SEMIDISCRETE INTEGRABLE SYSTEMS INSPIRED BY THE DAVYDOV–KYSLUKHA MODEL

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S u m m a r y

Intending to mimicry certain physical features of the Davydov–Kyslukha exciton-phonon system, we have suggested four distinct combinations of ansätze for matrix-valued Lax operators capable to generate a number of semidiscrete integrable nonlinear systems in the framework of the zero-curvature approach.

Dealing with Taylor-like ansätze for Lax operators, two types of general nonlinear integrable systems on infinite quasione-dimensional regular lattices are proposed. In accordance with the Mikhailov reduction group theory, both general systems turn out to be underdetermined, thereby permitting numerous reduced systems written in terms of true field variables. Each of the obtained reduced systems can be considered as an integrable version of two particular coupled subsystems and demonstrates the symmetry under the space and time reversal ($\mathcal{PT}$-symmetry). Thus, we have managed to unify the Toda-like vibrational subsystem and the self-trapping lattice subsystem into the single integrable system, thereby substantially extending the range of realistic physical problems that can be rigorously modeled. The several lowest conserved densities associated with either of the possible infinite hierarchies of local conservation laws are found explicitly in terms of prototype field functions.

When considering the Laurent-like ansätze for Lax operators, we have isolated four new semidiscrete nonlinear integrable systems interesting for physical applications. Thus, we have coupled the Toda-like subsystem with the induced-trapping subsystem of $\mathcal{PT}$-symmetric excitations. Another integrable system is set up as the subsystem of Frenkel-like excitons coupled with the subsystem of essentially nontrivial vibrations. We also have revealed the integrable system of two self-trapping subsystems coupled together by means of a mutually induced nonlinearity. At last, we have obtained the integrable system, where the Toda-like subsystem and the self-trapping subsystem are coupled akin to a charged particle with an electromagnetic field. In so doing, the vector-potential part of the Hamiltonian function is appeared as the density of excitations in the self-trapping subsystem. Each of the proposed systems admits the clear Hamiltonian representation characterized by the two pairs of canonical field variables with the standard (undeformed) Poisson structure. Several general local conserved densities having been found in the framework of a generalized direct procedure are presented explicitly. These conserved densities are readily adaptable to any integrable system under consideration.