I.O. ANISIMOV, S.V. SOROKA
Radio Physics Faculty, Taras Shevchenko National University of Kyiv
(4g, Prosp. Glushkova, Kyiv 03022, Ukraine; e-mail: ioa@univ.kiev.ua)

PACS 52.35.Mw

FORMATION OF A DENSITY JUMP IN THE INHOMOGENEOUS ISOTHERMAL PLASMA INTERACTING WITH MODULATED ELECTRON BEAM

The results of analytical estimation and numerical simulation of the interaction of the inhomogeneous isothermal plasma with a modulated electron beam are presented. The formation of a fuzzy jump of the plasma density in the late phase of the interaction in a region, where the local plasma resonance point initially existed, is demonstrated. The jump is moving to the dense plasma, when the current density amplitude of the beam is large.

Keywords: modulated electron beam, local plasma resonance region, ponderomotive force, inhomogeneous plasma

1. Introduction

The problem of the interaction of a modulated electron beam with the inhomogeneous plasma attracts the considerable interest in the field of plasma electronics. The electron beam excites an HF electric field in a local plasma resonance region (LPRR) [1–3]. The deformation of the plasma density profile is caused by a ponderomotive force of the HF electric field and has an influence on the efficiency of a linear transformation of the beam modes into electromagnetic and Langmuir waves [4, 5]. This fact is important for the development of beam-plasma amplifiers and generators with the direct radiation, as well as for the interpretation of the radiation of modulated electron beams in the experiments in the upper atmosphere and the space.

A number of such deformation features have been studied in our previous papers using the numerical solution of modeling equations [2] and the simulation within the particles-in-cells (PIC) method [3, 8, 9]. A burst of the HF electric field was shown to be formed at the early time points in LPRR in a plasma with initially linear density profile. Then a density pit (caviton) arises in its position. The further scenario depends on the ratio of electron and ion temperatures in the plasma. If the electron temperature is much more than the ion one, the quasiperiodic caviton generation takes place in the LPRR. Ion-sound pulses propagate both forward and backward from the LPRR. The plasma density jump is formed on the LPRR position at the late time points in the isothermal plasma. The formation of a similar jump in the case of the interaction of the inhomogeneous plasma with an HF electric field was foreseen in [10, 11].

The aim of this study is the detailed examination of a deformation of the isotropic plasma density profile under the action of a modulated electron beam at the late time instants. In particular, the comparison of the results of analytical estimation and those of numerical simulation using the PIC method was carried out.

2. Model Description, Simulation Method, and Choice of Parameters

Let us consider a warm isotropic planarly stratified plasma with $T_e = T_i$ and the initially linear density profile

$$n(z) = n_c(\omega)\left(1 + \frac{z}{L}\right), \quad n_c(\omega) = \frac{m\omega^2}{4\pi e^2}, \qquad (1)$$

where L is the characteristic length of a plasma inhomogeneity. The modulated electron beam with initial velocity v_b is moving in parallel to the plasma density gradient. Initially, the beam density is modulated. The modulation is sinusoidal with frequency ω.

The computer simulation of this system was carried out, by using the PIC method. A program package

PDP1 [12] was used. The modified version of the package [13] makes it possible to specify the initial modulation of the electron beam and the initial inhomogeneous distribution of a plasma and to store and to analyze intermediate data of the simulation.

The parameters of the beam-plasma system were chosen so that density profile of the beam in LPRR was quasisinusoidal [14]. In all cases, the beam density was much less than the plasma density.

Consequently, the following simulation parameters were chosen:
– beam velocity – 7×10^9 cm/s;
– beam modulation frequency – 1.27×10^8 Hz (critical density – 2×10^8 cm^{-3});
– plasma density varies linearly within the range $(1 \div 3) \times 10^8$ cm^{-3} (electron plasma frequency $f_{pe} = (0.74 \div 2.20) \times 10^8$ Hz);
– length of the simulation interval – 200 cm (characteristic dimension of the inhomogeneity – 200 cm or $\omega L/c = 3.3$ in dimensionless units, $\Lambda \equiv L/r_{\rm D} = 800$);
– plasma temperature – 11 eV ($v_{T_e} = 2 \times 10^8$ cm/s);
– ratio of ion and electron masses $M/m = 1836$ corresponds to hydrogen.

