THE LOCALIZATION OF A SPHERICAL FERROMAGNETIC MICROPARTICLE IN AN OSCILLATING MAGNETIC FIELD IN A LIQUID STREAM

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The model of magnetic tweezers in a rapidly oscillating alternating magnetic field, which allows one to estimate the movement of a ferromagnetic magnetically soft microparticle in a magnetic trap and to localize it in a liquid, is offered. The estimations of the strength and the frequency of an oscillating alternating magnetic field and the size of a spherical microparticle are carried out. The movement trajectory of a spherical microparticle is calculated as well.

1. Introduction

The localization of microparticles by means of a magnetic trap is one of the perspective methods of researches, which finds applications in different areas of science [1,2,3]. In comparison with other methods, the possibility to manipulate by separate microparticles allows one to investigate the physical properties of biological cells in natural media, to study the mechanical and adsorption properties of DNA, *etc.* [4]. However, for maintenance of the conditions necessary to alive cells, such experiments are often carried out in a flowing liquid [5-7]. Therefore, during the design and in the use of magnetic traps, one needs to consider the additional force caused by the available hydraulic stream. Thus, the development of new magnetic traps and the improvement of existing ones with regard for the forces acting on a microparticle are actual scientific problems.

To localize a ferromagnetic microparticle, which is under the influence of a magnetic field, the condition of existence of a point, where the potential energy reaches its local minimum, i.e. the potential hole appears, must be satisfied. Because it is impossible to reach such a local minimum for a ferromagnetic microparticle in a constant magnetic field [8], we use, in the present work, a variable magnetic field analogously to the proposition of Kapitsa [9] for the stabilization of a mathematical pendulum in the upper unstable position. As it will be shown further, this gives possibility to solve the problem of the localization of a spherical ferromagnetic microparticle by means of a variable oscillating magnetic field by creating an effective local minimum.

2. Theory

To research the possibility of the practical realization of a magnetic field with a local minimum of energy, first of all, we will consider a ferromagnetic microparticle in a magnetic field, which depends on a spatial coordinate $\mathbf{r}$. Then the energy U of this microparticle is [10]

$$U(\mathbf{r}) = -\mathbf{MH},$$

where $\mathbf{H}$ is the intensity of the magnetic field, and $\mathbf{M}$ is the microparticle magnetic moment.

In the first approximation, we consider that if the constant z-component of the magnetic field is stronger as compared with the x- and y-components, then the microparticle is magnetized homogeneously along the z axis, and its size is much less, than the characteristic scale of a change of the magnetic field. The magnetic moment of the microparticle M_0 is fixed along Oz due to the magnetic bias. Then, in the non-uniform oscillating magnetic field, the oscillating force

$$\mathbf{f}(\mathbf{r},t) = -\frac{d}{d\mathbf{r}}\left(M_0 H_z\left(x,y,z,t\right)\right)$$

appears. Here, the z-component of the external magnetic field is defined as $H_z = h_z(x,y,z)\cos(\omega t)$.

Taking the above reasoning into account, the equation of motion will have the well-known form

$$m\ddot{\mathbf{r}} + \alpha(\dot{\mathbf{r}} - \mathbf{V}_0) = \mathbf{f}(\mathbf{r},t), \tag{1}$$

where m is the microparticle weight, α is the dissipative multiplier, $\dot{\mathbf{r}}$ is the first derivative of the spatial coordinate with respect to the time, $\ddot{\mathbf{r}}$ is the second derivative with respect to the time of the spatial coordinate, and $\mathbf{V}_0$ is the liquid stream velocity. According to the above-introduced notations, we have

$$\mathbf{f}(\mathbf{r},t) = -\frac{dU}{d\mathbf{r}} = M_0 \frac{d}{d\mathbf{r}} h_z(x,y,z)\cos(\omega t), \tag{2}$$

where $\frac{d}{d\mathbf{r}} h_z(x,y,z)$ is a coordinate function.

