INFLUENCE OF LIGHT-BEAM CONFINEMENT ON THE HYSTERESIS AT THE FRIEDERICKSZ TRANSITION IN A NEMATIC LIQUID CRYSTAL CELL

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PACS 61.30.Cz; 61.30.Gd; 61.30.Hn; 64.70.Md

The influence of the shape and the finite transverse size of an incident light beam on the hysteresis at the light-induced Friedericksz transition in a homeotropically oriented nematic liquid crystal cell has been considered. The cases of light beams confined in one and two dimensions have been examined. The orientational instability threshold and the jump of a director deviation angle at the transition, as well as their dependences on the transverse size of the incident light beam, were found in the regimes of incident light intensity growth and reduction. Conditions, under which the width of the hysteresis loop is maximal, have been determined.

1. Introduction

An enhanced interest in the phenomena of a director reorientation in nematic liquid crystal (NLC) cells subjected to the action of external light fields and, in particular in the threshold director reorientation known as the light-induced Friedericksz transition (LIFT) [1, 2], is associated with a wide application of NLC cells in various electron-optical devices. An important characteristic of the LIFT at that is the light intensity threshold, above which the light beam incident on a cell induces the transformation of the NLC director state from a homogeneously oriented into an inhomogeneous one or *vice versa* [3–7]. As was shown in works [1, 8], the LIFT can be accompanied by a hysteresis phenomenon, i.e. the threshold magnitude $I_{\rm th}$ of the director reorientation, when the incident light intensity increases, can differ from the threshold magnitude $I'_{\rm th}$ of the inverse director reorientation, when the light intensity diminishes. In this case, when the light intensity achieves the corresponding threshold value, the NLC director abruptly changes its state from the homogeneously oriented into an inhomogeneous one and *vice versa* [8, 9]. The LIFT hysteresis was observed experimentally under the action of external static magnetic [10] and quasistatic electric [11] fields. However, the conditions needed for the LIFT hysteresis to be observed for sure remain not clear enough till now.

As a rule, while analyzing the LIFT, light beams that fall onto a cell are assumed unconfined, with the intensity being distributed uniformly. The effects of the light-beam shape and the finiteness of the corresponding transverse dimension were considered in works [1, 2, 12]; however, the magnitude of LIFT threshold was examined there only in the regime where the incident light intensity increased. In work [13], the influence of the light beam transverse finiteness on the LIFT hysteresis was considered as well, but only in the special case of a one-dimensional intensity distribution.

In this work, the influence of the parameters of confined light beams on the LIFT hysteresis is studied in order to find the optimal conditions for its experimental observation. Both one- and two-dimensional confinements of a light beam are analyzed.

2. Equations Describing the NLC Director in the Field of Light Beams Confined in one Dimension

Let us consider a homeotropic plane-parallel NLC cell located between the planes $z = 0$ and $z = L$, with absolutely rigid boundary conditions. Let a light wave linearly polarized along the Ox axis fall normally along the Oz axis onto the cell. For the sake of definiteness, let us consider the incident light beam to be confined along the Oy axis. When the threshold of orientational instability is achieved, the reorientation of the director takes place in the xOz plane [2], whereas the system remains uniform along the Ox axis. Therefore, the expression for the director in the cell bulk should be sought in the form

$$\mathbf{n} = \mathbf{e}_x \cdot \sin\varphi(y,z) + \mathbf{e}_z \cdot \cos\varphi(y,z), \tag{1}$$

where $\mathbf{e}_x$ and $\mathbf{e}_z$ are the unit vectors of the Cartesian coordinate system, φ is the angle of the director deviation from its initial unperturbed direction (along the Oz axis).

Let the light intensity distribution $I(y)$ over the transverse cross-section of an incident light beam have the form $I(y) = I_0 f(y)$, where I_0 is the scale factor, and the function $f(y)$ defines the beam shape.