These values are rather typical of laboratory experiments.

3. Analytical Estimations

The calculation of a stationary density profile of the plasma interacting with an HF electric field was carried out in [10]. It was based on the assumption of the conservation of the product

$$\varepsilon(z)E(z) = D_0, \tag{2}$$

where $\varepsilon(z)$ is the cold plasma electric permittivity, $E(z)$ is the longitudinal electric field, D_0 is the induction of the given HF electric field. In the model analyzed, this assumption is also satisfied if the alternating current of the beam is harmonic. Then, according to Ampere's circuital law, we have

$$\varepsilon(z)E(z) = i\frac{4\pi j}{\omega}, \tag{3}$$

where j is the beam alternating current density with amplitude j_m.

Thus, we can use the results of [10] in the case of a deformation of the density profile in the inhomogeneous plasma in LPRR by a modulated electron beam field.

Strictly speaking, the results of calculation [10] are valid only if the effective collision frequency at a non-zero plasma temperature [15]

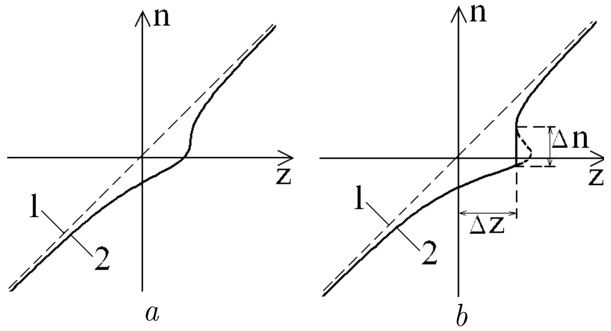

Fig. 1. Initial plasma density profile (*1*) and a stationary profile formed by the electric field excited by the given modulated beam current (*2*): *a* – beam current amplitude is less than the threshold value, *b* – beam current amplitude exceeds the threshold value (8). The coordinate origin corresponds to the local plasma resonance point of the initial density profile

$$\frac{\nu_{\rm eff}}{\omega} = \left(\frac{r_{\rm D}}{L}\right)^{2/3}, \tag{4}$$

where $r_{\rm D} = v_{T_e}/\omega$ is the Debye radius, is small in comparison with the collision frequency ν of electrons and heavy particles. But it is possible to obtain a number of simple estimations that are valid even if this condition is not satisfied [11].

The results in [10] can be used in the case analyzed if

$$j_m^2 \ll j_{\rm max}^2, \tag{5}$$

where

$$j_{\rm max} = \frac{\omega\, \nu_{\rm eff} m v_{T_e}}{2pie}. \tag{6}$$

For the chosen simulation parameters, $j_{\rm max} = 1.5$ A/m^2. According to [10], for

$$j_m < j_{\rm thr}, \tag{7}$$

where

$$j_{\rm thr} = \frac{8 j_{\rm max}}{3\sqrt{3}}\sqrt{\frac{\nu_{\rm eff}}{\omega}}, \tag{8}$$

the plasma density profile remains continuous. But, for

$$j_m > j_{\rm thr}, \tag{9}$$

the density jump is formed on this profile (Fig. 1).

For chosen simulation parameters, the threshold value of current density is $j_{\rm thr} = 0.25$ A/m^2.