The analysis of the motion of a microparticle can be simplified similarly to Kapitsa's problem [9]. We assume that the microparticle moves along a smooth trajectory with simultaneous small oscillations (with a frequency ω) around it. In this case, the function $\mathbf{r}$ can be represented like a sum

$$\mathbf{r} = \mathbf{R} + \boldsymbol{\xi}, \tag{3}$$

where $\mathbf{R}$ is the function, which describes the "smooth" movement of the particle, and $\boldsymbol{\xi}$ is the rapidly oscillating component, $|\xi| << |\mathbf{R}|$.

Let us substitute (3) in (1). By expanding $\frac{d}{d\mathbf{r}}U(\mathbf{r})$ and $\mathbf{f}(\mathbf{r},t)$ in small displacements ξ, we obtain, to within the terms squared in ξ,

$$m\ddot{\mathbf{R}} + \alpha\left(\dot{\mathbf{R}} - \mathbf{V}_0\right) + m\ddot{\boldsymbol{\xi}} + \alpha\dot{\boldsymbol{\xi}} = \mathbf{f}(\mathbf{R},t) + \frac{\partial}{\partial x_i}\mathbf{f}(\mathbf{R},t)\,\boldsymbol{\xi}_i, \tag{4}$$

where x_i are the components of the spatial vector $\mathbf{r}$, $i = 1,2,3$.

By separating the oscillating terms from expression (4), we obtain

$$m\ddot{\boldsymbol{\xi}} + \alpha\dot{\boldsymbol{\xi}} = \mathbf{f}(\mathbf{R},t). \tag{5}$$

The solution of this differential equation is searched in the form

$$\boldsymbol{\xi} = \mathbf{A}\cos\omega t + \mathbf{B}\sin\omega t. \tag{6}$$

Differentiating Eq. (6) and substituting the result in (5), we obtain

$$\begin{cases} -\mathbf{A}m\omega^2 + \mathbf{B}\alpha\omega = M_0 \frac{d}{d\mathbf{r}} h_z(x,y,z), \\ -\mathbf{B}m\omega^2 - \mathbf{A}\alpha\omega = 0. \end{cases} \tag{7}$$

By solving (7), we have the coefficients $\mathbf{A}$ and $\mathbf{B}$:

$$\mathbf{A} = -\frac{mM_0 \frac{d}{d\mathbf{r}} h_z(x,y,z)}{m^2\omega^2 + \alpha^2}, \quad \mathbf{B} = \frac{\alpha M_0 \frac{d}{d\mathbf{r}} h_z(x,y,z)}{m^2\omega^3 + \alpha^2\omega}.$$

By substituting the coefficients $\mathbf{A}$ and $\mathbf{B}$ in (6), we obtain the expression for the oscillating component:

$$\boldsymbol{\xi}(t) = -\frac{mM_0 \frac{d}{d\mathbf{r}} h_z(x,y,z)}{m^2\omega^2 + \alpha^2} \times$$

$$\times \cos\omega t + \frac{\alpha M_0 \frac{d}{d\mathbf{r}} h_z(x,y,z)}{m^2\omega^3 + \alpha^2\omega}\sin\omega t. \tag{8}$$

Let $|\sigma| = \frac{mM_0^2}{4(m^2\omega^2+\alpha^2)}$. Then the coefficients $\mathbf{A}$ and $\mathbf{B}$ take the form

$$\mathbf{A} = \frac{4|\sigma| \frac{d}{d\mathbf{r}} h_z(x,y,z)}{M_0}, \quad \mathbf{B} = \frac{4\alpha|\sigma| \frac{d}{d\mathbf{r}} h_z(x,y,z)}{mM_0\omega}.$$

Let us rewrite expression (6) as

$$\boldsymbol{\xi} = (A_0\cos\omega t + B_0\sin\omega t)\frac{d}{d\mathbf{r}} h_z(x,y,z), \tag{9}$$

where $A_0 = \frac{4|\sigma|}{M_0}$, $B_0 = \frac{4|\sigma|}{M_0}\frac{\alpha}{m\omega}$.

