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**COMMENT ON “INVESTIGATION
OF HADRON MULTIPLICITY AND HADRON
YIELD RATIOS IN HEAVY-ION COLLISIONS”**

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*Oliinychenko, Bugaev, and Sorin [arXiv:1204.0103 [hep-ph]] considered the role of conservation laws in discussing possible weaknesses of thermal models which are utilized in describing the hadron multiplicities measured in central nucleus-nucleus collisions. They argued to analyze the criteria for chemical freeze-out and to conclude that none of them was robust. Based on this, they suggested a new chemical freeze-out criterion. They assigned to the entropy per hadron the ad hoc value 7.18 and supposed to remain unchanged over the whole range of the baryo-chemical potentials. Due to the unawareness of the recent literature, the constant entropy per hadron has been discussed in Refs. [Fizika B **18**, 141 (2009), Europhys. Lett. **75**, 420 (2006), Phys. Rev. C **85**, 014908 (2012) and nucl-ph/1306.3291]. Furthermore, it has been shown that the constant entropy per hadron is equivalent to the constant entropy normalized to the cubic temperature, an earlier criterion for the chemical freeze-out introduced in Refs. [Europhys. Lett. **75**, 420 (2006), Nucl. Phys. A **764**, 387 (2006)]. In this comment, we list out the ignored literature, compare between the entropy-number density ratio and two criteria of averaged energy per averaged particle number and constant entropy per cubic temperature. All these criteria are confronted to the experimental results. The physics of constant entropy per number density is elaborated. It is concluded that this ratio cannot remain constant, especially at large chemical potentials related to the AGS and SIS energies.*

Keywords: hadron multiplicity, hadron yield ratios, heavy-ion collisions.

1. Introduction

In the preprint [1], Oliinychenko, Bugaev, and Sorin have considered the role of conservation laws, the values of hard core radii along with the effects of the Lorentz contraction of hadron eigenvolumes in discussing the weaknesses of thermal models, which are utilized in describing the hadron multiplicities measured in the central nucleus-nucleus collisions. Regardless the unawareness of earlier literature, the authors concluded that none of the criteria for the chemical freeze-out is robust. In doing this, they entirely disregarded the indirect experimental results in baryo-chemical potentials μ_b and their corresponding temperatures T . A systematic analysis of the four criteria describing the chemical freeze-out is introduced in [2–4]. Furthermore, a comparison between these four criteria is elaborated in [2–4].

Starting from phenomenological observations at the SIS energy, it was found that the averaged energy per averaged particle $\epsilon/n \approx 1$ GeV [5], where the

Boltzmann approximations are applied to calculating ϵ/n , this constant ratio is assumed to describe the whole $T - \mu_b$ diagram. For completeness, we mention that the authors assumed that the pions and rho-mesons get dominant, at high T and small μ_b . The second criterion assumes that the total baryon number density $n_b + n_{\bar{b}} \approx 0.12$ fm⁻³ [6]. In the framework of percolation theory, the authors of Ref. [7] have suggested the third criterion. As shown in Fig. 2 of [3], the last two criteria seem to give almost identical results. All of them are stemming from the phenomenological observation. The fourth criterion based on lattice QCD simulations was introduced in Ref. [2,3]. Accordingly, the entropy normalized to the cubic temperature is assumed to remain constant over the whole range of baryo-chemical potentials, which is related to the nucleus-nucleus center-of-mass energies $\sqrt{s_{NN}}$ [4]. An extensive comparison between constant ϵ/n and constant s/T^3 is given in [2,3].

The thermodynamic quantities deriving the chemical freeze-out in the framework of hadron resonance gas are deduced in [2,3]. Explicit expressions for s/n

at vanishing and finite temperatures are introduced in [2, 8]. The motivation of suggesting the constant normalized entropy is the comparison to the lattice QCD simulations with two and three flavors. We simply found the $s/T^3 = 5$ for two flavors and $s/T^3 = 7$ for three flavors. Furthermore, we confront the hadron resonance gas results to the experimental estimation for the freeze-out parameters, T and μ_b .