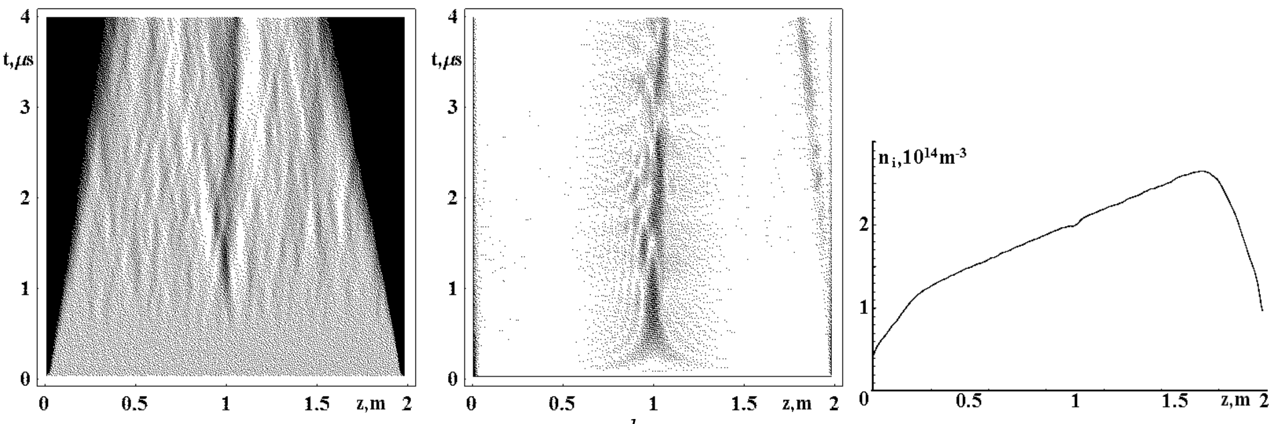

Fig. 2. Space-time distributions of ion density perturbations (a) and the electric field magnitude (b) and the spatial distribution of the ion density at the time moment $t = 3$ μs (c) for $j_m = 0.03$ mA/cm^2

In the initial-boundary problem, the electric field in LPRR grows from zero value. Consequently, the relation between the length Δz of the perturbed plasma region and the corresponding plasma density jump Δn (Fig. 1,b) is as follows:

$$\frac{\Delta z}{L} \sim \frac{\Delta n}{n_c(\omega)} \sim \left(\frac{2j_m}{j_{\max}} \frac{\nu_{\text{eff}}}{\omega}\right)^{2/3}. \tag{10}$$

The last relation can be rewritten as the equilibrium condition between the gas-kinetic pressure jump $\Delta p = k_{\text{B}} T \Delta n$ corresponding to the plasma density jump Δn and pressure jump caused by a jump of the HF electric field ponderomotive potential $\Phi = e^2|E|^2/(4m\omega^2)$:

$$\Delta p = n_c \Delta\Phi, \tag{11}$$

where

$$E \sim \frac{4\pi j_m}{\omega\Delta\varepsilon}, \quad \Delta\varepsilon \sim \frac{\Delta n}{n_c(\omega)}. \tag{12}$$

Let us estimate the density jump formation time. At the first stage, the cavity is formed during the time [9]

$$\tau_{\text{f}} \sim l_0/v_s, \tag{13}$$

where $v_s \simeq (k_{\text{B}}T/M)^{1/2}$ is the ion-acoustic velocity, $l_0 = L(\nu_{\text{eff}}/\omega)$ is the characteristic length of LPRR, and $(\nu_{\text{eff}}/\omega) = (L/r_{\text{D}})^{2/3}$ is the normalized effective collision frequency. At the same time, a couple of ion-acoustic waves running away from the cavity is excited. The characteristic damping time of these waves is

$$\tau_{\text{d}} \sim 1/\gamma, \tag{14}$$

where

$$\gamma \approx \gamma_i = \frac{0.38 k v_{T_i}}{(1 + k^2 r_{\text{D}}^2)^2} \tag{15}$$

is the decrement of ion-acoustic waves in the isothermal plasma [16]. Then the total formation time for a plasma density jump is

$$\tau = \tau_{\text{f}} + \tau_{\text{d}}. \tag{16}$$

Substituting the chosen simulation parameters, we obtain $\tau_{\text{f}} \sim 0.5$ μs, $\tau_{\text{d}} \sim 1.4$ μs, $\tau \sim 1.9$ μs.

4. Numerical Simulation Results

The simulation was carried out for four values of beam current density: $j_{m1} = 0.01$ mA/cm^2, when condition (7) is satisfied; $j_{m2} = 0.03$ mA/cm^2, when condition (9) is satisfied, but condition (4) is also satisfied; $j_{m3} = 0.1$ mA/cm^2 corresponds to the breaking point of condition (4); and $j_{m1} = 0.5$ mA/cm^2, when condition (4) considerably fails.

When $j_{m1} = 0.01$ mA/cm^2, the simulation results completely correspond to the linear regime [17]. The plasma density profile in LPRR is not perturbed.