Minimizing the free energy of a NLC cell with respect to the angle φ and, in so doing, using the solution of Maxwell equations for the electric field of the incident light wave obtained in the geometric optics approximation, we arrive at the following stationary equation [13]:

$$(1 - k\sin^2\varphi)\frac{\partial^2\varphi}{\partial z^2} + m\frac{\partial^2\varphi}{\partial y^2} - k\sin\varphi\cos\varphi\left(\frac{\partial\varphi}{\partial z}\right)^2 +$$

$$+\frac{\pi^2}{L^2}\frac{I_0}{I_{\mathrm{Fr}}}\frac{\varepsilon_\parallel^{3/2} f(y)\sin\varphi\cos\varphi}{(\varepsilon_\perp + \varepsilon_a\cos^2\varphi)^{3/2}} = 0\,, \tag{2}$$

where $k = (K_3 - K_1)/K_3$, $m = K_2/K_3$, and $I_{\mathrm{Fr}} = \frac{8\pi^3\varepsilon_\parallel K_3}{\varepsilon_a\varepsilon_\perp L^2}$ is the threshold of the Fredericksz transition in the field of a uniform spatially unconfined light beam, provided that the anchoring between the director and the cell surface is absolutely rigid [1].

Let the director deviations can be considered small in a vicinity of the orientational instability threshold. Let φ_m mean the maximum angle of the director deviation, which is reached in the middle of the cell and in the center of the light beam, i.e. $\varphi_m = \varphi(y=0, z=L/2)$. The value of φ_m evidently depends on I_0: $\varphi_m = \varphi_m(I_0)$. It is convenient, however, to examine the inverse relation $I_0 = I_0(\varphi_m)$. This relation is single-valued and, since the angle φ_m is small, can be presented as a series

$$I_0/I_{\mathrm{Fr}} = \rho + \sigma\varphi_m^2 + \tau\varphi_m^4 + o(\varphi_m^4), \tag{3}$$

where ρ, σ, and τ are constant coefficients of expansion which depend on the form of the function $f(y)$ and which are to be determined. The series expansion (3) takes the relation $I_0(-\varphi_m) = I_0(\varphi_m)$ into account, because the sign of a deviation angle φ is governed only by fluctuations of the director and does not depend on the incident light intensity.

It is also convenient to present the solution of Eq. (2) as an expansion in the small parameter φ_m:

$$\varphi(y,z) = A(y,z)\varphi_m + B(y,z)\varphi_m^3 + C(y,z)\varphi_m^5 + o(\varphi_m^5), \tag{4}$$

where A, B, and C are unknown expansion coefficients. Series (4) contains only odd powers of φ_m, because the signs of the angle φ_m and the function $\varphi(\varphi_m)$ must be identical. Provided that series (4) satisfies the boundary conditions corresponding to the absolutely rigid anchoring of the director on the cell surfaces, we obtain

$$A(y; z=0,L) = B(y; z=0,L) = C(y; z=0,L) = 0. \tag{5}$$

In addition, in accordance with the definition of φ_m as the maximal angle of a director deviation (i.e. the value of φ at $y=0$ and $z=L/2$), we also obtain

$$A(y=0, z=L/2) = 1,$$

$$B(y=0, z=L/2) = C(y=0, z=L/2) = 0. \tag{6}$$

Substituting expansions (3) and (4) into Eq. (2) and zeroing the coefficients at the corresponding powers of φ_m, we obtain—to an accuracy of small terms of the order of φ_m^5—the following system of linear differential equations for the functions $A(y,z)$, $B(y,z)$, and $C(y,z)$:

$$A''_{zz} + mA''_{yy} + \frac{\pi^2}{L^2}\rho A f(y) = 0, \tag{7}$$

$$B''_{zz} + mB''_{yy} + \frac{\pi^2}{L^2}\rho B f(y) =$$

$$= kA^2A''_{zz} + kA'^2_z A - \frac{\pi^2}{L^2}\left(\rho\alpha A^3 + \sigma A\right) f(y), \tag{8}$$