Then relation (9) yields

$$\boldsymbol{\xi} = \sqrt{A_0^2 + B_0^2}\left(\frac{A_0}{\sqrt{A_0^2+B_0^2}}\cos\omega t + \right.$$

$$\left. + \frac{B_0}{\sqrt{A_0^2+B_0^2}}\sin\omega t\right)\frac{d}{d\mathbf{r}} h_z(x,y,z),$$

where $\cos\alpha_0 = \frac{A_0}{\sqrt{A_0^2+B_0^2}} = \frac{m\omega}{\sqrt{m^2\omega^2+\alpha^2}}$.

After the contraction of the previous expression by trigonometric transformations, we obtain

$$\boldsymbol{\xi} = \sqrt{A_0^2 + B_0^2}\cos(\omega t - \alpha_0)\frac{d}{d\mathbf{r}} h_z(x,y,z), \tag{10}$$

where α_0 is a phase shift. After the substitution of the coefficients $A_0 = \frac{4|\sigma|}{M_0}$, $B_0 = \frac{4|\sigma|}{M_0}\frac{\alpha}{m\omega}$, and $|\sigma| = \frac{mM_0^2}{4(m^2\omega^2+\alpha^2)}$, the final equation for $\boldsymbol{\xi}$ takes the form

$$\boldsymbol{\xi} = \frac{M_0}{\omega\sqrt{m^2\omega^2+\alpha^2}}\cos(\omega t - \alpha_0)\frac{d}{d\mathbf{r}} h_z(x,y,z)$$

or

$$\boldsymbol{\xi} = \frac{4\delta}{M_0}\cos(\omega t - \alpha_0)\frac{d}{d\mathbf{r}} h_z(x,y,z), \tag{11}$$

where $\delta = \frac{M_0^2}{4\omega\sqrt{m^2\omega^2+\alpha^2}}$.

Let us substitute (11) in the equation of motion (4). After averaging, we have

$$\frac{\partial}{\partial x_i}\mathbf{f}\left(\mathbf{R},t\right)\boldsymbol{\xi}_i = 2\delta\frac{d}{d\mathbf{r}}\left[\frac{d}{d\mathbf{r}}h_z\left(x,y,z\right)\right]^2\times$$

$$\times\left[\overline{\cos^2\left(\omega t\right)}\cos\alpha_0 - \overline{\cos\left(\omega t\right)\sin\left(\omega t\right)}\sin\alpha_0\right],$$

whence

$$\frac{\partial}{\partial x_i}\mathbf{f}\left(\mathbf{R},t\right)\boldsymbol{\xi}_i = \delta\frac{d}{d\mathbf{R}}\left[\frac{d}{d\mathbf{R}}h_z\left(x,y,z\right)\right]^2\times$$

$$\times\cos\alpha_0 = \delta_0\frac{d}{d\mathbf{R}}\left[\frac{d}{d\mathbf{R}}h_z\left(x,y,z\right)\right]^2,$$

where $\delta_0 = \frac{mM_0^2}{4(m^2\omega^2+\alpha^2)}$.

Then the equations describing the averaged motion become

$$m\ddot{\mathbf{R}} + \alpha\left(\dot{\mathbf{R}} - \mathbf{V}\right) = -\delta_0\frac{d}{d\mathbf{R}}\left(\frac{dh_z}{d\mathbf{R}}\right)^2. \quad (12)$$

Let us set a coordinate dependence $h_z\left(x,y,z\right) = a\left(-z^2 + \frac{x^2+y^2}{3}\right)$. Then, for the stationary point Z_0, we have

$$\alpha V_{0z} = 8\delta_0 a^2 Z_0.$$

Hence,

$$Z_0 = \frac{\alpha V_{0z}}{8\delta_0 a^2}. \quad (13)$$

Analogously for the stationary points X_0=Y_0, we have $\alpha V_{0x} = \frac{8}{9}\delta_0 a^2 X_0$.

