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GENERATION OF THE LEPTONIC ASYMMETRY IN THE STERILE NEUTRINO HADRONIC DECAYS

We consider the leptonic asymmetry generation in the νMSM via hadronic decays of sterile neutrinos at $T \ll T_{EW}$, when the masses of two heavier sterile neutrinos are between m_π and 2 GeV. The choice of the upper mass bound is motivated by the absence of direct experimental searches for singlet fermions with greater mass. We carried out computations at zero temperature and ignored the background effects. Combining constraints of a sufficient value of the leptonic asymmetry for the production of dark matter particles, the condition for sterile neutrino to be out of thermal equilibrium, and existing experimental data, we conclude that it can be satisfied only for the mass of a heavier sterile neutrino in the range 1.4 GeV $\lesssim M <$ 2 GeV and only for the case of a normal hierarchy for the active neutrino mass.

Keywords: leptonic asymmetry, sterile neutrino, hadronic decays.

1. Introduction

The Standard Model (SM) is a minimal relativistic field theory, which is able to explain almost all experimental data in particle physics [1]. However, there are several observable facts that cannot be explained in the SM frame. First, the neutrinos of SM are strictly massless, which contradicts the experimental fact of the neutrinos oscillations [2, 3]. The second problem is the impossibility to explain the baryon asymmetry of the Universe (BAU) within the SM. Finally, the SM does not provide the dark matter (DM) candidate. The SM cannot also solve the strong CP problem in particle physics, the problem of primordial perturbations, the horizon problem in cosmology, *etc.*

The solutions of the above-mentioned problems of the SM require some new physics between the electroweak and the Planck scales. An important challenge for the theoretical physics is to see if it is possible to solve them using only the extensions of the SM below the electroweak scale [4].

The Neutrino Minimal Standard Model (νMSM) is an extension of the SM by three massive right-handed neutrinos (sterile neutrinos), which do not take part in the gauge interactions of the SM[1]. The model was suggested by M. Shaposhnikov and T. Asaka [5, 6]. The masses of sterile neutrinos are predicted to be smaller than those on the electroweak scale, and, thus, there is no new energy scale introduced in the theory. The parameters of the νMSM can be chosen in order to explain simultaneously the masses of active neutrinos, the nature of DM, and BAU.

The lightest sterile neutrino (the mass is expected to be in the keV range [4]) can be intensively produced in the early Universe and have a cosmologically long life-time. So, it might be a viable DM candidate. The sufficient amount of these neutrinos can be generated through an efficient resonant mechanism proposed by Shi and Fuller [7].

In the νMSM, the required amount of leptonic asymmetry (in accordance with the Shi–Fuller mech-

[1] This is why these neutrinos are called sterile neutrinos. The left-handed neutrinos of the SM are called active neutrinos.

anism) can be created due to decays of the two heavier sterile neutrinos. These particles are generated at temperatures $T > T_{EW}$, and their masses are expected to be in the range $m_\pi < M_I < T_{EW}$ [8], where m_π is the pion mass, and M_I is the mass of an I-sterile neutrino. The leptonic asymmetry at the temperature of the sphaleron freeze-out ($T \sim T_{EW}$) is related to the baryon asymmetry of the Universe. At temperatures $T < T_{EW}$, the leptonic asymmetry from decays of heavier sterile neutrinos cannot convert into the baryon asymmetry and is accumulated. As was shown in [4,9], the required amount of the leptonic asymmetry $\Delta = \Delta L/L = (n_L - n_{\bar{L}})/(n_L + n_{\bar{L}})$,

$$10^{-3} < \Delta < 2/11, \tag{1}$$

has to already exist in the Universe at the moment of the beginning of the production of DM particles (it takes place at a temperature around 0.1 GeV).

We consider here the leptonic asymmetry generation at $T \ll T_{EW}$, when the masses of two heavier sterile neutrinos are between m_π and 2 GeV. The motivation is following. The mass of a heavier sterile neutrino cannot be less than m_π (the constraint is coming from accelerator experiments combined with Big-Bang Nucleosynthesis (BBN) bounds [10, 11]), and there is no direct experimental searches for singlet fermions with mass more than 2 Gev [10].

Since the masses of active neutrinos in the νMSM are produced by the "see-saw" mechanism [12], some constraints on the parameters of the νMSM come from active neutrino parameters that can be found from the experiments on neutrino oscillations. Namely, these are the mass squared differences of active neutrinos and the mixing angles. Till recently, the mixing angle θ_{13} was supposed to have a value close to zero. But the new observations indicate its essential difference from zero [13].

The aim of this work is to obtain constraints on the parameters of the νMSM from the required amount of the leptonic asymmetry and cosmology conditions. We want also to investigate the influence of a non-zero mixing angle θ_{13} on also space of allowed parameters of the νMSM. We do it following [14] with the use of a simple model: we ignore the background effects and do computations at zero temperature.

The paper is organized as follows. In Section 2, we present the Lagrangian of the νMSM, make its convenient parametrization, and present the Yukawa couplings in terms of parameters of the mass matrix of active neutrinos. In Section 3, we derive the expression for the leptonic asymmetry. The limitations on the νMSM parameters are imposed in Section 4. Section 5 is devoted to the analysis and conclusions.

2. Basic Formalism of the νMSM

In the νMSM [5, 6], the following terms are added to the Lagrangian of the SM (without taking the kinetic terms into account):

$$\mathcal{L}^{ad} = -F_{\alpha I}\bar{L}_\alpha \tilde{\Phi}\nu_{IR} - \frac{M_{IJ}}{2}\bar{\nu}^c_{IR}\nu_{JR} + \text{h.c.} \tag{2}$$

Here, the index $\alpha = e, \mu, \tau$ corresponds to the active neutrino flavors, indices I, J run from 1 to 3, L_α is for the lepton doublet of left-handed particles, ν_{IR} is for the field functions of sterile right-handed neutrinos, the superscript "c" means the charge conjugation, $F_{\alpha I}$ is for the new (neutrino) matrix of the Yukawa constants, M_{IJ} is for the Majorana mass matrix of the right-handed neutrinos, Φ is for the field of the Higgs doublet, $\tilde{\Phi} = i\sigma_2\Phi^*$.

After the spontaneous symmetry breaking, the field of a Higgs doublet in the unitary gauge is

$$\Phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix},$$

where h is the neutral Higgs field, and the parameter v determines the minimum of the Higgs field potential ($v \cong 247$ GeV). In this case, Lagrangian (2) acquires the Dirac–Majorana neutrino mass terms:

$$\mathcal{L}^{\rm DM} = -\frac{v}{\sqrt{2}}F_{\alpha I}\bar{\nu}_\alpha\nu_{IR} - \frac{M_{IJ}}{2}\bar{\nu}^c_{IR}\nu_{JR} + \text{h.c.}, \tag{3}$$

or, in the conventional form [15],

$$\mathcal{L}^{\rm DM} = -\left(\overline{(N_L)^c}\,\frac{M^{\rm DM}}{2}\,N_L + \text{h.c.}\right), \tag{4}$$

where

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix};\; N_L^c = \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix};\; M^{DM} = \begin{pmatrix} M_L & {M_D}^T \\ M_D & M_R \end{pmatrix} \tag{5}$$

and

$$M_L = 0, \quad M_D = F^+\frac{v}{\sqrt{2}}, \quad M_R = M^*, \tag{6}$$

where M, F are square matrices of the third order with elements $F_{\alpha I}$ and M_{IJ}.

In the zero approximation, the νMSM Lagrangian is assumed to be invariant under $U(1)_e \times U(1)_\mu \times$ $\times U(1)_\tau$ transformations, that provides the preservation of the e, μ, τ lepton numbers separately. It is also assumed that two heavier sterile neutrinos interact with the active neutrinos, but the third (lightest) sterile neutrino does not interact [2]. This assumption can be realized by the following matrix $M^{\rm DM}$ [16]:

$$M_R^{(0)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}, \quad M_D^{(0)+} = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & h_{12} & 0 \\ 0 & h_{22} & 0 \\ 0 & h_{32} & 0 \end{pmatrix},$$
$$M_L^{(0)} = 0. \tag{7}$$

In this approximation, we have two massive sterile neutrinos with equal mass M, the third neutrino is massless, and all active neutrinos have zero mass. It contradicts the observable data [2,3]. To adjust it, the next small terms are added to the matrix $M^{\rm DM}$ [16]:

$$M_R^{(1)} = \Delta M = \begin{pmatrix} m_{11}e^{-i\alpha} & m_{12} & m_{13} \\ m_{12} & m_{22}e^{-i\beta} & 0 \\ m_{13} & 0 & m_{33}e^{-i\gamma} \end{pmatrix},$$

$$M_D^{(1)+} = \frac{v}{\sqrt{2}} \begin{pmatrix} h_{11} & 0 & h_{13} \\ h_{21} & 0 & h_{23} \\ h_{31} & 0 & h_{33} \end{pmatrix}, \quad M_L^{(1)} = 0. \tag{8}$$

This correction violates the $U(1)_e \times U(1)_\mu \times U(1)_\tau$ symmetry, leads to the appearance of the mass of the third sterile neutrino, and takes off the mass degeneracy for two heavier sterile neutrinos. It also leads to the appearance of the extra small masses of the active neutrinos and nonzero mixing angles among them.