2. The Hadron Resonance Gas Model

The hadron resonances treated as a free gas [9–13] are conjectured to add to the thermodynamic pressure in the hadronic phase (below T_c). This statement is valid for free resonances, as well as for strong interactions between the resonances themselves. It has been shown that the thermodynamics of a strongly interacting system can also be approximated to an ideal gas composed of hadron resonances with masses ≤ 2 GeV [12, 14]. Such a mass cut-off is implemented to avoid the Hagedorn singularity [16]. Therefore, the confined phase of QCD, the hadronic phase, is modelled as a non-interacting gas of resonances. The grand canonical partition function reads

$$Z(T, V) = \mathbf{Tr} \left[\exp^{-H/T} \right], \quad (1)$$

where H is the Hamiltonian of the system, and T is the temperature. The Hamiltonian is given by the sum of the kinetic energies of relativistic Fermi and Bose particles. The main motivation of using this Hamiltonian is that it contains all relevant degrees of freedom of confined and strongly interacting matter. Obviously, it can be characterized by various – but a complete – set of microscopic states. Therefore, the physical properties of the quantum systems turn to be accessible in the approximation of non-correlated *free* hadron resonances. Each of them is conjectured to add to the overall thermodynamic pressure of the *strongly* interacting hadronic matter. It includes implicitly the interactions that result in the resonance formation. In addition, it has been shown that this model can submit a quite satisfactory description of the particle production in heavy-ion collisions [9–13]. With the above assumptions about the dynamics, the partition function can be calculated exactly and be expressed as a sum over *single-particle partition* functions Z_i^1 of all hadrons and their resonances.

$$\ln Z(T, \mu_i, V) = \sum_i \ln Z_i^1(T, V) =$$

$$= \sum_i \pm \frac{V g_i}{2\pi^2} \int_0^\infty k^2 dk \ln \left(1 \pm \exp \left[\frac{\mu_i - \epsilon_i}{T} \right] \right), \quad (2)$$

where $\epsilon_i(k) = (k^2 + m_i^2)^{1/2}$ is the i -th particle dispersion relation, g_i is the spin-isospin degeneracy factor, and \pm stands for bosons and fermions, respectively.

The switching between hadron and quark chemistry is given by the relations between the *hadronic* chemical potentials and the quark constituents; $\mu_i = 3n_b \mu_q + n_s \mu_s$, where $n_b(n_s)$ being the baryon (strange) quantum number. The chemical potential assigned to the light quarks is $\mu_q = (\mu_u + \mu_d)/2$, and the one assigned to a strange quark reads $\mu_s = \mu_q - \mu_s$. The strangeness chemical potential μ_s is calculated as a function of T and μ_i under the assumption that the overall strange quantum number has to remain conserved in heavy-ion collisions [12].

The HRG calculations assume quantum statistics and an overall strangeness conservation. With this regard, the strangeness chemical potential μ_s is calculated at each value of T and μ_b assuring that the number of strange particles should be the same as that of the anti-strange particles. It is worthwhile to mention that no statistical fitting has been applied in determining all thermodynamic quantities, including entropy and number density derived from Eq. (2).

As introduced in Ref. [12], the whole spectrum of possible interactions is to be represented by the S -matrix. A recent review on the estimation of the excluded volume, which reflects the repulsive interaction, as a function of $\sqrt{s_{NN}}$ is given in [15] and references therein. According to [12], the fugacity term can be expanded to include various kinds of interactions. In such a way, the S -matrix gives plausible scattering processes taking place in the system of interest. It is found that including hadron resonances with some effective masses has almost the same effect as that of a free particle with same mass. At high energy, the effective mass approaches the physical value. In other words, even strong interactions are taken into consideration via heavy resonances. These conclusions suggest that the grand canonical partition function is able to simulate various types of interactions, when resonances with masses up to 2 GeV are included. As elaborated previously, this mass cut-off is supposed to avoid Hagedorn's singularity. A conclusive convincing proof has been presented through confronting HRG results to LGT [9–13].

3. Physics of Constant Entropy Per Number Density

From the entropy and equilibrium, the Gibbs condition simply leads to

$$\frac{s}{n} = \frac{1}{T} \left(\frac{p}{n} + \frac{\epsilon}{n} - \mu_b \right), \quad (3)$$

the rhs is positive as long as $\mu_b < p/n + \epsilon/n$, where the thermodynamic quantities, p , ϵ and n are supposed to be calculated at the $T - \mu_b$ diagram of the chemical freeze-out. Figure 1 shows the experimental estimation for the freeze-out parameters T and μ_b . It is obvious that increasing μ_b leads to decreasing T , and, therefore, all values of the thermodynamic quantities decrease as well. Cleymans *et al.* [5] suggested an empirical $T - \mu_b$ relation

$$T = a - b\mu_b^2 - c\mu_b^4, \quad (4)$$

where a , b and c are fitting parameters. Almost same kind of restriction would be valid for ϵ/n . According to Eq. (3),

$$\frac{\epsilon}{n} = T \frac{s}{n} + \mu_b - \frac{p}{n}. \quad (5)$$

The physics of constant s/T^3 has been discussed in Ref. [2, 3]. It combines the three thermodynamic quantities, p/T^4 , ϵ/T^4 , and n/T^3 :

$$\frac{s}{T^3} = \frac{p}{T^4} + \frac{\epsilon}{T^4} - \mu_b \frac{n}{T^3}. \quad (6)$$