$$C''_{zz} + mC''_{yy} + \frac{\pi^2}{L^2}\rho C f(y) = kA^2B''_{zz} + 2kABA''_{zz} -$$

$$-\frac{k}{3}A^4A''_{zz} + kA'^2_z B - \frac{2}{3}kA'^2_z A^3 + 2kA'_z B'_z A -$$

$$-\frac{\pi^2}{L^2}\left(3\rho\alpha A^2 B + \rho\beta A^5 + \sigma B + \sigma\alpha A^3 + \tau A\right) f(y). \tag{9}$$

Here, $\alpha = \frac{3}{2}\frac{\varepsilon_a}{\varepsilon_\parallel} - \frac{2}{3}$, $\beta = \frac{15}{8}\frac{\varepsilon_a^2}{\varepsilon_\parallel^2} - \frac{3}{2}\frac{\varepsilon_a}{\varepsilon_\parallel} + \frac{2}{15}$, and the primed quantities denote partial derivatives with respect to the variables indicated as subscripts.

The solution of the obtained system of equations (7)–(9) must satisfy the boundary conditions (5) and conditions (6) in the cell middle at the center of a light beam. Satisfying those conditions, we find the unknown coefficients ρ, σ, and τ for expansion (3).

 ISSN 2071-0194. Ukr. J. Phys. 2011. Vol. 56, No. 1

3. LIFT Induced by Light Beams Confined in one Dimension

First, let us analyze the series expansion (3) not solving Eqs. (7)–(9). Taking the terms to the order of φ_m^4 inclusive into account, we find the following solution of the biquadratic equation derived for φ_m:

$$\varphi_m^2 = \left[-\sigma \pm \sqrt{\sigma^2 + 4\tau\left(I_0/I_{\rm Fr} - \rho\right)}\right] \Big/ (2\tau), \tag{10}$$

where the parameters ρ, σ, and τ depend on the intensity distribution over the transverse cross-section of an incident light beam.

Let $\sigma > 0$. Then, when the incident light intensity I_0 grows from zero and achieves the threshold of orientational instability $I_{\rm th} = \rho I_{\rm Fr}$, the system continuously transforms from the uniform state ($\varphi_m = 0$) into a non-uniform one ($\varphi_m \neq 0$). There is no hysteresis in the system, and the LIFT is a phase transition of the second kind. This problem, as was said above, was considered in a number of works [1–7].

However, if $\sigma < 0$, and the LIFT threshold $I_0 = I_{\rm th}$ is attained, the system—according to Eq. (10) and similarly to what was obtained in work [8]—demonstrates a jump-like transition from the uniform state into a non-uniform one with $\varphi_m = \sqrt{-\frac{\sigma}{\tau}}$. At the inverse transition, i.e. at a reduction of the light intensity I_0 from the region $I_0 > I_{\rm th}$, the transition of the system into the initial uniform state takes place at a lower intensity $I_0 = I'_{\rm th} < I_{\rm th}$. The magnitude of this threshold, as was done in work [8], can be found from the condition that the radicand in expression (10) is positive:

$$I'_{\rm th} = I_{\rm Fr}\left(\rho - \frac{\sigma^2}{4\tau}\right) = I_{\rm th} - \Delta I_{\rm th},$$

$$\Delta I_{\rm th} = I_{\rm Fr}\,\frac{\sigma^2}{4\tau} > 0. \tag{11}$$

In this case, when achieving $I_0 = I'_{\rm th}$, the system jumps into the initial uniform state from the non-uniform one with $\varphi_m = \sqrt{-\frac{\sigma}{2\tau}}$. Hence, the system demonstrates a hysteresis behavior, and the corresponding LIFT is a phase transition of the first kind.