In terms of the introduced corrections, Lagrangian (2) takes the form

$$\mathcal{L}^{ad} = -h_{\alpha I}\bar{L}_\alpha \tilde{N}_I \tilde{\Phi} - M\bar{\tilde{N}}_2^c \tilde{N}_3 - \frac{\Delta M_{IJ}}{2}\bar{\tilde{N}}_I^c \tilde{N}_J + \text{h.c.}, \tag{9}$$

where $\tilde{N}_I$ are right-handed neutrinos in the gauge basis.

In order to find the masses of the active neutrino, one has to make diagonalization of the matrix $M^{\rm DM}$. The diagonalization is realized in two steps. First, $M^{\rm DM}$ matrix is reduced to the block-diagonal form via the unitary transformation [17] in the "see-saw" approach:

$$M_{\rm block} = W^T M^{\rm DM} W =$$

$$= \begin{pmatrix} -(M_D)^T (M_R)^{-1} M_D & 0 \\ 0 & M_R \end{pmatrix} = \begin{pmatrix} M_{\rm light} & 0 \\ 0 & M_{\rm heavy} \end{pmatrix}, \tag{10}$$

where

$$W = \begin{pmatrix} 1 - \frac{1}{2}\varepsilon^+\varepsilon & \varepsilon^+ \\ -\varepsilon & 1 - \frac{1}{2}\varepsilon\varepsilon^+ \end{pmatrix}, \quad \varepsilon = M_R^{-1} M_D \ll 1 \tag{11}$$

and $M_{\rm light} = -(M_D)^T (M_R)^{-1} M_D$ and $M_{\rm heavy} = M_R$ are the mass matrices of the active and sterile neutrinos, respectively. Each block of the matrix $M^{\rm DM}$ can be diagonalized now independently by the matrix

$$U = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix}. \tag{12}$$

The mass matrix of the active and sterile neutrinos is diagonalized by the unitary transformation $U_{1(2)}$:

$$U_1^T M_{\rm light} U_1 = {\rm diag}(m_1, m_2, m_3),$$

$$U_2^T M_{\rm heavy} U_2 = {\rm diag}(M_1, M_2, M_3). \tag{13}$$

There is a standard parametrization [2] for $U_{1(2)}$:

$$U_{1(2)} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{14}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, θ_{12}, θ_{13}, and θ_{23} are the three mixing angles; δ is the Dirac phase, and α_1 and α_2 are the Majorana phases. The angles θ_{ij} can be in the region $0 \le \theta_{ij} \le \pi/2$, and the phases $\delta, \alpha_1, \alpha_2$ vary from 0 to 2π. Each of the matrices U_1 and U_2 contains its own independent angles and phases.

[2] Therefore, the lightest sterile neutrino in the νMSM is a candidate for the DM particle.

Then the elements of $M_{\rm light}$ can be defined by masses and elements of the mixing matrix U of the active neutrinos:

$$[M_{\rm light}]_{\alpha\beta} = m_1 U^*_{\alpha 1} U^*_{\beta 1} + m_2 U^*_{\alpha 2} U^*_{\beta 2} + m_3 U^*_{\alpha 3} U^*_{\beta 3}. \quad (15)$$

The data that come from the neutrino oscillation experiments are presented in Table 1.

On the other hand, the "see-saw" formula (in the approximation where the elements of the first column of the Yukawa matrix are neglected and $M \gg \gg m_{ij}$) immediately implies that the mass of the lightest sterile neutrino is zero, and the mass matrix of the active neutrinos has the form [16]

$$[M_{\rm light}]_{\alpha\beta} = -\frac{v^2}{2M}(h_{\alpha 2} h_{\beta 3} + h_{\alpha 3} h_{\beta 2}). \quad (16)$$

Its eigenvalues are

$$m_a = 0, \quad m_{\binom{b}{c}} = \frac{v^2 (F_2 F_3 \mp |h^+ h|_{23})}{2M}, \quad (17)$$

where $F_I^2 = (h^+ h)_{II}$, m_a is the mass of the lightest active neutrino, and m_c is the mass of the heaviest active neutrino. The sum over the neutrino masses is given by

$$\frac{v^2 F_2 F_3}{M} = \sum_{i=1}^{3} m_i. \quad (18)$$

System (16) has the infinite number of solutions. Indeed, the replacement $h_{\alpha 2} \to z h_{\alpha 2}$, $h_{\alpha 3} \to h_{\alpha 3}/z$ (z is an any complex number) does not change the system. Then one can define the real quantity ε

$$\varepsilon = F_3/F_2, \quad \varepsilon = |z| \quad (19)$$

as an independent parameter of the model.

Table 1. **Experimental constraints on the parameters of active neutrinos [3]**

Central value	Confidence interval
$\Delta m^2_{21} = (7.58 \pm 0.21) \times 10^{-5}$ eV^2	(7.1–8.1) $\times 10^{-5}$ eV^2
$\lvert\Delta m^2_{23}\rvert = (2.40 \pm 0.15) \times 10^{-3}$ eV^2	(2.1–2.8) $\times 10^{-3}$ eV^2
$\tan^2\theta_{12} = 0.484 \pm 0.048$	$31^0 < \theta_{12} < 39^0$
$\sin^2 2\theta_{23} = 1.02 \pm 0.04$	$37^0 < \theta_{23} < 53^0$
$^*\sin^2 2\theta_{13} = 0.11$ ($\theta_{13} = 10^0$)	

*Results of T2K Collaboration [13]: $0.03 < \sin^2 2\theta_{13} < 0.28$ in the case of the normal hierarchy and $0.04 < \sin^2 2\theta_{13} < 0.34$ in the case of the inverted hierarchy.

As was shown in [18], system (16) has good solutions for the ratios of elements of the second column of the Yukawa matrix:

$$\begin{cases} A_{12} = \dfrac{M_{12}}{M_{22}}\left(1 \pm \sqrt{1 - \dfrac{M_{11} M_{22}}{{M_{12}}^2}}\right), \\ A_{13} = \dfrac{M_{13}}{M_{33}}\left(1 \pm \sqrt{1 - \dfrac{M_{11} M_{33}}{{M_{13}}^2}}\right), \\ A_{23} = \dfrac{M_{23}}{M_{33}}\left(1 \pm \sqrt{1 - \dfrac{M_{22} M_{33}}{{M_{23}}^2}}\right), \end{cases} \quad (20)$$

where $A_{12} = h_{12}/h_{22}$, $A_{13} = h_{12}/h_{32}$, $A_{23} = h_{22}/h_{32}$, and M_{IJ} are elements of the M_{light} matrix. The ratios of the third-column elements of the Yukawa matrix are expressed through the A_{ij} elements:

$$\frac{h_{23}}{h_{13}} = A_{12}\frac{M_{22}}{M_{11}}; \quad \frac{h_{33}}{h_{13}} = A_{13}\frac{M_{33}}{M_{11}}. \quad (21)$$

Though, formally, there are eight different choices for solutions (20), only four ones are independent. For example, if we fix the sign before the square roots in the expressions for A_{12} and A_{13}, then A_{23} is unambiguously determined by the relation

$$A_{23} = A_{13}/A_{12}. \quad (22)$$

Solutions (20) allow one to find the ratios of elements of the Yukawa matrix [18]:

$$\frac{(h_{12}; h_{22}; h_{32})}{F_2} = = \frac{e^{i\arg(h_{12})}}{\sqrt{1 + |A_{12}|^{-2} + |A_{13}|^{-2}}}\left(1; A_{12}^{-1}; A_{13}^{-1}\right) \quad (23)$$

$$\frac{(h_{13}; h_{23}; h_{33})}{F_3} = \frac{e^{i\arg(h_{13})}}{\sqrt{1 + |A_{12}\frac{M_{22}}{M_{11}}|^2 + |A_{13}\frac{M_{33}}{M_{11}}|^2}} \times \times \left(1; A_{12}\frac{M_{22}}{M_{11}}; A_{13}\frac{M_{33}}{M_{11}}\right), \quad (24)$$

where the phases of h_{12} and h_{13} are connected by the condition

$$\arg(h_{12}) + \arg(h_{13}) = \arg(M_{11}). \quad (25)$$

This is the exact solution of (16) that definitely expresses the ratio of elements of the Yukawa matrix via parameters of the active neutrino mass matrix. For the fixed values of active neutrino parameters, there are only two choices for the placing of the signs in the expressions for A_{12}, A_{13}, and A_{23} (20) which are not inconsistent with condition (22). These two variants are distinguished from each other by the simultaneous replacement of the sign in front of the square roots in the expressions for A_{12}, A_{13}, and A_{23}. It can be shown that such replacement of the signs leads to interchanging and conjugating the ratios of elements of the second and third columns of the Yukawa matrix, notably $h_{22}/h_{12} \leftrightarrow h^*_{23}/h^*_{13}$, $h_{32}/h_{12} \leftrightarrow h^*_{33}/h^*_{13}$ [18].

As was announced in Introduction, only the two heavier sterile neutrinos take part in the production of the leptonic asymmetry. Therefore, we will exclude the lightest sterile neutrino from consideration, so hereinafter the indices I, J take the value 2 or 3 referring to the two heavy sterile neutrinos. In this case, there are 11 additional parameters in the νMSM as compared with SM. Seven of them will be identified with elements of the active neutrino mass matrix (m_2, m_3, θ_{12}, θ_{13}, θ_{23}, δ, and α_2). The remaining four parameters will be defined as follows: the average mass of two heavier sterile neutrinos $M = \frac{M_2+M_3}{2}$, their mass splitting $\Delta M = \frac{M_3-M_2}{2}$, the parameter ε, and the phase $\xi = \arg(h_{12})$.