At the chemical equilibrium, the particle production at the freeze-out is conjuncted to fully fulfil the laws of thermodynamics, as Eq. (3). The hadronic abundances observed in the final state of heavy-ion collisions are settled when s/T^3 drops to 7, i.e., the degrees of freedom drop to $7\pi^2/4$. Meanwhile the changing in the particle number with the changing in the collision energy is given by μ_b , the energy that produces no additional work, i.e. the stage of vanishing free energy, gives the entropy at the chemical equilibrium. At the chemical freeze-out, the equilibrium entropy represents the amount of energy that cannot be used to produce an additional work. In this context, the entropy is defined as a degree of sharing and spreading the energy inside the system, that is in chemical equilibrium [3].

4. Constant Entropy per Number in Lattice QCD Simulations and Heavy-ion Collisions

For completeness, we analyze s/n as measured in LGT. Once again, the related literature on lattice QCD simulations is not cited in [1]. For example, Borsanyi *et al.* [17] studied the trajectories of constant s/n , where $s = S/V$ and $n = N/V$, on the phase diagram and thermodynamic observables along these isentropic lines. This was not the only work devoted to such line of constant physics [18]. In the Stefan-Boltzmann limit, the ratio s/n is assumed to remain unchanged with increasing μ_b (Appendix A of [17]). In doing this, the lowest order in perturbation theory is assumed, where the strangeness chemical potential μ_S likely vanishes. For μ_b/T , a limiting behavior for the isentropic lines on the phase diagram is obtained. The ratio s/n has been measured at various $\sqrt{s_{NN}}$ [19]. It is concluded that, in limits of low temperatures, increasing the chemical potential results in an overestimation for the ratio s/n even beyond the applicability region of the Taylor-expansion method, which is applied in lattice QCD simulations at a finite chemical potential. Two remarks are now in order. First, the values of s/n seem to depend on the chemical potential μ_b or $\sqrt{s_{NN}}$. This is confirmed in different experiments [19] and lattice gauge theory [17]. Second, the ratio s/n as calculated in the lattice QCD simulations [17] is suggested to characterized the QCD phase diagram [12],

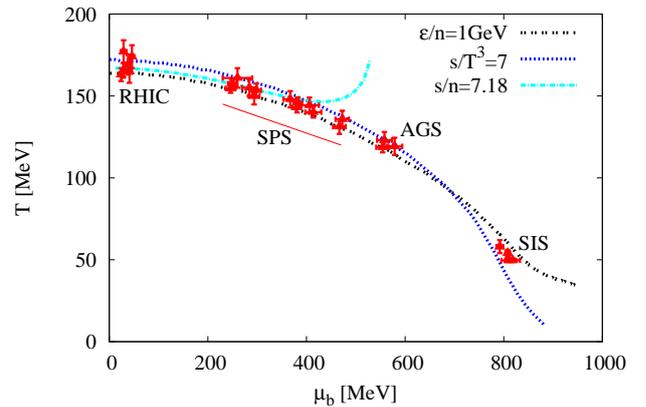


Fig. 1. The freeze-out parameters, T and μ_b , measured in various heavy-ion collisions experiments (labelled) are compared with the three criteria, $\epsilon/n = 1$ GeV (double-dotted line), $s/n = 7.18$ (dash-dotted line) and $s/T^3 = 7$ (solid line). The s/n dotted curve seems to diverge at $\mu_b > 500$ MeV, i.e., much high temperatures are needed to fulfil the condition

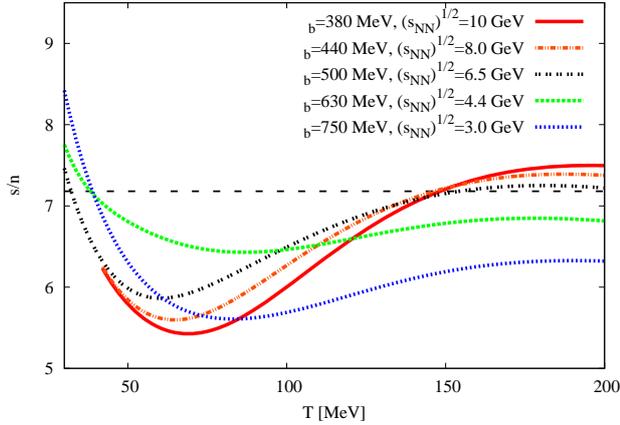


Fig. 2. The thermal evolution of s/n at $\mu_b = 500, 630$ and 750 MeV is presented. The horizontal dashed line indicates $s/n = 7.18$. The singularities at low temperatures are stemming from the almost vanishing number density

i.e., the deconfinement phase transition, while likely differs from the freeze-out diagram [2, 3], especially at large chemical potential μ_b or small $\sqrt{s_{NN}}$. At fixed μ_b , the critical (deconfinement) differs from the freeze-out (hadronization) temperature. As per LGT, the ratio s/n characterizes the deconfinement phase transition, especially at a large chemical potential.