Figure 2 presents the simulation results for $j_{m2} = 0.03$ mA/cm^2, when both conditions (4) and (9) are satisfied, i.e. the space-time distributions of ion density perturbations (Fig. 2,a) and the electric field magnitude (Fig. 2,b) and the spatial distribution of the ion density for the time moment $t = 3$ μs (Fig. 2,c). Dark regions in Fig. 2,a correspond to negative density perturbations, and light regions to positive ones. Dark regions in Fig. 2,b correspond to the larger field.

ISSN 2071-0194. Ukr. J. Phys. 2013. Vol. 58, No. 1

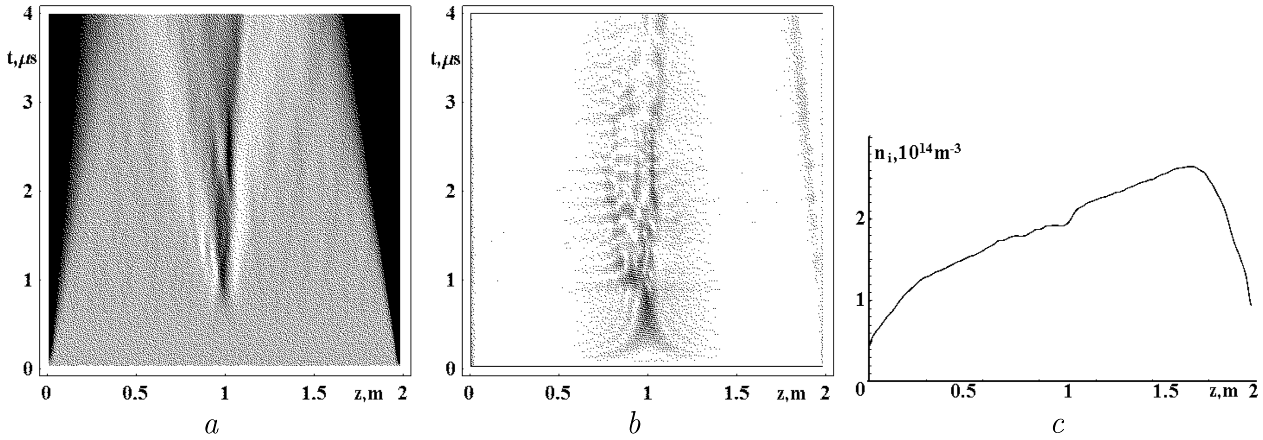

Fig. 3. The same as in Fig. 2, but for $j_m = 0.1$ mA/cm^2

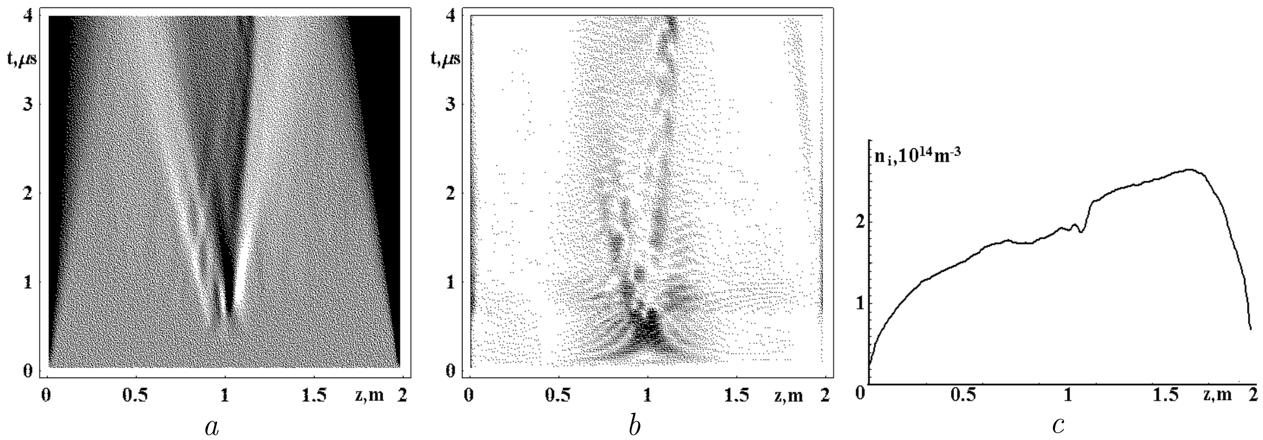

Fig. 4. The same as in Fig. 2, but for $j_m = 0.5$ mA/cm^2

One can see that, during the time interval of about 0.5 μs (compare with estimation of $\tau_{\rm f}$), the ion density cavity surrounded with two peaks is formed in the LPRR. During about 1.9 μs (compare with estimation of τ), a peak from the side of a subcritical plasma vanishes gradually, and a slightly fuzzy density jump is formed. Its position is shifted to the larger density region relative to the initial local plasma resonance point.

The comparison of Fig. 2,*a* and Fig. 2,*b* shows that local bursts of the electric field are excited at local maxima of the subcritical plasma density. These bursts give rise to the formation of new plasma density minima. These effects are similar to those observed in a plasma with hot electrons [8]. They can result in some irregularity of the plasma density profile in a vicinity of the density jump in the subcritical plasma region (Fig. 2,*c*).

One can also see a non-stationary perturbation of the electron density profile corresponding to a Langmuir wave running to the subcritical plasma region. Thereby, the strictly stationary plasma density profile predicted in [11] is not observed.

The width of the formed step is approximately 6 cm, which is close to the analytical estimate (9 cm) based on (10).

Figure 3 presents the results of simulation for $j_{m3} = 0.1$ mA/cm^2. The main difference from the previous case is that the density jump position shifts now to the dense plasma with time. One can compare this result with the well-known effect of the plasma barriers' transillumination by strong electromagnetic waves [18]. According to the simulation results, the jump propagation velocity is 2.6×10^6 cm/s, which is of the order of the ion thermal velocity ($v_{T_i} = 4.5 \times 10^6$ cm/s). The jump formation time in this case is 3 μs. This value approximately corresponds to the analytical estimate. It is worth to note that, although relation (10) is not already valid, the value of density "step" calculated from this relation (19.6 cm) is in good agreement with the results of simulation (20.4 cm). Similarly to the previous case, local bursts of the electric field are observed on the local maxima of the subcritical plasma after the cavity destruction.