Thus, we can parametrize Lagrangian (9) in the following way:

$$\mathcal{L}=\left(\frac{M\sum m_{\nu_i}}{v^2}\right)^{1/2}\left[\frac{1}{F_2\sqrt{\varepsilon}}h_{\alpha 2}\bar{L}_\alpha\tilde{N}_\alpha+\frac{\sqrt{\varepsilon}}{F_3}h_{\alpha 3}\bar{L}_\alpha\tilde{N}_3\right]\tilde{\Phi}- -M\bar{\tilde{N}}^c_2\tilde{N}_3-\frac{1}{2}\Delta M\left(\bar{\tilde{N}}^c_2\tilde{N}_2+\bar{\tilde{N}}^c_3\tilde{N}_3\right)+\text{h.c.}, \tag{26}$$

where $a_{\alpha I} = h_{\alpha I}/F_I$ are defined by Eqs. (23) and (24).

Lagrangian (2) can be written in another basis, namely if the mass matrix of sterile right-handed neutrinos is diagonal. In this case, the Lagrangian reads

$$\mathcal{L}^{ad} = -g_{\alpha I}\bar{L}_\alpha N'_I\tilde{\Phi} - \frac{M_I}{2}\bar{N}'^c_I N'_I + \text{h.c.}, \tag{27}$$

where N'_I are right-handed neutrinos, and $g_{\alpha I}$ are elements of the Yukawa matrix in this basis.

The transition from the presentation of Lagrangian (2) in the gauge and mass bases can be made with a unitary transformation that transfers the mass matrix of a right-handed neutrino to the diagonal form [14, 16]:

$$V^*\begin{pmatrix}\Delta M & M\\ M & \Delta M\end{pmatrix}V=\begin{pmatrix}M-\Delta M & 0\\ 0 & M+\Delta M\end{pmatrix};$$

$$V=\frac{1}{\sqrt{2}}\begin{pmatrix}-i & i\\ 1 & 1\end{pmatrix}. \tag{28}$$

So, the transition can be made by

$$\tilde{N}_I = V_{IJ}N'_J, \quad g_{\alpha I} = h_{\alpha J}V_{JI}. \tag{29}$$

With help of these relations, we can express Lagrangian (26) in terms of the right-handed neutrino functions of Lagrangian (27):

$$\mathcal{L}^{ad}=-\left(\frac{M\sum m_{\nu_i}}{2v^2}\right)^{1/2}\left[\left(\frac{ia_{\alpha 2}}{\sqrt{\varepsilon}}-i\sqrt{\varepsilon}a_{\alpha 3}\right)\bar{L}_\alpha N'_2+ +\left(\frac{a_{\alpha 2}}{\sqrt{\varepsilon}}+\sqrt{\varepsilon}a_{\alpha 3}\right)\bar{L}_\alpha N'_3\right]\tilde{\Phi}- -\frac{1}{2}(M-\Delta M)\bar{N'}^c_2N'_2-\frac{1}{2}(M+\Delta M)\bar{N'}^c_3N'_3. \tag{30}$$

By comparing (30) and (27), one can express the Yukawa couplings in different forms

$$g_{\alpha 2}=\left(\frac{M\sum_i m_{\nu_i}}{2v^2}\right)^{1/2}\left(\frac{ia_{\alpha 2}}{\sqrt{\varepsilon}}-i\sqrt{\varepsilon}a_{\alpha 3}\right), \tag{31}$$

$$g_{\alpha 3}=\left(\frac{M\sum_i m_{\nu_i}}{2v^2}\right)^{1/2}\left(\frac{a_{\alpha 2}}{\sqrt{\varepsilon}}+\sqrt{\varepsilon}a_{\alpha 3}\right). \tag{32}$$

The mass eigenstates of neutrinos for the Lagrangian with the mass matrix M^{DM} (5) can be easily expressed through the neutrino states of Lagrangian (27), particularly:

$$N^c=\left(1-\frac{1}{2}\varepsilon\varepsilon^+\right)N'^c+\varepsilon\nu_L\simeq N'^c+\varepsilon\nu_L. \tag{33}$$

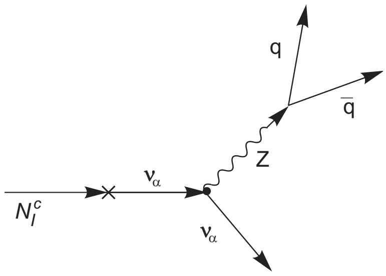
Here, N are the mass eigenstates of the right-handed neutrinos in which they are produced and decay, ν_L are the active neutrinos of the SM in the flavor basis, and

$$\varepsilon_{\alpha I}\equiv\Theta_{\alpha I}=\frac{v}{\sqrt{2}}\frac{g_{\alpha I}}{M_I} \tag{34}$$

is the mixing angle ($\varepsilon_{\alpha I} \ll 1$).

3. The Computation of the Leptonic Asymmetry

As was indicated in Section 1, the leptonic asymmetry in the νMSM is generated due to decays of the heavier sterile neutrinos into SM particles. At temperatures $T \ll T_{EW}$, the interaction of sterile neutrinos with SM particles via the neutral Higgs field can



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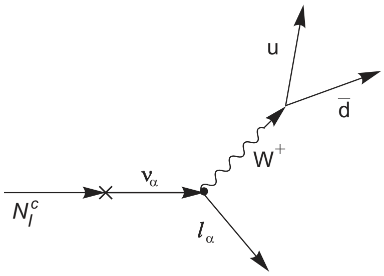

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Fig. 1. Decay of a sterile neutrino via a Z-boson (a) and a W^+-boson (b) (the cross on the line of a sterile neutrino means an oscillation of a sterile neutrino to an active one)

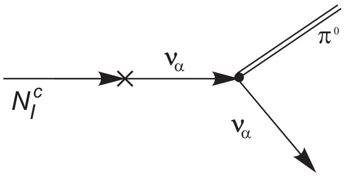

Fig. 2. Effective low-energy decay of a sterile neutrino into a π^0 meson and an active neutrino

be neglected. The only possible way for the interaction of a sterile neutrino with matter is through the mixing with active neutrinos (33).

For a sterile neutrino with mass $m_\pi < M_I <$ < 2 GeV, the channels for the decay into a two-body final state are as follows:

$$N_I \to \pi^0\nu_\alpha, \pi^+e^-_\alpha, \pi^-e^+_\alpha, K^+e^-_\alpha, K^-e^+_\alpha, \eta\nu_\alpha,$$

$$\eta'\nu_\alpha, \rho^0\nu_\alpha, \rho^+e^-_\alpha, \rho^-e^+_\alpha. \tag{35}$$

The decay channel $N_{2,3} \to N_1 + ...$ is strongly suppressed because of the small Yukawa coupling constants of N_1. The decay of a sterile neutrino into the K^0 state is forbidden, because the composition of K^0 $(d\bar{s})$ cannot be obtained through the decay of a Z-boson.

The three-body final state, as well as a many-hadron final state, can be safely neglected [10]. These last decay channels contribute less than 10% for $M_I <$ < 2 GeV. For $m_\pi < M_I < 2$ GeV, the decays into a D-meson can also be neglected because its mass is not much smaller than 2 GeV.

Let us consider the decay of a sterile neutrino in the νMSM. A sterile neutrino oscillates into an active neutrino that decays into a Z-boson and an active neutrino (or a $W^\pm$-boson and a charged lepton) in accordance with the SM. The Z-boson (or $W^\pm$-boson) decays hereafter into a quark-antiquark pair, see Fig. 1. Since the kinetic energy of these quarks are small enough, the quark pair will form a bound state. Since $M_I < 2$ GeV $\ll \ll M_{Z(W)}$, we can use the low-energy Fermi theory, shrink the heavy boson propagator into an effective vertex, and use a meson for the final state (see Fig. 2).

The decay of a sterile neutrino into a charged lepton and a charged meson through a $W^\pm$-boson is described by the charged current interaction

$$\mathcal{L}_C = \frac{G_{\rm F}}{\sqrt{2}} \left(j_\nu^{CC}\right)^+ j^{\nu\, CC}, \tag{36}$$

where $j_\nu^{CC} = j_\nu^{l\,CC} + j_\nu^{h\,CC}$ is the sum of charged lepton and hadron currents:

$$j_\nu^{l\,CC} = \sum_\alpha \bar{e}_\alpha\gamma_\nu(1-\gamma^5)\nu_\alpha,$$

$$j_\nu^{h\,CC} = \sum_{n,m} V^*_{n,m}\bar{d}_m\gamma_\nu(1-\gamma^5)u_n. \tag{37}$$

The indices m, n run over the quark generation, $\alpha = e, \mu, \tau$ and V is Cabbibo–Kobayashi–Maskawa (CKM) matrix. Similarly, the decay of a sterile neutrino into an active neutrino and a neutral meson through the Z-boson is described by the neutral current interaction

$$\mathcal{L}_N = \sqrt{2}G_{\rm F} \left(j_\nu^{NC}\right)^+ j^{\nu\, NC}, \tag{38}$$

where $j_\nu^{NC} = j_\nu^{l\,NC} + j_\nu^{h\,NC}$ is the sum of active neutrino and hadron currents:

$$j_\nu^{l\,NC} = \sum_\alpha \bar{\nu}_\alpha\gamma_\nu\frac{1-\gamma^5}{2}\nu_\alpha,$$

$$j_\nu^{h\,NC} = \sum_f \bar{f}\gamma_\nu\left(t_3^f(1-\gamma^5) - 2q_f\sin^2\theta_W\right)f, \tag{39}$$

where the sum over f means the sum over all quarks, t_3^f is the weak isospin of a quark, q_f is the electric charge of a quark in proton charge units, notably $t_3^f = 1/2$, $q_f = +2/3$ for u, c, t and $t_3^f = -1/2$, $q_f = -1/3$ for d, s, b quarks.