5. Results and Conclusions

In Fig. 1, the freeze-out parameters, T and μ_b , measured in various heavy-ion collisions experiments are compared with the three criteria, $\epsilon/n = 1$ GeV (double dashed line), $s/n = 7.18$ (dash-dotted line) and $s/T^3 = 7$ (solid line). The experimental data are taken from [4] and the reference therein. The quality of each criterion in describing the experimental data is presented. All conditions are almost equivalent at a very high energy or low chemical potential. The ability of the condition $\epsilon/n = 1$ GeV at very low energies are not as much as that of $s/T^3 = 7$. As discussed in Section 3, $s/n = 7.18$ seems to fail to reproduce the freeze-out parameters at $\mu_b > 500$ MeV.

The dependence of the freeze-out temperature T on μ_b starts to be non-monotonic at $\mu_b \sim 400$ MeV (compare solid with double dot-dashed curves in the right panel of Fig. 2). Starting from this value, the resulting T increases with μ_b . At fixed μ_b values, high freeze-out temperatures are needed to fulfil the condition. This would mean that the freeze-out temperatures at large μ_b become much higher than those

at vanishing μ_b . At higher chemical potentials, the resulting dash-dotted curve diverges. To have a detailed illustration for this observation, we firstly study the thermal evolution of s/n at a very high chemical potential, right panel of Fig. 2. Details on the utilized HRG model are elaborated in Section 2. In this figure, the ratio s/n is calculated a function of T , at five fixed values of μ_b . We observe that $s/n = 7.18$ is eventually achieved at $\mu_b \leq 500$ MeV. At higher chemical potentials, the condition $s/n = 7.18$ is apparently not fulfilled. The ratio s/n never reaches the value 7.18, i.e. singularity. Furthermore, we notice that the thermal evolution of s/n is non-monotonic. Three remarks are now in order. First, due to a small particle number n at very small T , the ratio s/n gets very large in this limit. In other words, the condition $s/n = 7.18$ might be fulfilled at very small T and finite μ_b . This would be a kind of artifact, as the physical interpretation is not related to the chemical freeze-out. Second, the ratio s/n raises with increasing T . The resulting peak seems never reach the value 7.18, i.e. there is no freeze-out temperature at the given μ_b (singularity). Third, with increasing T , the ratio s/n slowly decreases, assuring the previous remark.

The second criteria to interpret the observation that s/n seems to diverge at large μ_b is based on an analytical treatment to find out the value of μ_b at which the freeze-out curve diverges. The latter means that $\partial T/\partial \mu_b = \infty$ or $\partial \mu_b/\partial T = 0$. In doing this, we start with $s/n = \text{const}$ or

$$d \frac{s}{n} = 0. \quad (7)$$

Then we get

$$\begin{aligned} n \left(\frac{\partial s}{\partial T} \right) dT + n \left(\frac{\partial s}{\partial \mu_b} \right) d\mu_b &= \\ = s \left(\frac{\partial n}{\partial T} \right) dT + s \left(\frac{\partial n}{\partial \mu_b} \right) d\mu_b, \end{aligned} \quad (8)$$

which can be solved as follows.

$$\frac{\partial \mu_b}{\partial T} = \frac{n \frac{\partial s}{\partial T} - s \frac{\partial n}{\partial T}}{s \frac{\partial n}{\partial \mu_b} - n \frac{\partial s}{\partial \mu_b}}. \quad (9)$$

