can also be done.

6. The design “two permanent magnets” allows one to obtain very diverse shapes of the force characteristic owing to the application of magnetic circuits, which satisfy almost any specific requirements.

1. K. Qian, P. Zeng, W.-M. Ru, and H.-Y. Yuan, *IEEE Trans. Magnetics* **39**, 1 (2003).
2. <http://www.linmot.com/products/magnetic-spring/>.
3. V.Yu. Tsivilitsin, Yu.V. Milman, V.A. Goncharuk, and I.B. Bondar, *Dopov. Nat. Akad. Nauk Ukr.* No. 9, 78 (2010).

4. V.Yu. Tsivilitsin, Yu.V. Milman, V.A. Goncharuk, and I.B. Bondar, *Dopov. Nat. Akad. Nauk Ukr.* No. 1, 81 (2011).
5. V.Yu. Tsivilitsin and A.P. Zhezherun, Patent for utility model No. 958 U F16F6/00, Ukraine (2001).
6. A.B. Altman, A.N. Gerberg, P.A. Gladyshev *et al.*, *Permanent Magnets. A Handbook* (Energiya, Moscow, 1980) (in Russian).
7. V.Yu. Tsivilitsin, Patent for utility model No. UA 83233 U, Ukraine (2013).

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РОЗРАХУНОК ТА ЕКСПЕРИМЕНТАЛЬНЕ  
ДОСЛІДЖЕННЯ СИЛИ ВТЯГУВАННЯ МАГНІТНИХ  
ПРУЖИН ДВОХ ТИПІВ

Резюме

Запропоновано конструкції магнітних пружин двох типів, розроблено методи розрахунку їх сили втягування. Отримано формули для підрахунку сили втягування на основній ділянці силової характеристики пружин. Експериментальні дані знаходяться у добрій відповідності до теоретичних розрахунків. Силкові характеристики запропонованих конструкцій магнітних пружин можна змінювати залежно від конкретного застосування. Проведено експериментальну перевірку отриманих формул. Показано шляхи зміни силових характеристик магнітних пружин відповідно до вимог конкретних застосувань.