NUCLEI AND NUCLEAR REACTIONS

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SHEAR AND BULK VISCOSITIES OF A HADRON GAS WITHIN RELAXATION TIME APPROXIMATION AND ITS TEST

We concentrate on the calculation of the shear and bulk viscosities of a hadron gas. They define its dissipative dynamics and influence its experimentally measurable elliptic flow. Due to the difficulty of this calculation, the relaxation time approximation (RTA) was used in previous works. As those results have approached the realistic ones, there is a need to find out how accurate RTA is. For this sake, we calculate the viscosities in RTA, by using the cross sections extracted from the ultrarelativistic quantum molecular dynamics (UrQMD) model and compare them with the same ones calculated without RTA. This allows us to find the estimates of errors due to the application of RTA in the calculations of viscosities, which are valid also for other similar models. For instance, in the temperature region 100 MeV ≲ T ≲ 160 MeV at zero chemical potentials, the shear viscosity becomes smaller up to 1.57 times or up to 1.45 times if the averaged relaxation time is used. This has important consequences for the interpretation of the previously made calculations of viscosities and some other related calculations. Within RTA, we also find estimation of the enhancement of the bulk viscosity of a hadron gas because of the nonconservation of particle numbers.

K e y w o r d s: relaxation time approximation, bulk viscosity, shear viscosity, hadron gas.

1. Introduction

The bulk and shear viscosities are required for the dissipative hydrodynamic description. This description finds applications to the strongly interacting matter created in heavy ion collisions. In particular, its elliptic flow can be measured, see review [1].

In this paper, we focus on the calculation of the shear and bulk viscosity coefficients of a hadron gas at zero chemical potentials. Ref. [2] provides these calculations, being close to the realistic ones, with unique advancement. However, they are done in RTA.

RTA for the Boltzmann equation (BE) is known for a long time, see, e.g., Ref. [3]. The advantage of using it is that it provides a simplification in calculations. However, in all its known realizations, the errors from its application are not controlled. There

are some tests and analysis of RTA [4–6], though it is not sufficient for the estimation of errors from RTA if the calculations for a hadron gas are required. Thus, there is a need to verify this approximation. When the error estimates for it are found, they can be used in other RTA-based calculations (directly or after a modification, derivation, or implementation of some ansatz), e.g., in Ref. [7], or hydrokinetic calculations [8]. The heat conductivity and diffusion coefficients, being rather closer by their properties to the shear viscosity, could be expected to have approximately the same errors at the chemical potentials small compared to the temperature and with approximately the same densities.

It turns out that the bulk viscosity enhances much after the introduction of inelastic (particle number changing) processes. Taking them into account may meet some difficulties, though they can be considered relatively easily in RTA. One could also speak

about the approximate conservation of particle numbers, which has a specific mathematical realization for the bulk viscosity. Making comparison between the maximal and minimal enhancements (or between the cases of minimal and maximal particle number conservations) within the same hadron gas model and within the same approximations, one could find the error estimates needed in Refs. [2, 7, 9].

2. RTA and Results

The system of Boltzmann equations in the local rest frame in RTA can be written as [3]

$$\frac{df_k(t, \mathbf{r}, p_k^0)}{dt} = -\frac{f_k(t, \mathbf{r}, p_k^0) - f_k^{(0)}}{\tau_{\mathrm{rel},k}(t, \mathbf{r}, p_k^0)}, \tag{1}$$

$$f_k^{(0)} \equiv f_k^{(0)}(t, \mathbf{r}, p_k^0) = \exp(-p_k^0/T(t, \mathbf{r})), \tag{2}$$

$$\tau_{\mathrm{rel},k}^{-1}(t, \mathbf{r}, p_k^0) = \sum_l \int \frac{d^3 p_l}{(2\pi)^3} f_l^{(0)} v_{kl} \sigma_{kl}^{\mathrm{tot}}(s), \tag{3}$$

$$v_{kl} \equiv \frac{\sqrt{(s - m_k^2 - m_l^2)^2 - 4 m_k^2 m_l^2}}{2 p_k^0 p_l^0}, \tag{4}$$

where $f_k(t, \mathbf{r}, p_k^0)$ and $f_k^{(0)}$ are the nonequilibrium and local equilibrium distribution functions, respectively, $\tau_{\mathrm{rel},k}(t, \mathbf{r}, p_k^0) \equiv \tau_{\mathrm{rel},k}(p_k^0)$ is the relaxation time depending on the one-particle energy p_k^0 of the k-th species (cf. Ref. [10]), v_{kl} is the relativistic relative velocity (cf. Ref. [11]), $\sigma_{kl}^{\mathrm{tot}}(s)$ is the total $2 \leftrightarrow 2$ cross-section (see also comments below), and s is the usual Mandelstam variable.