The matrix element corresponding to the Feynman diagram of the sterile neutrino decay (see, e.g., Figs. 1 and 2) can be obtained from the interactive effective Lagrangian [14]. For example, the effective Lagrangian of the decay of an I sterile neutrino into the $\pi^\pm, \pi^0$ final states is

$$\mathcal{L}^{\pi}_{\rm eff} = \frac{G_{\rm F}}{2} M_I f_\pi \Theta_{\alpha I} \bar{\nu}_\alpha (1+\gamma_5) N_I \pi^0 + \Bigg[\frac{G_{\rm F}}{\sqrt{2}} M_I f_\pi V_{ud} \times \times \Theta_{\alpha I} \bar{e}_\alpha \left((1+\gamma_5) - \frac{m_\alpha}{M_I}(1-\gamma_5)\right) N_I \pi^- + \text{h.c.}\Bigg]. \quad (40)$$

Here, $G_{\rm F}$ is the Fermi coupling constant, M_I is the mass of an I-sterile neutrino, m_α is the mass of a charged lepton of the α generation, and f_π is the π-meson decay constant that is defined as

$$\langle \pi^+ | \bar{u}(1+\gamma^5)\gamma_\nu d | 0\rangle = -f_\pi (p_\pi)_\mu, \quad (41)$$

where p_π is the pion 4-momentum.

The leptonic asymmetry ϵ can be defined as

$$\epsilon = \frac{\Gamma_{N\to l} - \Gamma_{N\to \bar{l}}}{\Gamma_{N\to l} + \Gamma_{N\to \bar{l}}}, \quad (42)$$

where $\Gamma_{N\to l}$ is the total decay rate of sterile neutrinos into leptons, and $\Gamma_{N\to \bar{l}}$ is the total decay rate of sterile neutrinos into antileptons.

At the tree level, the decay rates of sterile neutrinos into leptons and antileptons are equal. Therefore, we must compute the one-loop diagrams (see Fig. 3). In the case of nearly degenerated sterile neutrinos, the contribution from the diagrams presented in Fig. 3, *b*) can be neglected as compared with the diagrams presented in Fig. 3, *a*). Indeed, the propagator of a sterile neutrino in diagrams *a)* is proportional to $1/\Delta M$ in the center-of-mass frame. The leading order contribution to the leptonic asymmetry comes from the interference between one-loop diagrams and tree-level diagrams [19]. In this case, $\Gamma_{N\to l} - \Gamma_{N\to \bar{l}} \sim \Theta^4$ and $\Gamma_{N\to l} + \Gamma_{N\to \bar{l}} \sim \Theta^2$, and the leptonic asymmetry is suppressed.

In our case where the mass splitting between two heavier sterile neutrinos is very small, and it is of the

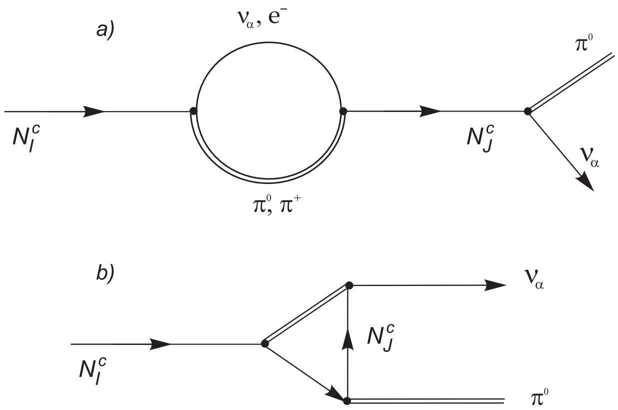

Fig. 3. Example of one-loop diagrams of the decay $N_I \to \nu_\alpha \pi^0$

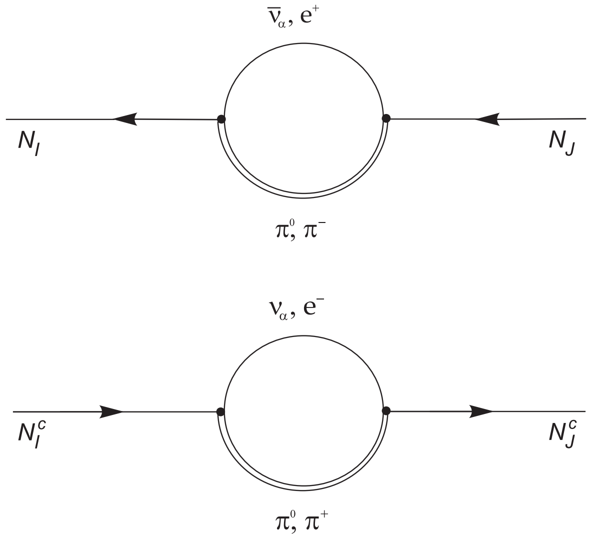

Fig. 4. Contributions to the effective Hamiltonian

same order as their decay rate (we will see it below), the oscillations between N_I and N_J are of importance (see Fig. 4). So, the corresponding mass eigenfunctions are no longer the N_I states, but are their mixture, namely ψ_I [14, 20]. It is these physical eigenstates that evolve in time with a definite frequency. The subsequent decay of these fields will produce the desired lepton asymmetry

$$\Delta = \frac{\Gamma_{\psi\to l} - \Gamma_{\psi\to \bar{l}}}{\Gamma_{\psi\to l} + \Gamma_{\psi\to \bar{l}}}, \quad (43)$$

where $\Gamma_{\psi\to l}$ and $\Gamma_{\psi\to \bar{l}}$ are the total decay rates of sterile neutrinos with eigenfunctions ψ_I into leptons and antileptons, respectively. In this case, the leading-order contribution to the leptonic asymmetry comes from tree-level diagrams.

In the general case, the correct description of the processes can be made in the frame of the density matrix formalism (see, e.g., [5]). We will follow a simpler way by considering a non-Hermitian Hamiltonian. The effective Hamiltonian in the basis of $\{N_2, N_3\}$ is $H = H_0 + \Delta H$, where H_0 is the diagonal Hamiltonian of equal-mass particles

$$H_0 = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}. \tag{44}$$

The corrections to this Hamiltonian are given by the one-loop diagrams (see Fig. 4):

$$\Delta H = \begin{pmatrix} -\Delta M - \frac{i}{2}\Gamma_2 & -\frac{i}{2}\Gamma_{23} \\ -\frac{i}{2}\Gamma_{23} & \Delta M - \frac{i}{2}\Gamma_3 \end{pmatrix}. \tag{45}$$

The dispersive part of these diagrams can be absorbed in the mass renormalization of the fields [20], and this causes the appearance of the mass splitting ΔM. The absorptive part of the diagrams will define the total decay rates of a sterile neutrino Γ_I and the rate of oscillations between sterile neutrinos Γ_{23}.

The total decay rates of I-sterile neutrinos into charged mesons and leptons of the α-generation are

$$\Gamma_I^{\alpha\pi^\pm} = \Gamma(N_I \to \pi^\pm + l_\alpha^\mp) =$$
$$= \frac{G_F^2 f_\pi^2 |V_{ud}|^2 M^3}{8\pi} |\Theta_{\alpha I}|^2 S(M, m_\alpha, m_\pi) \times$$
$$\times \left[\left(1 - \frac{m_\alpha^2}{M^2}\right)^2 - \frac{m_\pi^2}{M^2}\left(1 + \frac{m_\alpha^2}{M^2}\right)\right], \tag{46}$$

$$\Gamma_I^{\alpha K} = \Gamma(N_I \to K^\pm + l_\alpha^\mp) =$$
$$= \frac{G_F^2 f_K^2 |V_{us}|^2 M^3}{8\pi} |\Theta_{\alpha I}|^2 S(M, m_\alpha, m_K) \times$$
$$\times \left[\left(1 - \frac{m_\alpha^2}{M^2}\right)^2 - \frac{m_K^2}{M^2}\left(1 + \frac{m_\alpha^2}{M^2}\right)\right], \tag{47}$$

$$\Gamma_I^{\alpha\rho^\pm} = \Gamma(N_I \to \rho^\pm + l_\alpha^\mp) =$$
$$= \frac{G_F^2 g_\rho^2 |V_{ud}|^2 M^3}{4\pi m_\rho^2} |\Theta_{\alpha I}|^2 S(M, m_\alpha, m_\rho) \times$$
$$\times \left[\left(1 - \frac{m_\alpha^2}{M^2}\right)^2 + \frac{m_\rho^2}{M^2}\left(1 + \frac{m_\alpha^2 - 2m_\rho^2}{M^2}\right)\right], \tag{48}$$

where

$$S(M_I, m_\alpha, m) =$$
$$= \sqrt{\left(1 - \frac{(m - m_\alpha)^2}{M_I^2}\right)\left(1 - \frac{(m + m_\alpha)^2}{M_I^2}\right)}, \tag{49}$$

and the values of decay constants and elements of the CKM matrix are given in [2]: $f_\pi = 0.131$ GeV, $f_K = 0.16$ GeV, $g_\rho = 0.102$ GeV^2, $|V_{ud}| = 0.97$, $|V_{us}| = 0.23$.