Let the intensity distribution over the transverse cross-section of an incident light beam be described by the function $I(y) = I_0 \cosh^{-2}(y/a)$. The solution of Eq. (7) that satisfies conditions (5) and (6) and is finite as $y \to \pm\infty$ looks like [15]

$$A(y,z) = \cosh^{-\varepsilon}\frac{y}{a}\,\sin\frac{\pi z}{L}\,, \tag{12}$$

where $\varepsilon = \frac{\pi a}{\sqrt{m}L}$. The magnitude of the dimensionless LIFT threshold at the light intensity growth, being identical to the coefficient ρ in series (3), is equal to

$$\frac{I_{\rm th}}{I_{\rm Fr}} = \rho = 1 + \frac{\sqrt{m}L}{\pi a}\,. \tag{13}$$

The solutions of Eqs. (8) and (9), as well as the corresponding values of the parameters σ and τ, are presented in Appendix (for σ and τ, see Eqs. (D1.5) and (D1.6), respectively).

In Fig. 1,*a*, the calculated dependences of the dimensionless LIFT threshold on the light beam width at the growing ($I_{\rm th}/I_{\rm Fr}$, curve *1*) and decreasing ($I'_{\rm th}/I_{\rm Fr}$, curve *2*) incident light intensities are depicted. They have qualitatively the same character as in the case of a beam with the intensity distribution $I(y) = I_0\Theta(a-|y|)$, which was considered by us earlier in work [13]. However, as is seen from Fig. 1,*b*, in the present case of the intensity distribution over the beam, the increase of the ratio a/L is accompanied by a monotonous growth of the hysteresis loop width from zero, for narrow beams, to a certain constant value, for wide ones ($a/L \gtrsim 20$). The maximal width of the hysteresis loop turns out approximately equal to that for beams with $I(y) = I_0\Theta(a-|y|)$ [13]. In numerical calculations, we used the following values for NLC parameters, which are close to typical ones [14]: $k = 0.6$, $m = 0.3$, $\varepsilon_\parallel = 3.06$, and $\varepsilon_\perp = 2.37$.

Figure 2 demonstrates that the jump $\Delta\varphi_m$ of the maximal angular director deviation monotonously grows with increase in the transverse beam size a/L and saturates at $a/L \gtrsim 10$. Qualitatively, this behavior reproduces a similar dependence of the hysteresis loop width. The maximal jumps $\Delta\varphi_m$ turn out almost the same as for the beam with $I(y) = I_0\Theta(a-|y|)$ [13].

The calculations show that the region of LIFT hysteresis existence is determined by the parameters k and m. For every ratio a/L and every m, there exists a critical value $k_{\rm th} = -6\alpha\rho\varepsilon/(4\varepsilon+1)$, which is determined by the equation $\sigma = 0$, so that the LIFT occurs without a hysteresis ($\sigma > 0$) at $k < k_{\rm th}$ and with a hysteresis ($\sigma < 0$) at $k_{\rm th} < k < 1$. The dependences of the critical value $k_{\rm th}(a/L)$ and $k_{\rm th}(m)$, which are depicted in Fig. 3, were plotted for a number of parameters m and a/L, respectively; qualitatively, they have the same character as for the beam with $I(y) = I_0\Theta(a-|y|)$ [13]. For every m, the critical value $k_{\rm th}$ monotonously falls down as the ratio a/L increases, approaching the finite value $k_{\rm th}^\infty = 1 - \frac{9}{4}\frac{\varepsilon_a}{\varepsilon_\parallel}$ in the limiting case of an indefinitely wide beam ($a/L \to \infty$), the value being independent of the light beam shape in the case of one-dimensional