We also consider the momentum averaged relaxation time as in Ref. [2]. In general, this approximation should not be better; however, we find it about as good as with the momentum dependent relaxation time in our calculations. In Ref. [2], the transport cross-sections as in Ref. [12] are used. The UrQMD hadronic cross-sections [13, 14], which we exploit [1], have some extrapolation of the angular dependence from nucleons, but we do not just adopt it here. The expected deviations are of 4% or less. With the improved transport cross-sections as in Refs. [4, 5], the isotropic total cross-sections would get the extra factor equal to 2/3 (and the viscosities would get the factor 3/2). This does not provide a better overall

[1] We use improved and extracted UrQMD cross sections as described in Ref. [9].

description of the viscosities, though it is better at high enough temperatures, see below.

We use the $\sigma_{kl}^{\mathrm{tot}}(s)$ as $2 \leftrightarrow n$ total cross-sections (TCSs), by considering also quasielastic and other than $2 \leftrightarrow 2$ processes, as in Ref. [9]. So that, some cross sections add up exactly and some ones add up approximately into the total ones. This approximation is a good one, see Ref. [9] for checkups. In addition to this, we also use the elastic plus quasielastic cross sections (EQCSs) [9] and find approximately the same error estimates as with the TCSs. Other approximations which we apply (ideal gas equation of state, no medium effects, classical statistics) result in small corrections [2, 9] at least in the temperature range 100 MeV $\lesssim T \lesssim$ 160 MeV at zero chemical potentials. If these corrections are not small, then, in the assumption of absent or weak correlations with the RTA corrections, the latter ones can be still applicable.

The relaxation time enters the shear η and bulk ξ viscosities as (cf. Ref. [10])

$$\eta = \frac{1}{15T} \sum_k \int \frac{d^3 p_k}{(2\pi)^3} \frac{\tau_{\mathrm{rel},k}(p_k^0)}{(p_k^0)^2} |\mathbf{p}|^4 f_k^{(0)}, \tag{5}$$

$$\xi = T^3 \sum_k \int \frac{d^3 p_k}{(2\pi)^3} \frac{\tau_{\mathrm{rel},k}(p_k^0)}{(p_k^0)^2} \hat{Q}_k^2 f_k^{(0)}, \tag{6}$$

where $\hat{Q}_k$ is the dimensionless bulk viscosity source term, which we take in the convenient form as in Ref. [15]. The approximation of maximal particle number conservation implies that particles' charges are equal to the Kronecker's delta functions, $q_{ak} = \delta_{ak}$. The approximation of maximal nonconservation is equivalent to the case $q_{ak} = 0$ at zero chemical potentials. There are also matching conditions, which can be satisfied modifying additionally the relaxation time [16]. We do not investigate whether this modification of RTA provides a better overall description. We are interested in the testing of RTA as the one in Ref. [2].

Figures 1 and 2 show the results of calculations of the shear viscosity and the bulk viscosity, respectively, at zero chemical potentials. In addition to the calculations in RTA, there are also depicted the results of calculations within the variational method with the application of the TCSs [9]. From these results, one can see the deviations from the application of RTA. To see how strong is the dependence of these deviations on the energy dependence of TCSs, we also

made the same calculations with EQCSs. This almost does not change the deviations, so that we do not show the results with EQCSs. As long as the temperature dependence of the viscosities in the SHMC model [2] and in the one of the present paper are similar to each other, the found error estimates from the application of RTA should be approximately the same for the SHMC model.