The total decay rates of I-sterile neutrinos into neutral mesons and active neutrinos are

$$\Gamma_I^{\alpha\pi^0} = \Gamma(N_I \to \pi^0 + \nu_\alpha) =$$
$$\frac{G_F^2 f_\pi^2 M^3}{16\pi} |\Theta_{\alpha I}|^2 \left(1 - \frac{m_\pi^2}{M^2}\right)^2, \tag{50}$$

$$\Gamma_I^{\alpha\rho^0} = \Gamma(N_I \to \rho_0 + \nu_\alpha) =$$
$$= \frac{G_F^2 g_\rho^2 M^3}{8\pi m_\rho^2} |\Theta_{\alpha I}|^2 \left(1 + 2\frac{m_\rho^2}{M^2}\right)\left(1 - \frac{m_\rho^2}{M^2}\right)^2, \tag{51}$$

$$\Gamma_I^{\alpha\eta} = \Gamma(N_I \to \eta + \nu_\alpha) =$$
$$= \frac{G_F^2 f_\eta^2 M^3}{16\pi} |\Theta_{\alpha I}|^2 \left(1 - \frac{m_\eta^2}{M^2}\right)^2, \tag{52}$$

$$\Gamma_I^{\alpha\eta'} = \Gamma(N_I \to \eta' + \nu_\alpha) =$$
$$= \frac{G_F^2 f_{\eta'}^2 M^3}{16\pi} |\Theta_{\alpha I}|^2 \left(1 - \frac{m_{\eta'}^2}{M^2}\right)^2, \tag{53}$$

where $f_\eta = 0.156$ GeV, $f'_\eta = -0.058$ GeV [2].

As one can see, the decay rates into $\rho^\pm, \rho^0$ mesons are slightly different, because they are vector mesons. The adduced decay rates (50) – (53) were obtained in [10, 21]. The total rate of the decay of a sterile neutrino into mesons and leptons is sum of the rates over all decay channels Λ (35) and over leptonic generations,

$$\Gamma_I = \sum_{\alpha,\Lambda} \Gamma_I^{\alpha\Lambda}\, \Theta(y_{\alpha\Lambda}), \tag{54}$$

where y_Λ is the difference of the I sterile neutrino mass and the total mass of all final particles of the decay channel Λ; and $\Theta(x)$ is the usual Heaviside function. The rate of oscillations between I and J sterile neutrinos (Γ_{IJ}) can be expressed through the decay rates

$$\Gamma_{IJ} = \sum_{\alpha,\Lambda} \frac{\mathrm{Re}(\Theta_{\alpha I}\Theta_{\alpha J}^*)}{|\Theta_{\alpha I}|^2}\, \Gamma_I^{\alpha\Lambda}\, \Theta(y_{\alpha\Lambda}). \tag{55}$$

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The eigenvalues and the corresponding eigenfunctions of the non-Hermitian Hamiltonian $H = H_0 + \Delta H$ are given by

$$\omega_2 = M - \frac{i}{4}(\Gamma_2+\Gamma_3) - \frac{1}{4}c, \quad \psi_2 = \frac{1}{\sqrt{N}}\begin{pmatrix} B \\ 1 \end{pmatrix}, \tag{56}$$

$$\omega_3 = M - \frac{i}{4}(\Gamma_2+\Gamma_3) + \frac{1}{4}c, \quad \psi_3 = \frac{1}{\sqrt{N}}\begin{pmatrix} 1 \\ -B \end{pmatrix}, \tag{57}$$

where N is a normalization factor, and

$$c = \sqrt{(4\Delta M - i(\Gamma_3 - \Gamma_2)^2 - 4(\Gamma_{23})^2},$$

$$B = (4i\Delta M + (\Gamma_3 - \Gamma_2) + ic)/(2\Gamma_{23}).$$

It should be noted that the sterile neutrinos are not initially in the states ψ_2 and ψ_3, but in the state N_2 and N_3. The fact is that the sterile neutrino were in a thermal equilibrium, before they propagated freely. The equilibrium was maintained by the weak interaction between the sterile neutrinos and particles in the background. The weak interaction eigenstates are N_2 and N_3; therefore, the sterile neutrinos are initially in the state N_2 or N_3. In general, the initial state of a sterile neutrino is a superposition of N_2 and N_3 states and can be described by the density matrix

$$\hat{\rho}_{\text{initial}} = \hat{\rho}(t=0) = \sum_{I=2,3} \alpha_I |N_I(0)\rangle\langle N_I(0)|, \tag{58}$$

where $\alpha_2 + \alpha_3 = 1$. It was shown in [14] that the leptonic asymmetry dependence on the parameter α_I can be neglected. We confirm this statement, and, hereafter, we will consider the symmetric initial state $\alpha_2 = \alpha_3 = 1/2$.

The time evolution of the density matrix can be obtain in a simple way. Since

$$|\psi_I\rangle = U_{IJ}|N_J\rangle, \tag{59}$$

where

$$U = \frac{1}{\sqrt{N}}\begin{pmatrix} B & 1 \\ 1 & -B \end{pmatrix}, \tag{60}$$

the time evolution of the $|N_I\rangle$ state is known:

$$|N_I(t)\rangle = U^{-1}_{IK} e^{-i\omega_K t}|\psi_K(0)\rangle =$$

$$= U^{-1}_{IK} e^{-i\omega_K t} U_{KJ}|N_J(0)\rangle = R_{IJ}|N_J(0)\rangle. \tag{61}$$

Thus,

$$\hat{\rho}(t) = \frac{1}{2}\sum_{I,J,K=2}^{3} R_{IK}(t)^* R_{IJ}(t)|N_J(0)\rangle\langle N_K(0)| =$$

$$= \frac{1}{2}\sum_{J,K=2}^{3} (R^\dagger R)_{KJ}|N_J(0)\rangle\langle N_K(0)|. \tag{62}$$

The average production rate of leptons is given by

$$\Gamma = \int_0^\infty dt \int d\Pi_2 \sum_l Tr\,[|l\rangle\langle l|\hat{\rho}(t)] = \frac{1}{2}\int_0^\infty dt \int d\Pi_2\, Tr \times$$

$$\times \left[\sum_{l,K,J} (R^\dagger R)_{KJ}\langle l|N_J(0)\rangle\langle N_K(0)|l\rangle\right] =$$

$$= \frac{1}{2}\int_0^\infty dt \int d\Pi_2 \sum_{l,J,K} (R^\dagger R)_{KJ} A_{Jl} A^*_{Kl}, \tag{63}$$

where the sum over l means the sum over all lepton generations and include charged leptons and active neutrinos, $\langle l|N_J(0)\rangle = A_{Jl}$ is the transition amplitude of the decay of an I sterile neutrino into a lepton at the tree level that includes all possible channels of the reaction, and $d\Pi_2$ is the differential two-body phase space

$$d\Pi_2 = \frac{d^3q}{(2\pi)^3 2E_q}\frac{d^3k}{(2\pi)^3 2E_k}(2\pi)^4\delta^4(p-q-k),$$

where p, q, k are the 4-momenta of initial and final particles in the decay.

Similarly, the production rate of antileptons is

$$\bar{\Gamma} = \frac{1}{2}\int_0^\infty dt \int d\Pi_2 \sum_{l,J,K} (R^\dagger R)_{KJ} A^*_{Jl} A_{KJ}. \tag{64}$$

The measure of the leptonic asymmetry is given by

$$\Delta = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} = \left[\int dt \int d\Pi_2 \mathrm{Im}((R^\dagger R)_{32})\mathrm{Im}(A^*_{2l}A_{3l})\right] /$$

$$/ \left[\int dt \int d\Pi_2((R^\dagger R)_{22}|A_2 l|^2 + (R^\dagger R)_{33}|A_3 l|^2 +\right.$$

$$\left. + 2\mathrm{Re}(A^*_{2l}A_{3l})\mathrm{Re}(R^\dagger R)_{23})\right]. \tag{65}$$

The integration over $d\Pi_2$ gives [14]

$$\Delta = \left[\int dt \mathrm{Im}((R^\dagger R)_{32}) \sum_\alpha \mathrm{Im}(\Theta^*_{\alpha 2}\Theta_{\alpha 3}) V_\alpha \right] /$$

$$/ \left[\int dt \sum_\alpha ((R^\dagger R)_{22} |\Theta_{\alpha 2}|^2 + (R^\dagger R)_{33} |\Theta_{\alpha 3}|^2 +$$

$$+ 2\mathrm{Re}(\Theta^*_{\alpha 2}\Theta_{\alpha 3}) \mathrm{Re}(R^\dagger R)_{23}) V_\alpha \right], \qquad (66)$$

where V_α is defined via the sum over all possible channels of the decay of a sterile neutrino into leptons of the generation α:

$$V_\alpha = \sum_\Lambda \frac{\Gamma_I^{\alpha\Lambda}}{|\Theta_{\alpha I}|^2} \Theta(y_{\alpha\Lambda}). \qquad (67)$$

4. Restrictions on the Parameters of the *νMSM*

As was pointed in Section 1, the leptonic asymmetry of the Universe has to be constrained by condition (1) at the moment of the beginning of the production of DM particles. It allows us to constrain the parameters of the *νMSM*. To do it, we can construct the leptonic asymmetry (66) as a function of only three parameters of *νMSM*: M, ΔM, and ε.