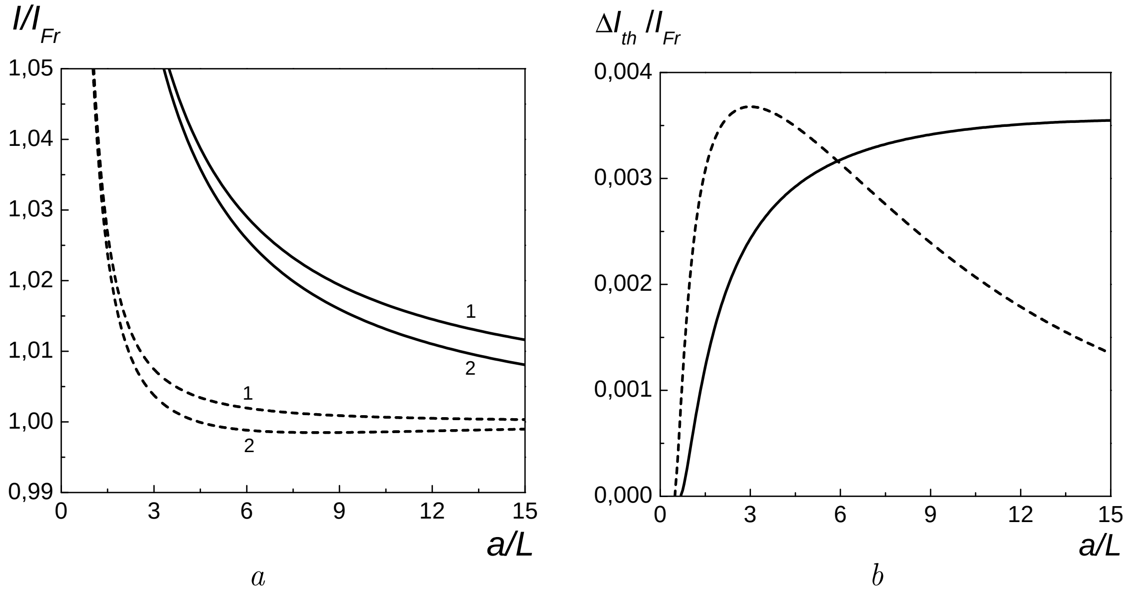


Fig. 1. Dependences of the LIFT threshold $I_{\rm th}/I_{\rm Fr}$ (*a*) and the hysteresis loop width $\Delta I_{\rm th}/I_{\rm Fr}$ (*b*) on the ratio a/L for the incident light beam at $k = 0.6$, $m = 0.3$, and $\varepsilon_a/\varepsilon_\parallel = 0.22$. Solid and dashed curves correspond to the beam intensity distributions $I(y) = I_0 \cosh^{-2}(y/a)$ and $I(y) = I_0\Theta(a - |y|)$, respectively. Curves *1* and *2* in panel (*a*) correspond to the regimes with increase and decrease in the intensity, respectively

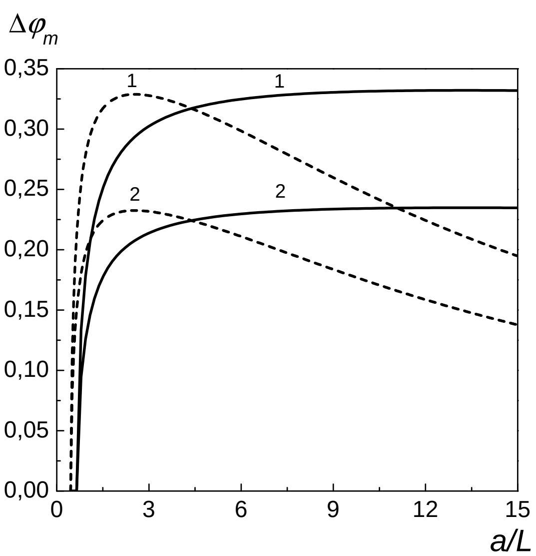


Fig. 2. Jumps $\Delta\varphi_m$ of the maximal director deviation angle as the functions of the ratio a/L at $k = 0.6$, $m = 0.3$, and $\varepsilon_a/\varepsilon_\parallel = 0.22$. Solid and dashed curves correspond to the beam intensity distributions $I(y) = I_0 \cosh^{-2}(y/a)$ and $I(y) = I_0\Theta(a - |y|)$, respectively. Curves *1* and *2* correspond to the regimes with increase and decrease in the intensity, respectively