From Fig. 1, one can see that, in the important temperature range[2] 100 MeV $\lesssim T \lesssim 160$ MeV, the shear viscosity becomes smaller up to 1.57 times because of the application of RTA. If the averaged relaxation time is used, these deviations are somewhat smaller and reach the factor 1.45, which is rather an accidental improvement. At smaller temperatures, in a vicinity of the shear viscosity minimum, these deviations are somewhat larger instead. Moreover, the 3/2 times larger shear viscosity in RTA (see above) would give a better description at higher temperatures, but a worse description in a vicinity of the shear viscosity minimum. This minimum for the hadron gas is attributed to resonant peaks in the quasielastic cross-sections of pions, dominating at those energies and temperatures. So this rapid change in the energy dependence of the cross-sections does not permit the improved RTA description. At higher temperatures, there are different cross-section energy dependences canceling each other approximately, which results in a better description with one constant cross-section[3] [9].

Figure 2 demonstrates the calculations of the bulk viscosity in the approximations of minimal and maximal particle number conservations (see comments above). The bulk viscosity calculations using the variational method are shown only in the approximation of the maximal conservation, because only this one is considered in Ref. [9]. As long as the calculations of the bulk viscosities using RTA with the averaged and not averaged relaxation times differ by 13% or less, we do not show the results with the averaged relax-
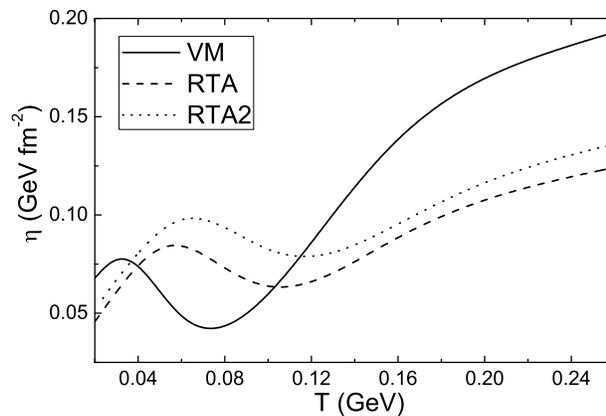

Fig. 1. Shear viscosity versus the temperature. The calculations are done using the variational method in the third order (solid line), using RTA (dashed line) and using RTA with the averaged relaxation time (dotted line)
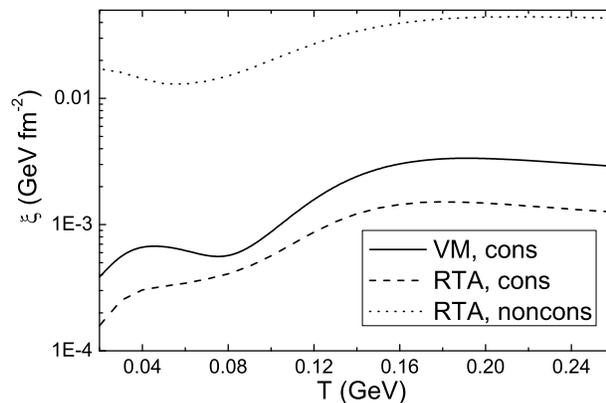

Fig. 2. Bulk viscosity versus the temperature. The calculations are done in the approximation of maximal conservation using the variational method in the fifth order (solid line) and using RTA (dashed line). The calculations in the approximation of maximal nonconservation using RTA are denoted by the dotted line

ation time. The bulk viscosity in RTA turns out to be smaller at all the temperatures by 1.4–2.4 times[4].

One can see from Fig. 2 that the enhancement of the bulk viscosity due to the maximal particle number nonconservation is large. At the chemical freeze-out temperature $T \approx 160$ MeV, the ratio of the RTA-based bulk viscosity with the maximal nonconservation to the one with the maximal conservation is equal to 27.27. On the chemical freeze-out line,

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[2] At zero chemical potentials, one has the kinetic freeze-out temperature $T \approx 120$ MeV [17]; and, for both the pseudo-critical temperature and the chemical freeze-out temperature, one has $T \approx 160$ MeV [17, 18]

[3] In this approximation the deviations from application of RTA are of the factor 1.6–1.7 in the whole considered temperature range.