We do it in the following way. The leptonic asymmetry function (66) is maximized over the phases δ, α_2, and ξ (and α_1 in case of the inverted hierarchy) and is taken at the central values of active neutrino mass matrix parameters[3] (see Table 1). This function contains the dependence on the ratios of the Yukawa matrix elements with a mixing angle $\Theta_{\alpha I}$ (34) that can be expressed through solutions (20) with two possible choices of the sign consistent with condition (22). So far as the relation for leptonic asymmetry (66) has no symmetry for interchanging and conjugating the ratios of elements of the second and third columns of the Yukawa matrix, we have to consider two variants of the solutions. For fixed values of mixing angles and phases, we will designate the allowed solution of (20) with two or more signs

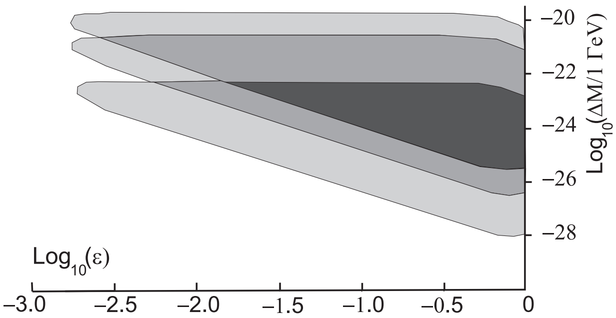

Fig. 5. Grey areas are the regions of parameters, where $\Delta > 10^{-3}$ in the case of the normal hierarchy. The areas correspond to $M = 0.3$ GeV (bottom), $M = 1$ GeV (middle), $M = 2$ GeV (top)

(+) as a solution of A type. *Vice versa*, the solution with two or more signs (−) will be designated as a solution of B type. It should be noted that our results (23), (24) for B type of a solution coincide with results of [8], where the ratios of elements were obtained in the particular case $\theta_{13} \to 0$, $\theta_{23} \to \pi/4$. We separately consider the cases where $\theta_{13} = 0$ and $\theta_{13} = 10°$.

Thereby, we construct the allowed regions ($\Delta > 10^{-3}$) in the plane of parameters ΔM and ε at fixed values of M.

In the case of the normal hierarchy, the difference between the case of $\theta_{13} = 0$ or $\theta_{13} = 10°$ and the case of a solution of A or B types is not essential, so we illustrate the allowed regions in Fig. 5. In the case of the inverted hierarchy, the difference between the case of a solution of A or B types is not essential, but the cases of $\theta_{13} = 0$ and $\theta_{13} = 10^{\circ}$ are substantially different. So, we illustrate the allowed regions in two parts of Fig. 6.

It should be noted that we have investigated the forms of allowed regions not only for the central value of θ_{13} angle, but for a range given by data of [13]. We conclude that, in case of the normal hierarchy, the regions are almost not sensitive to the value of θ_{13} in the range $0 < < \theta_{13} < 16°$. In case of the inverted hierarchy, it is true for the regions b and $\theta_{13} < 18°$ in Fig. 6. But, for $\theta_{13} = 0$, the allowed regions are appreciably different.

We also illustrate the regions, where the maximum of Δ can be more than 2/11 in Fig. 7 (white inner figures) for the both hierarchies. We do it only for the mass $M = 1$ GeV, because these regions are at

[3] In case of the normal hierarchy, we have $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{21}} = 0.009\,\mathrm{eV}$, $m_3 = \sqrt{|\Delta m^2_{23}| + \Delta m^2_{21}} = 0.05\,\mathrm{eV}$. In case of the inverted hierarchy, we have $m_1 = \sqrt{|\Delta m^2_{23}| - \Delta m^2_{21}} = 0.048\,\mathrm{eV}$, $m_2 = \sqrt{|\Delta m^2_{23}|} = 0.049\,\mathrm{eV}$, $m_3 = 0$

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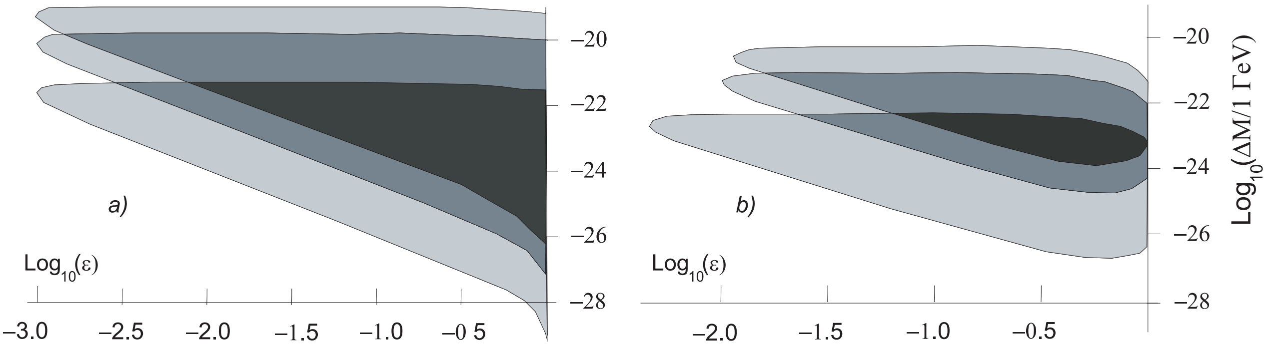

Fig. 6. Grey areas are the regions of parameters, where $\Delta > 10^{-3}$ in the case of the inverted hierarchy. The areas correspond to $M = 0.3$ GeV (bottom), $M = 1$ GeV (middle), $M = 2$ GeV (top). Parts *a*) and *b*) represent the case of $\theta_{13} = 0°$ and $10°$, respectively

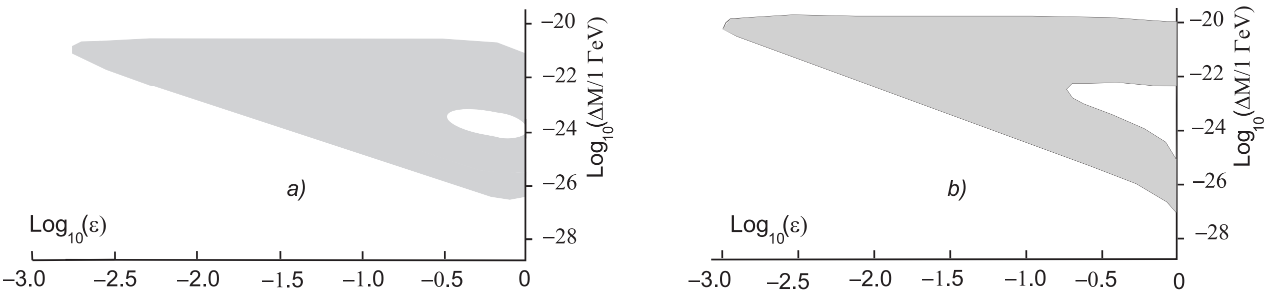

Fig. 7. Grey areas represent the regions of parameters, where $10^{-3} < \Delta < 2/11$ for $M = 1$ GeV in case of the normal (*a*) and inverted hierarchies (*b*)

small values of ε, and it will not intersect with other subsequent constrains. Moreover, at some values of phases, the leptonic asymmetry in this region can be less than 2/11, and we cannot exclude this region ultimately.

By examples, we present the possible values of I sterile neutrino decay rate Γ_I (54) and the rate of oscillations between I and J sterile neutrinos Γ_{IJ} (55) for $M = 1$ GeV and $\theta_{13} = 10°$ in Fig. 8. As one can see, the values of Γ_I and Γ_{IJ} are really of the same order as ΔM. This confirms the previous assumption about the necessity to consider the oscillations between sterile neutrinos.

In order to create the leptonic asymmetry, the sterile neutrinos should be out of a thermal equilibrium. This means that

$$\Gamma_2 \lesssim H, \tag{68}$$

where H is the Hubble parameter that determines the expansion rate of the Universe. In the radiation-dominated epoch, the Hubble parameter is given by

$$H = \frac{T^2}{M^*_{\rm PL}}, \tag{69}$$

where $M^*_{\rm PL} = \sqrt{\frac{90}{8\pi^3 g^*(T)}} M_{\rm PL}$, $M_{\rm PL} = 1.22\times10^{19}$ GeV is the Planck mass, and $g^*(T)$ is the internal degree of freedom [22]. At temperatures $T \sim 1$ GeV, we can take $g^* \simeq 65$.