Hence,

$$X_0 = Y_0 = \frac{9}{8}\frac{\alpha V_{0x}}{\delta_0 a^2}.$$

The final relations for the stationary points are as follows:

$$X_0 = Y_0 = \frac{9}{2}\frac{\alpha V_{0x}\left(m^2\omega^2+\alpha^2\right)}{a^2 M_0^2 m},$$

$$Z_0 = \frac{1}{2}\frac{\alpha V_{0x}\left(m^2\omega^2+\alpha^2\right)}{a^2 M_0^2 m}.$$

Let us consider the oscillating amplitudes for these stationary points. We take the expression for the small oscillating amplitudes around the stationary point $X \approx X_0$ from (11) for $\boldsymbol{\xi}$:

$$\xi_x = \xi_y = \frac{4\delta}{M_0}\frac{d}{dx}h_z\left(x,y,z\right) = \frac{3\alpha V_{0x}\sqrt{m^2\omega^2+\alpha^2}}{m\omega a M_0},$$

$$\xi_z = \frac{4\delta}{M_0}\frac{d}{dz}h_z\left(x,y,z\right) = \frac{3\alpha V_{0x}\sqrt{m^2\omega^2+\alpha^2}}{m\omega a M_0}.$$

From (12), we find the frequencies of small oscillations around the stationary point along the axis $0x$. We have

$$m\ddot{x}_1 + \alpha\left(\dot{x}_1 - V_{0x}\right) = -\frac{8}{9}a^2\delta_0 X_0 - \frac{8}{9}a^2\delta_0 x_1,$$

whence we obtain

$$\ddot{x}_1 + \frac{\alpha}{m}\dot{x}_1 + \frac{8}{9}\frac{a^2\delta_0}{m}x_1 = 0.$$

The frequency of the small oscillations

$$\Omega_x = \sqrt{\frac{8}{9}\frac{a^2\delta_0}{m} - \left(\frac{\alpha}{2m}\right)^2}$$

with $\frac{\alpha}{2} \ll \frac{8a^2\delta_0}{9}$. That's why $\Omega_x \approx \Omega_y \approx \sqrt{\frac{8}{9}\frac{a^2\delta_0}{m}} \approx \sqrt{\frac{2}{9}\frac{aM_0}{m\omega}}$ according to $\lim\limits_{\alpha\to 0}\delta_0 \to \frac{M_0^2}{4\omega^2 m}$.

Since $\frac{\alpha}{2} \ll 8a^2\delta_0$, we have $\Omega_z \approx \sqrt{\frac{8a^2\delta_0}{m}} \approx \sqrt{2}\frac{aM_0}{m\omega}$.

The condition for the application of Kapitsa's method requires that the frequency of slow small oscillations be much less than the frequency of fast oscillations for $\boldsymbol{\xi}$, i.e., $\Omega \ll \omega$. Than

$$\sqrt{\frac{2}{9}\frac{aM_0}{m\omega}} \ll \omega, \quad (14)$$

which yields $\omega \gg \sqrt{\frac{aM_0}{m}}$.

3. Numerical Modeling of a Magnetic Trap

The microparticle motion equation in the magnetic field (3) underlies the magnetic trap model. Let us do the numerical simulation of a magnetic trap with the oscillating force (3). The equation of motion of a microparticle in the magnetic field reads

$$m\ddot{\mathbf{r}} + \alpha(\dot{\mathbf{r}} - \mathbf{V}_0) = M_0\frac{d}{d\mathbf{r}}h_z\left(x,y,z\right)\cos(\omega t). \quad (15)$$

Let us consider the case with a spherical microparticle. To simplify the magnetic trap model, we normalize Eq.

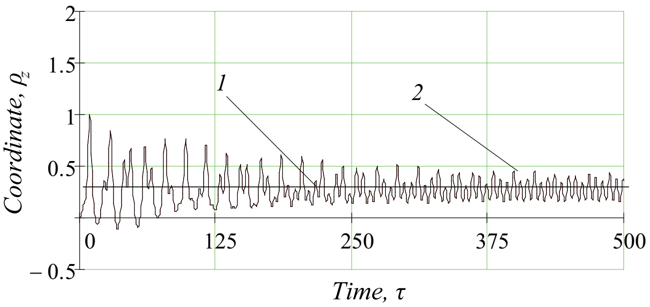

Fig. 1. Trajectory of motion of a microparticle along the axis Oz in a stream of blood. *1* – the stationary point of the trajectory calculated by the analytical expression (13), *2* – the trajectory calculated based on the results of numerical calculations of Eq. (14). (Coordinates are dimensionless)