The results of simulation for $j_{m3} = 0.5$ mA/cm^2 (Fig. 4) qualitatively replicate those for $j_{m3} = 0.1$ mA/cm^2. The difference is only in the disappearance of field bursts on local maxima of the subcritical plasma density. An appreciable field burst is observed only at the initial time, while the plasma density profile remains linear (Fig. 4,*c*). The formation time of the plasma density jump is 3 μs. In this case, the analytically calculated width of the "step" (57 cm) is still in good agreement with the result of simulation (48 cm). The density jump velocity (4×10^6 cm/s) has the same order of magnitude as that in the previous case.

The increase of the jump formation time with the beam current density amplitude can be explained by an increase of the cavity formation time at the first stage of the process due to the growth of the cavity. Ion-acoustic pulses with larger amplitude also have a longer damping time. Both reasons were not taken into account in the simplified estimation (16).

5. Conclusion

1. The density cavity is formed in the LPRR at the first stage of the interaction of a modulated electron beam with the inhomogeneous isothermal plasma. Then its edge from the side of the subcritical plasma smears, and the fuzzy density jump is formed. An appreciable deformation of the plasma density profile starts from the threshold value of current density amplitude defined by (8).
2. The first stage of this process hypothetically corresponds to the self-consistent solution for the plasma density and the field profile obtained in [2], when the plasma density profile perturbation is caused by the structure of the electric field. This field is formed by the beating between electric fields of the modulated beam and a Langmuir wave excited by this beam. Obviously, this solution is unstable, because the corresponding density distribution collapses. The slightly fuzzy density jump is formed in its position in the isothermal plasma. This jump corresponds to the stationary solution obtained in [10].
3. The excitation of electric field bursts on the local maxima of the subcritical plasma density can cause irregularities of the plasma density profile in the region before the density jump. The similar effect takes place in a plasma with hot electrons [8].
4. If the beam current density amplitude exceeds the threshold value (4) and the conditions [10] for estimations to be valid do not hold, the gradual shift of the density jump to the dense plasma with the velocity of the order of the ion thermal velocity is observed. This result corresponds to the well-known effect of the plasma barriers forcing by strong electromagnetic waves.