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[4] These numbers are replaced with 2.1–2.6 if the approximation of one constant cross-section is used.

where the elastic plus quasielastic rates are equal to the total rates, we do not know *a priori* which approximation dominates in the bulk viscosity, so that we need to divide this number by two to use it as an error estimate in the calculations with either of the approximations. None of the approximations should be valid beyond its region delimited by the chemical freeze-out line. At smaller temperatures (and the same chemical potentials), the same error estimate amounted to 13.64 can be used, because the inelastic processes become weaker there [9, 19]. One could also connect the fading of the enhancement of the bulk viscosity to the collision rates and to multiply the number 27.27 by $1 - \text{rate}_{\text{EQCSs}}/\text{rate}_{\text{TCSs}}$ (or to use the corresponding expression with the shear viscosities or the bulk viscosities, being approximately inversely proportional to the collision rates, from Ref. [9]; we do not present this result explicitly here for compactness). The temperatures $T \gtrsim 160$ MeV are less interesting, because they are above the pseudo-critical temperature [18]; however, a similar extrapolation could be made at these temperatures. We present these estimates with caveat, because the calculations based on the Chapman–Enskog and variational methods [15] give notably larger estimates.

3. Conclusions

In the direct comparison, we have found that one can normally expect to have deviations in the viscosities from the application of RTA up to 2–3 times with the energy dependence of cross-sections as that for the hadron gas at the considered temperatures 20 MeV $\leq$ $\leq T \leq 260$ MeV and zero chemical potentials.

At the temperatures $T \gtrsim 100$ MeV and zero chemical potentials, the application of RTA decreases the shear viscosity η. The factor of this deviation reaches 1.57 at $T = 160$ MeV or 1.45 if the averaged relaxation time is used. As long as the temperature dependence of η in the SHMC model [2] and in the present paper are similar to each other, the found error estimates should be approximately the same. This confirms that the multihadron production processes and some other ones, which are seemingly not taken into account in Ref. [2], are important to get η/s (s is the entropy density) well consistent with the experimental data [9].

Using TCSs, we have found that the ratio of the bulk viscosity with the maximal nonconservation to the one with the maximal conservation is equal to 27.27 at the chemical freeze-out temperature $T \approx$ 160 MeV. This number should be divided by two on the chemical freeze-out line, where neither conservation nor nonconservation is preferred *a priori*. At other temperatures, some extrapolations could be used. For this goal, the further investigations are desirable. We present that estimate with caveat, because the calculations based on the Chapman–Enskog and variational methods [15] give notably larger estimates. If they are notably larger, this may be the case of the largest deviations from the application of RTA.

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ЗСУВНА ТА ОБ'ЄМНА В'ЯЗКОСТІ
ҐАДРОННОГО ГАЗУ В НАБЛИЖЕННІ
ЧАСУ РЕЛАКСАЦІЇ ТА ЙОГО ПЕРЕВІРКА

Р е з ю м е

Обчислено зсувну та об'ємну в'язкості ґадронного газу. Вони визначають його дисипативну динаміку і впливають на еліптичний потік, який вимірюється експериментально. Внаслідок складності цього обчислення наближення часу релаксації (НЧР) було використано в попередніх роботах. Оскільки ці результати наблизились до реалістичних результатів, є необхідність знайти, наскільки точним є НЧР. Для цього обчислено в'язкості в НЧР, використовуючи перетини розсіяння, витягнуті з моделі ультрарелятивістської квантової молекулярної динаміки (УрКМД), та порівняно їх з цими самими обчисленнями, але без НЧР. Це дозволяє нам знайти оцінки похибок через застосування НЧР в обчисленнях в'язкостей, які є вірними і для інших схожих моделей. Наприклад, в інтервалі температур $100 \text{ МеВ} \lesssim T \lesssim 160 \text{ МеВ}$ при нульових хімічних потенціалах зсувна в'язкість стає меншою до 1,57 разів через застосування НЧР або до 1,45 разів, якщо використовується усереднений час релаксації. Це має важливі наслідки для інтерпретації попередньо зроблених обчислень в'язкостей та для деяких інших споріднених обчислень. В рамках НЧР також знаходимо оцінку збільшення об'ємної в'язкості ґадронного газу через незбереження чисел частинок.