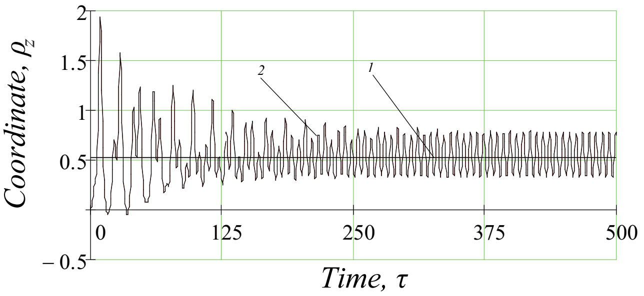

Fig. 2. Trajectory of motion of a microparticle along the axis Oz in a stream of water. *1* – the stationary point of the trajectory calculated by the analytical expression (13), *2* – the trajectory calculated based on the results of numerical calculations of Eq. (14). (Coordinates are dimensionless)

(13), by making such substitutions: $\mathbf{r} = \beta\boldsymbol{\rho}$ (where $\boldsymbol{\rho}$ is the radius-vector), $\omega t = \tau$, and $\alpha = 6\pi\eta b$ (η is the liquid dynamic viscosity factor) [11]. The final normalized equation of motion of the microparticle is

$$\ddot{\boldsymbol{\rho}} + K(\dot{\boldsymbol{\rho}} - \upsilon_0) = D\frac{d}{d\boldsymbol{\rho}} h_z\left(\rho_x, \rho_y, \rho_z\right)\cos(\tau),$$

where $K = \frac{\alpha}{m\omega}$ and $D = \frac{2}{9}\frac{M_0 a}{m\omega^2}$ are controlling dimensionless parameters, which determine the dynamic movement of the microparticle.

We consider that the velocity vector $\mathbf{V} = \dot{\mathbf{r}}$ of microparticle's translational motion at every time moment τ directed along the force direction is defined by the expression on the right-hand side of Eq. (15).

As the bright example, we consider the behavior of a magnetic trap with regard for the conditions which are inherent in a blood vessel of a human bloodstream. A vein will be served as an example. The vein diameter is about 0.5 cm, the bloodstream velocity is about 20–50 cm/s [12]. The system parameters: $b = 0.05$ cm, $\rho_{\mathrm{Fe}} = 7.87$ g/cm^3, $\mu_{\mathrm{Fe}} = 1600$ Oe, $\omega = 100$ Hz, $a = 10$ g/cm^{5}s^2, $z_0 = 0$, $V_{z0} = 2$ cm/s, $\beta = 2$. The dynamic blood viscosity $\eta = 5 \times 10^{-3}$ g/cm·s. The microparticle radius is 10% of the vein diameter. A movement trajectories of an iron spherical microparticle under the condition that $H_0 \gg 3$ Oe, are shown in Figs. 1 and 2.

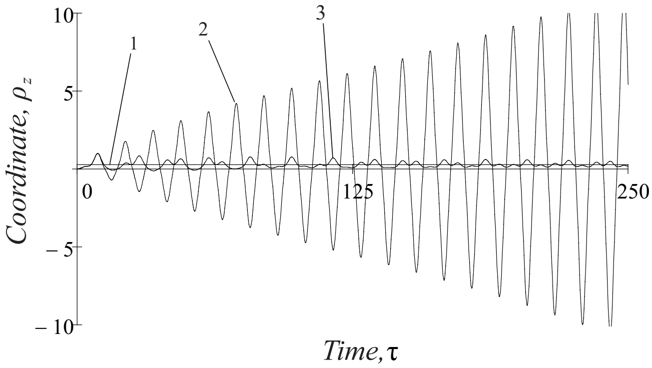

Fig. 3. Trajectory of motion of a microparticle along the axis Oz in a stream of blood. *1* – the stationary point of the trajectory calculated by the analytical expression (13), *2* – the trajectory calculated on the basis of the results of numerical calculations of Eq. (15), when condition (14) is not valid, *3* – the trajectory calculated, when condition (14) holds, on the basis the results of numerical calculations of Eq. (15). (Coordinates are dimensionless)