The authors are grateful to V.B. Gil'denburg (Nizhnii Novgorod, Russia) for the fruitful discussion of a part of this paper.

1. I.O. Anisimov and D.G. Stefanovsky, Ukr. Fiz. Zh. **33**, 38 (1988).
2. I.O. Anisimov, O.A. Borisov, O.I. Kelnyk, and S.V. Soroka, Probl. of Atom. Sci. and Techn. Ser. Plasma Physics **5**, 107 (2002).
3. I.O. Anisimov, O.I. Kelnyk, T.V. Siversky, and S.V. Soroka, Probl. of Atom. Sci. and Techn. Ser. Plasma Electr. New Accel. Meth. **4**, 91 (2003).
4. I.O. Anisimov and S.M. Levitsky, Ukr. Fiz. Zh. **9**, 1336 (1989).
5. I.O. Anisimov, T.V. Siversky, and S.V. Soroka, Bull. Kiev Univ. Ser. Phys. Math. **3**, 307 (2004).
6. E.G. Shustin and I.F. Kharchenko, *Excitation and Output of a Microwave Signal in Plasma Amplifiers and Generators*, preprint (Institute of Radioelectronics of the AS of the USSR, Moscow, 1968).
7. I.O. Anisimov and S.M. Levitsky, Plasma Phys. **9**, 824 (1994).
8. I.O. Anisimov, O.I. Kelnyk, and S.V. Soroka, Bull. Kiev Univ. Ser. Phys. Math. **1**, 297 (2004).
9. I.O. Anisimov and S.V. Soroka, Bull. Kiev Univ. Ser. Phys. Math. **2**, 385 (2004).
10. V.B. Gil'denburg, JETPh **46**, 2156 (1964).
11. V.B. Gil'denburg and G.M. Fraiman, JETPh **69**, 1601 (1975).
12. Ch.K. Birdsall and A.B. Langdon, *Plasma Physics via Computer Simulation* (McGraw-Hill, New York, 1985).
13. I.O. Anisimov, O.I. Kelnyk, S.M. Levitsky, D. Sasyuk, T. Siversky, and S.V. Soroka, Bull. Kiev Univ. Ser. Radiophys. Electr. **6**, 48 (2004).
14. I.O. Anisimov, O.I. Kelnyk, and S.V. Soroka, in *Proceedings of the 10th Int. Conference and School on Plasma Physics and Controlled Fusion, Alushta, Ukraine, 2004*, p. 141.

15. V.L. Ginzburg, *Propagation of Electromagnetic Waves in Plasma* (Pergamon, Oxford, 1970).
16. A.I. Akhiezer, I.A. Akhiezer, R.V. Polovin, A.G. Sitenko, and K.N. Stepanov, *Plasma Electrodynamics* (Pergamon Press, New York, 1975), Vols. 1-2.
17. I.O. Anisimov, O.I. Kelnyk, T.V. Siversky, and S.V. Soroka, Bull. Kiev Univ. Ser. Phys. Math. **1**, 229 (2003).
18. *The Interaction of Strong Electromagnetic Waves in Collisionless Plasma*, edited by A.G. Litvak, (Institute of Applied Physics of the AS of the USSR, Gor'kii, 1980) (in Russian).

Received 23.01.12

І.О. Анісімов, С.В. Сорока

ФОРМУВАННЯ СТРИБКА
КОНЦЕНТРАЦІЇ В НЕОДНОРІДНІЙ
ІЗОТЕРМІЧНІЙ ПЛАЗМІ ПРИ ВЗАЄМОДІЇ
З МОДУЛЬОВАНИМ ЕЛЕКТРОННИМ ПУЧКОМ

Р е з ю м е

У роботі наведено аналітичні оцінки та результати числових розрахунків взаємодії неоднорідної ізотермічної плазми з модульованим електронним пучком. Показано, що на пізніх стадіях взаємодії в області, де первісно існувала точка локального плазмового резонансу, формується розмитий стрибок концентрації плазми. При великих амплітудах густини струму пучка стрибок починає рухатися в бік щільної плазми.