Thus, the divergence in the freeze-out parameter, T , under the condition that $s/n = \text{const}$ is to be fulfilled when we are succeeded in determining μ_b values, at which

$$n \frac{\partial s}{\partial T} = s \frac{\partial n}{\partial T}, \quad (10)$$

is valid. Such μ_b values assure that the freeze-out temperature diverges. In doing this, we may use the classical limit

$$\ln Z(T, \mu_b) = \sum_i^N \frac{g_i}{2\pi^2} T^3 \left(\frac{m_i}{T}\right)^2 \exp\left(\frac{\mu_b^{(i)}}{T}\right) K_2\left(\frac{m_i}{T}\right), \quad (11)$$

where K_n is the n -th rank modified Bessel function. The baryo-chemical potential for the quark constituents of each baryon $\mu_b = 3n_b\mu_q$, where n_b (μ_q) being the baryon number (quark baryo-chemical potential). At the chemical freeze-out line, it is conjectured that the partonic matter should absolve the hadronization process into an equilibrated hadronic matter. In light of this, we might assume that the energy of each state in the phase space could be given by m , where $m \simeq T$. Then, the freeze-out temperature diverges at the quark baryo-chemical potential

$$\mu_q \geq T. \quad (12)$$

When μ_q becomes as higher as the temperature, then the freeze-out temperature calculated according to $s/n = 7.18$ rapidly diverges with increasing μ_b and or $\sqrt{s_{NN}}$. A quantitative estimation for μ_q is only possible when taking the quantum statistics into consideration.

The HRG calculations are performed as follows. Starting with a certain μ_b , the temperature T is increased very slowly. At this value of μ_b and each raise in T , the strangeness chemical potential μ_S is determined under the condition that the strange quantum numbers should remain conserved in heavy-ion collisions. Having the three values of μ_b , T , and μ_S , then all thermodynamic quantities are calculated. At each step, the ratio s/T^3 is checked. When it reaches the value 7, then the quantities like ϵ/n , $n_b + n_{\bar{b}}$, and s/n are registered. This procedure is repeated over all values of μ_b . Examples of such calculations can be seen from the thermal evolution presented in Fig. 2. So far we conclude that the applicability of $s/n = 7.18$ is limited to $\mu_b < 500$ MeV.

Furthermore, the robustness of $s/n = 7.18$ is very much limited in comparison to the four criteria: percolation [7], baryon number [6], energy per particle [5] and normalized entropy [2, 3].

The coincidence that s/T^3 is accompanied with constant s/n has been introduced in Ref. [2]. That

the authors of [1] argue that $s/n = 7.18$ is novel likely ignores the related literature. The four criteria [2, 3, 5–7] are based on the physical observation either phenomenological and/or theoretical, while the authors of [1] suggest an *ad hoc* value for s/n . It is inapplicable at the AGS and SIS energies. Its relation to s/T^3 is apparently overseen. The same is valid for the comparison with other criteria (some of them are ignored, completely) and for ignoring the experimental measurements. The *ad hoc* value assigned to s/n is obviously not much robust than the other criterion. The unawareness of literature and underestimating or even ignoring the previous work would seem as a violation of the rules of scientific research.

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КОМЕНТАР ДО СТАТТІ
“ДОСЛІДЖЕННЯ ВІДНОСИН МНОЖИННОСТІ
І ВИХОДУ АДРОНІВ У ЗІТКНЕННЯХ ВАЖКИХ ІОНІВ”

Р е з ю м е

Олійниченко, Бугаєв і Сорін в [arXiv: 1204.0103 [hep-ph]] розглянули роль законів збереження для обговорюваних можливих недоліків температурних моделей, що використовуються для опису адронних множинностей, вимірюваних у центральних ядер-ядерних зіткненнях. Вони проаналізували критерії хімічного фрізауту і зробили висновок, що жоден з них не є надійним. На цій підставі вони запропонували новий критерій, взявши безпідставно величину 7,18 для ентропії на адрон, вважаючи, що вона незмінна в всьому діапазоні баріон-хімічних потенціалів. Через недостатню інформованість про поточну літературу, тема

постійної ентропії на адрон обговорювалася в *Fizika B* **18**, 141 (2009), *Europhys. Lett.* **75**, 420 (2006), *Phys. Rev. C* **85** 014908 (2012) та *nucl-ph/1306.3291*. Було показано, що постійна ентропія на адрон еквівалентна постійній ентропії, нормованої на температуру в кубі. Цей відомий критерій хімічного фрізауту було введено в *Europhys. Lett.* **75**, 420 (2006), *Nucl. Phys. A* **764**, 387 (2006). У цьому коментарі ми перераховуємо ігноровані літературні джерела та порівнюємо величину відношення ентропії до щільності числа частинок і два критерії: середня енергія, нормована на середнє число частинок, і постійна ентропія, нормована на температуру в кубі. Ці критерії суперечать експерименту. Ми розглянули фізику постійної ентропії, нормованої на щільність числа частинок, і зробили висновок, що це відношення не залишається постійним, особливо при великих хімічних потенціалах для енергій синхротронів AGS і SIS.