Analyzing the microparticle coordinates on the basis of the results given above, it is clear that if the microparticle is injected into a vein under conditions of a bloodstream (Fig. 1), the localization of the microparticle takes 5 s, the maximum amplitude of oscillations around the stationary point with a coordinate of 0.28 cm is 0.88 cm and 0.28 cm in the stable oscillating regime – takes 3 s. After changing the blood viscosity coefficient by the water viscosity coefficient $\eta = 10^{-2}$ g/sm·s (Fig. 2), the position of the stationary point becomes equal to 0.55 cm, and the maximum amplitude of oscillations around the stationary point has increased to 1.88 cm and to 0.49 in the stable oscillating regime. Approaching the stabilized regime takes 2.5 s.

The figures above represent the microparticle trajectory motion in a liquid stream, when condition (14) is satisfied. The case where condition (14) is not valid is given in Fig. 3.

In Fig. 3, we can see that, if condition (14) is satisfied, the microparticle localization takes place (trajectory 3) in a vicinity of the stationary point Z_0 (line 1), and the oscillation amplitude stay within the narrow range of the Z_0 area. In addition, the oscillations occur in the stable regime with a clear damping after 4 s, which testifies that the microparticle has been trapped. In the case where condition (14) is not valid, the influence of the magnetic trap on the magnetic particle is insignificant, and the oscillation amplitude goes beyond the permissible range.

The analysis of the plots shows that, in the chosen range of initial data, the system really manifests the properties of a magnetic trap, by keeping the magnetic microparticle in a localized area of the magnetic field. It is clear that the time required for the system to approach the stable localized regime is changeable and depends on the initial conditions, which depend, in turn, on parameters of the system such as the microparticle radius, liquid dynamic viscosity, material density, external magnetic field frequency *etc.* Moreover, a negligible change of one of the parameters causes to the significant changes, for instance, of the amplitude and the coordinate, where the oscillation of the the microparticle takes place.

The figures show that the switching-on of the magnetic field leads to the localization of a microparticle. It carries out oscillations within a narrow interval of coordinates around the local position. The range of radii of the spherical microparticle, for which it can be localized, is 3×10^{-2}–6×10^{-2} cm (6–12% of the vein diameter). In this case, the particle localization is possible within $0 < z < 30$ cm and $0 < x < 0.5$ cm. The time of localization is from 2 s to 5 s for the field $H_0 \gg 6$ Oe at a particle radius of 3×10^{-2} cm and $H_0 \gg 2$ Oe at a particle radius of 6×10^{-2} cm.

4. Conclusion

The analytical and numerical results of studies of the dynamics of the motion of a ferromagnetic microparticle show that it is possible to reach a local minimum of the potential energy, by using the method of rapidly oscillating fields [9]. This allows one to carry out the localization of a spherical ferromagnetic microparticle at the given spatial point by a magnetic trap for a wide range of parameters such as the microparticle radius, density, saturation magnetization of a microparticle material, oscillation frequency of a magnetic field, and speed and viscosity of flowing blood in a vein. The localization of a ferromagnetic microparticle can be reached at the magnetic field frequency $\omega = 100$ Hz and its strength $H_0 \gg 2$ Oe for vessels, whose diameters are close to the diameter of a human vein.

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ЛОКАЛIЗАЦIЯ СФЕРИЧНОЇ ФЕРОМАГНIТНОЇ МIКРОЧАСТИНКИ ПIД ДIЄЮ ШВИДКО ОСЦИЛЮЮЧОГО МАГНIТНОГО ПОЛЯ В ПОТОЦI РIДИНИ

О.Ю. Горобець, М.М. Потьомкiн

Р е з ю м е

Запропоновано модель магнiтного пiнцета в зовнiшньому швидко осцилюючому магнiтному полi, яка дозволяє дослiджувати рух магнiтом'якої феромагнiтної мiкрочастинки в магнiтнiй пасцi та локалiзувати її в умовах потоку рiдини. Зроблено оцiнки сили та частоти зовнiшнього швидко осцилюючого магнiтного поля, а також розмiри сферичної мiкрочастинки. Розраховано траєкторiю руху сферичної мiкрочастинки.