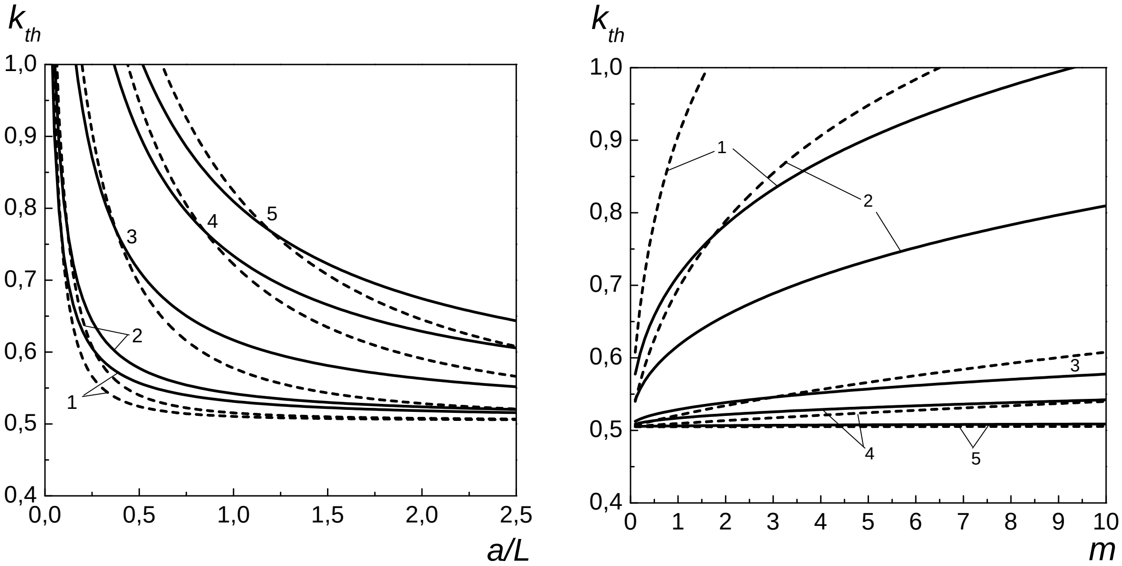
confinement. Note that, at $k < k_{\rm th}^{\infty}$, the LIFT takes place without a hysteresis, irrespective of the m-value and the beam width. As Fig. 3 illustrates, a reduction of the ratio a/L and a growth of the parameter m make the range of parameter k, where the hysteresis of LIFT exists, narrower.

In Fig. 4, the dependences of the critical value $m_{\rm th} = (\pi a/L)^2(4 + 6\alpha\rho/k)^2$ for the parameter m (those values correspond to $\sigma = 0$) on the ratio a/L for various k-values are exhibited. Here, the interval $0 < m < m_{\rm th}$ is a range, where the LIFT is accompanied by a hysteresis. If $m > m_{\rm th}$, the LIFT takes place without a hysteresis. As is seen from Fig. 4, a reduction of the ratio a/L and the parameter k (within the interval $k_{\rm th} < k < 1$) results in the narrowing of the parameter m range, where the LIFT hysteresis exists.

As follows from Figs. 3 and 4, the ranges of parameters k and m, where the LIFT hysteresis exists, weakly depend on the shape of a light beam confined in one dimension.