So, we get the condition

$$\sqrt{M^*_{\rm PL}\Gamma_2} \lesssim T. \tag{70}$$

The out-of-equilibrium condition means that the sterile neutrinos should decay at a temperature smaller than their mass ($T \lesssim M$). Moreover, the sterile neutrinos should decay before the creation of DM so that the leptonic asymmetry enhances the DM production. DM is created at $T \sim 0.1$ GeV. Therefore,

$$0.1 \lesssim \frac{\sqrt{M_{\rm PL^*}\Gamma_2}}{1\rm GeV} \lesssim \frac{M}{1\rm GeV}. \tag{71}$$

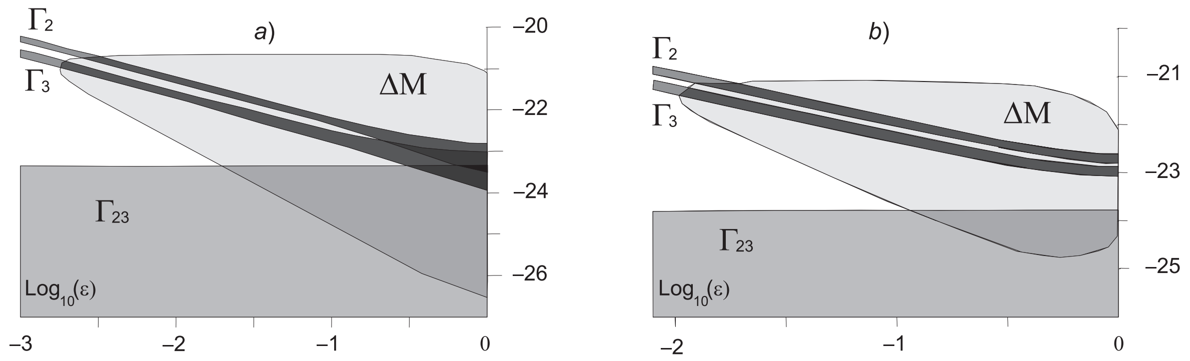

Fig. 8. Values of rates $\mathrm{Log}_{10}(\Gamma_I/1\ \mathrm{GeV})$, $\mathrm{Log}_{10}(\Gamma_{23}/1\ \mathrm{GeV})$, and $\mathrm{Log}_{10}(\Delta M/1\ \mathrm{GeV})$ for the leptonic asymmetry $\Delta > 10^{-3}$ in the case of $M = 1$ GeV and $\theta_{13} = 10^{\mathrm{o}}$ are on the ordinate axis: *a*) is the case of the normal hierarchy (A type of a solution), and *b*) is the case of inverted hierarchy (B type of a solution)

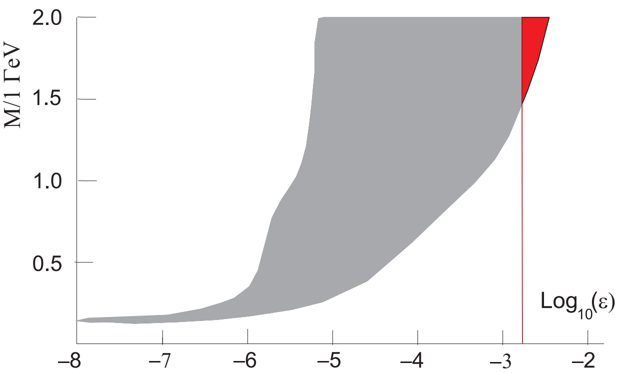

Fig. 9. Case of the normal hierarchy. The points on grey and red regions satisfy constraint (71). The region on the right from the vertical red line satisfies also condition (1)

In Fig. 9, we show the region of values of M and ε, where condition (71) is satisfied in the case of the normal hierarchy. On the scale of the parameters presented in Fig. 9, the differences between the case of $\theta_{13} = 0$ and $\theta_{13} = 10°$ and between the cases of a solution of A and B types are small, so we present only one figure. It is not true for the case of the inverted hierarchy, see Fig. 10.

It should be noted that the region in Fig. 9 is almost not sensitive to the value of θ_{13} in the interval $0 < \theta_{13} < 16°$. In case of the inverted hierarchy, it is true for the regions in Fig. 10, *b*) and $\theta_{13} < 18°$. But, in the case where $\theta_{13} = 0$, the regions are appreciably different.

As one can see, there are regions in Fig. 9 (red) and Fig. 10, *a*) (red and blue), where conditions (1) and (71) are satisfied simultaneously. This region of parameters is suitable for the DM production in the νMSM. For the case of the inverted hierarchy and the nonzero value of θ_{13}, we have no region that is suitable for the DM production. So, in ν MSM for physical nonzero values of θ_{13} and the mass of a sterile neutrino $m_\pi < M < 2$ GeV, the DM production can be realized only in case of the normal mass hierarchy of active neutrinos.

The region suitable for the DM production (case of the normal hierarchy and nonzero θ_{13}) can be used to obtain constraints for the mass splitting of a sterile neutrino. Fixing the mass of a sterile neutrino, one obtains possible values of ε (see Fig. 9). By using Fig. 5, one can obtain the possible values of mass splitting for the sterile neutrino with mass M. If the mass of a sterile neutrino is on the lower boundary of the allowed mass range ($M \simeq 1.4$ GeV), then the value of ΔM is exactly known ($\Delta M \approx 510^{-21}$ GeV). If the mass of a sterile neutrino is on the upper bound of the allowed mass range ($M = 2$ GeV), then ΔM can possess the values from the range $10^{-21} \lesssim \Delta M/1\mathrm{GeV} \lesssim 10^{-20}$.

Some existing experimental data restrict the area of parameters of νMSM. For $M < 0.45$ GeV, the best constraints come from the CERN PS191 experiment. For $0.45 < M < 2$ GeV, the constraints come from the NuTeV, CHARM, and BEBC experiments. The range of parameters admitted by these experimental data is summarized in [23]. These parameters are the mixing angle $(\Theta^+\Theta)_{22}$ (it defines the range of reactions with sterile neutrino) and the mass of the heavier sterile neutrino M.

To compare the constraints obtained in the present paper on the νMSM parameters (see Figs. 9 and

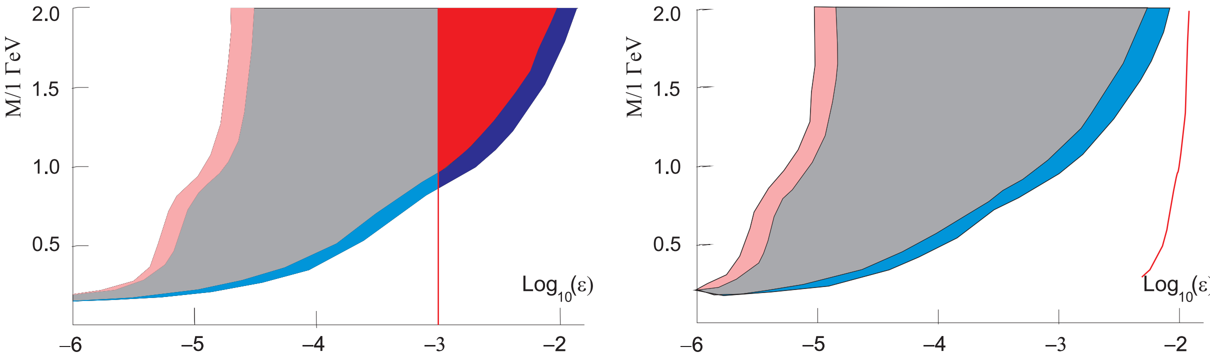

Fig. 10. Case of the inverted hierarchy: a) $\theta_{13} = 0$, *b*) $\theta_{13} = 10°$. The pink, grey, and red regions correspond to the A type of solutions. The grey, red, sky blue, and blue regions correspond to the B type of solutions. The points on these regions satisfy constraint (71). The region on the right from the red line satisfies also condition (1)

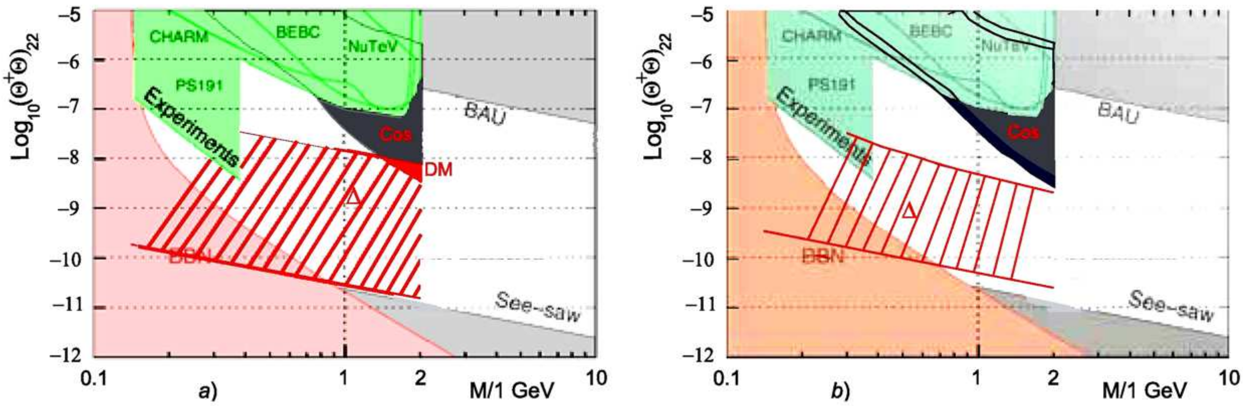

Fig. 11. Imposition of our constraints and summarized constraints from [23]: *a*) the case of the normal hierarchy, *b*) the case of the inverted hierarchy

10) with constraints summarized in [23], one has to rebuild the allowed regions in the space of parameters M and $\theta^2_{\nu N_2} = (\Theta^+\Theta)_{22}$.

In the general case, the relation between $(\Theta^+\Theta)_{22}$ and ε is quite difficult to be obtained. Really, in accordance with (29) and (34), we have

$$(\Theta^+\Theta)_{22} = \frac{\nu^2}{2M^2}(V^+h^+hV)_{22} =$$

$$= \frac{\nu^2}{2M^2}(F_2^2 + F_3^2 - 2|h^+h|_{23}\sin\chi), \qquad (72)$$

where $\chi = \arg[(h^+h)_{23}]$. Using (17)–(19), we get

$$F_2^2 = \frac{M}{\nu^2\varepsilon}(m_c + m_b), \quad F_3 = \varepsilon F_2,$$

$$|h^+h|_{23} = \frac{M}{\nu^2}(m_c - m_b) \qquad (73)$$

and

$$(\Theta^+\Theta)_{22} = \frac{m_c + m_b}{2M\varepsilon}\left(1 + \varepsilon^2 - 2\frac{m_c - m_b}{m_c + m_b}\varepsilon\sin\chi\right). \quad (74)$$

The problem consists in the parameter χ that is a complicated function of many parameters. But, in our case (see Figs. 9 and 10, $\epsilon < 0.16$), we can use the approximate relation

$$(\Theta^+\Theta)_{22} = \frac{m_c + m_b}{2M\varepsilon}. \qquad (75)$$

The imposition of our constraints presented in Figs. 9 and 10 for a nonzero value of θ_{13} and the summarized constraints from [23] is given in Fig. 11. Above the line marked "BAU", the baryogenesis is not possible: here, sterile neutrinos come to the thermal equilibrium above the T_{EW} temperature. Be-

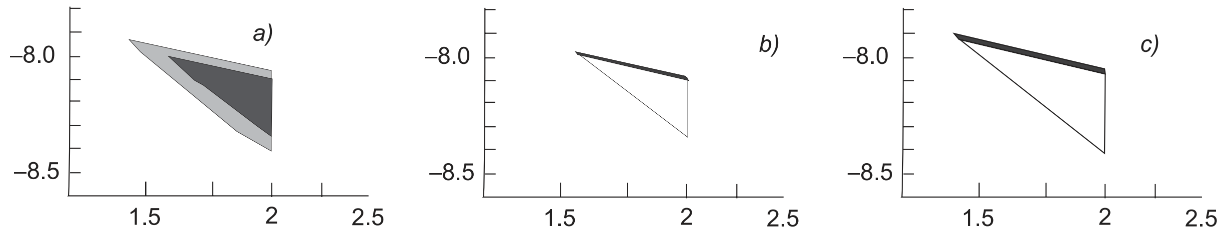

Fig. 12. Red region "DM" from Fig. 10 in the scaled-up form: *a*) $\theta_{13} = 0^0$ type A (dark) and type B (light); *b*) type A: $\theta_{13} = 0^0$ (white) and $\theta_{13} = 10^0$ (black); *c*) type B: $\theta_{13} = 0^0$ (white) and $\theta_{13} = 10^0$ (black). The variable M/1 GeV is along the abscissa axis, and the variable $\mathrm{Log}_{10}(\Theta^{+}\Theta)_{22}$ is along the ordinate axis

low the line marked "See-saw", the data on the neutrino masses and the mixing cannot be explained by the "see-saw" mechanism. The region noted as "BBN" is disfavored by the considerations of the Big Bang nucleosynthesis. The region marked "Experiment" shows a part of the parameter space excluded by direct searches for singlet fermions. The regions marked "Cos" "Δ," and "DM" are built in this paper. The grey and blue region "Cos" shows the parametric space allowed by the cosmological constraint (71) (grey region corresponds to A and B types of a solution, blue region corresponds to B type of a solution), the dashed region marked "Δ" shows the parametric space allowed by constraint (1), the red region marked "DM" shows the parametric space, where constraints (1) and (71) are noncontradictory. The last region is preferred for the DM production according to calculations of the present paper.

The red region marked "DM" is shown in Fig. 12 in the scaled-up form. The differences between the cases of $\theta_{13} = 0°$ and $\theta_{13} = 10°$ and between types of A and B of solutions are illustrated. As is seen, the choice of solutions of A or B type makes a greater change in the allowed region than the choice of $\theta_{13} = 0°$ or $\theta_{13} = 10°$.

5. Conclusion

In the present paper, we consider the leptonic asymmetry generation at $T \ll T_{EW}$, when the masses of two heavier sterile neutrinos are between m_π and 2 GeV.

We conclude that the oscillations and the decays of sterile neutrinos can produce a leptonic asymmetry that is large enough to enhance the DM production sufficiently to explain the observed DM in the Universe, but only in the case of the normal hierarchy of the active neutrino mass. The allowed range of parameters is narrow, and it is presented in Figs. 11 and 12. It should be noted that the allowed mass range for a heavier sterile neutrino is $1.42(1.55) \lesssim M < 2$ GeV for B (A) type of solutions, and the mixing angle between active and sterile neutrinos is $-7.91(-7.98) \lesssim \lesssim \mathrm{Log}_{10}(\Theta^{+}\Theta)_{22} \lesssim -8.41(-8.35)$ for B (A) type of solutions. If the sterile neutrino mass is on the lower boundary of the allowed mass range, then the value of ΔM is exactly known ($\Delta M \approx 5\times \times 10^{-21}$ GeV). If the sterile neutrino mass is on the upper bound of the allowed mass range, then ΔM can possess the values from the range $10^{-21} \lesssim \lesssim \Delta M/1$ GeV $\lesssim 10^{-20}$. In the case of the inverted hierarchy, there is no region suitable for the DM production.

The big range of parameters of the νMSM is not forbidden by the existing experimental data (see Fig. 11). Combining this range with our constraints (red region "DM" in Fig. 11) leads to the conclusion that the improvement of previous experiments, as NuTeV or CHARM, by one or two orders of magnitude can exclude the νMSM with $M < 2$ GeV or allow one to detect the right-handed neutrinos.

It should be noted that our constraints are quite rough and can be used only for estimations. Really, the form of red region "DM" is very sensitive to cosmological constraints. The applied condition $0.1\ \mathrm{GeV} < \sqrt{M_{\mathrm{PL}^*}\Gamma_2} < M$ is very approximate. The correct description of the processes can be made in the frame of the density matrix formalism or Boltzmann equations. Our computation is not valid for $M > 2$ GeV. However, the extrapolation of our results (see Fig. 11) suggests that the range of admitted parameters in the case of the normal hierarchy becomes bigger for masses above 2 GeV. We expect that, for masses above 2 GeV, the DM production can be realized in the case of the inverted hierarchy as well.

During computations, we used two types (A or B) of solutions (20). This is due to the fact that the ratios of the Yukawa matrix elements (they enter into the expression for the mixing angle $\Theta_{\alpha I}$) can be expressed through solutions (20) with two possible choices of the sign consistent with condition (22). It is closely related to the symmetry of (16) under replacing the elements of the second column of the Yukawa matrix by elements of the third column. These two variants are equal in rights.

The computation of the leptonic asymmetry in the applied simple model allows us to make some conclusions that, seemingly, will be correct also under a more rigorous consideration. Namely, the initial state of the right-handed neutrino in form (58) is not important for the lepton asymmetry generation (the final results are not sensitive to the values of constants α_I). In the case of the normal hierarchy, a deviation of the mixing angle θ_{13} from its zero value (up to the value 16°) almost does not change the region suitable for the DM production. In the case of the inverted hierarchy, the results are different for $\theta_{13} = 0$ and $\theta_{13} \neq 0$. Our calculations indicate that the case of $\theta_{13} = 0$ leads to the existence of a region suitable for the DM production. But, at nonzero values of θ_{13}, this region does not exist. Values of θ_{13} in the range $\theta_{13} < 18°$ ($\theta_{13} \neq 0$) almost do not change the region suitable for the DM production.

It is essential to note that, during computations, we have used functions maximized over unknown parameters of the model (phases $\delta, \xi, \alpha_2, \alpha_1$). If the maximization procedure was not performed, the final functions are sensitive to the values of mentioned phases. So, the obtained results are very optimistic. But if the proposed region of parameters "DM" in Fig. 11 will be forbidden by experimental data, it will mean that the mass of heavier sterile neutrinos must be larger than 2 GeV.

An essential assumption we have made is that the background effects are negligible. We do not have justify that it can be neglected in the thermal bath of the Universe. For simplicity, the computations were made at zero temperature. A rigorous justification of this assumptions is needed.

It should be noted that the region suitable for in Fig. 11 DM production in νMSM was recently calculated in the frame of a more general formalism in [24] (all details of calculations are presented in [25]). Certainly, the results of [24, 25] somewhat differ from our simple calculations.

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ГЕНЕРАЦІЯ ЛЕПТОННОЇ
АСИМЕТРІЇ В АДРОННИХ РОЗПАДАХ
СТЕРИЛЬНИХ НЕЙТРИНО

Р е з ю м е

Розглянуто утворення лептонної асиметрії при адронних розпадах стерильних нейтрино в моделі νMSM при $T \ll T_{EW}$ за умови, що маси двох найважчих стерильних нейтрино знаходяться в межах від m_π до 2 ГеВ. Верхня межа по масі зумовлена відсутністю прямих експериментів по виявленню стерильних нейтрино з більшими масами. Розрахунки були проведені при нульовій температурі без врахування впливу середовища. Вимога необхідного значення лептонної асиметрії для утворення частинок темної матерії, умова не перебування стерильних нейтрино в стані теплової рівноваги та наявні експериментальні обмеження є несуперечливими лише в області мас найважчих стерильних нейтрино 1,4 ГеВ $\lesssim M < 2$ ГеВ та при нормальній ієрархії мас активних нейтрино.