4. LIFT Induced by Light Beams Confined in Two Dimensions

Let the light intensity distribution over the transverse cross-section of incident beam be confined along both coordinates, x and y. For the sake of definiteness, let the distribution look like $I(r) = I_0\Theta(R - r)$, where R is the beam radius, and r is the distance reckoned from the beam axis. We use the one-constant approximation for elastic Frank constants, $K_1 = K_2 = K_3 = K$, be-



a *b*

Fig. 3. Dependences of the critical value of the parameter $k_{\rm th}$ on (a) the ratio a/L for various $m = 0.05$ (*1*), 0.1 (*2*), 1 (*3*), 5 (*4*), and 10 (*5*), and (b) on m for various $a/L = 0.5$ (*1*), 1 (*2*), 5 (*3*), 10 (*4*), and 100 (*5*). Solid and dashed curves correspond to the beam intensity distributions $I(y) = I_0 \cosh^{-2}(y/a)$ and $I(y) = I_0\Theta(a - |y|)$, respectively

cause the deformations of the director remain plane in this case [1, 2]. Taking the solution of the Maxwell equations into account, the stationary equation for the director looks like Eq. (2), in which $k = 0$ and $m = 1$, whereas the director deviation angle depends now on r and z. Substituting the series expansions (3) and (4) into the equation for the director, we obtain a system of differential equations for the unknown functions $A(r, z)$, $B(r, z)$, and $C(r, z)$ which is similar to system (7)–(9), but the Laplace operator is now expressed in terms of the cylindrical coordinates r and z.

According to Eq. (7), the function $A(r, z)$ which satisfies conditions (5) and (6) is finite as $r \to \infty$ and continuous together with its first derivative at the beam boundary $r = R$ looks like

$$A(r,z) = \begin{cases} J_0(qr)\sin\dfrac{\pi z}{L}\,, & \text{if } r \leqslant R, \\ \dfrac{J_0(qR)}{K_0(\tilde{q}R)}K_0(\tilde{q}r)\sin\dfrac{\pi z}{L}\,, & \text{if } r > R, \end{cases} \qquad (14)$$

where $q = \tilde{q}\sqrt{\rho - 1}$; $\tilde{q} = \pi/L$; and $J_n(x)$ and $K_n(x)$ are the ordinary and modified Bessel functions, respectively. The parameter ρ which determines the value of LIFT threshold $I_{\rm th}$ at the growing intensity is the first nontrivial root of the equation

$$\frac{qJ_1(qR)}{J_0(qR)} = \frac{\tilde{q}K_1(\tilde{q}R)}{K_0(\tilde{q}R)}\,. \qquad (15)$$

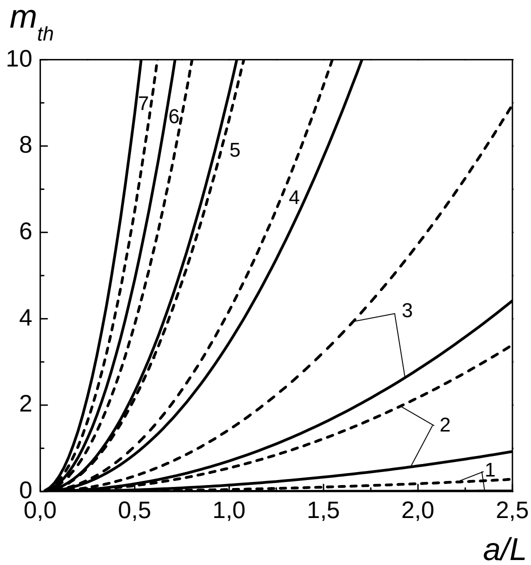


Fig. 4. Dependences of the critical value of the parameter $m_{\rm th}$ on the ratio a/L for various $k = 0.51$ (*1*), 0.55 (*2*), 0.6 (*3*), 0.7 (*4*), 0.8 (*5*), 0.9 (*6*), and 0.99 (*7*). Solid and dashed curves correspond to the beam intensity distributions $I(y) = I_0 \cosh^{-2}(y/a)$ and $I(y) = I_0\Theta(a - |y|)$, respectively

By solving Eqs. (8) and (9), we determine the parameters σ,

$$\sigma = -\frac{3}{2}\frac{\alpha\rho}{J_0^2(qR) + J_1^2(qR)}\int\limits_0^1 J_0^4(qRt)t\,dt\,, \qquad (16)$$

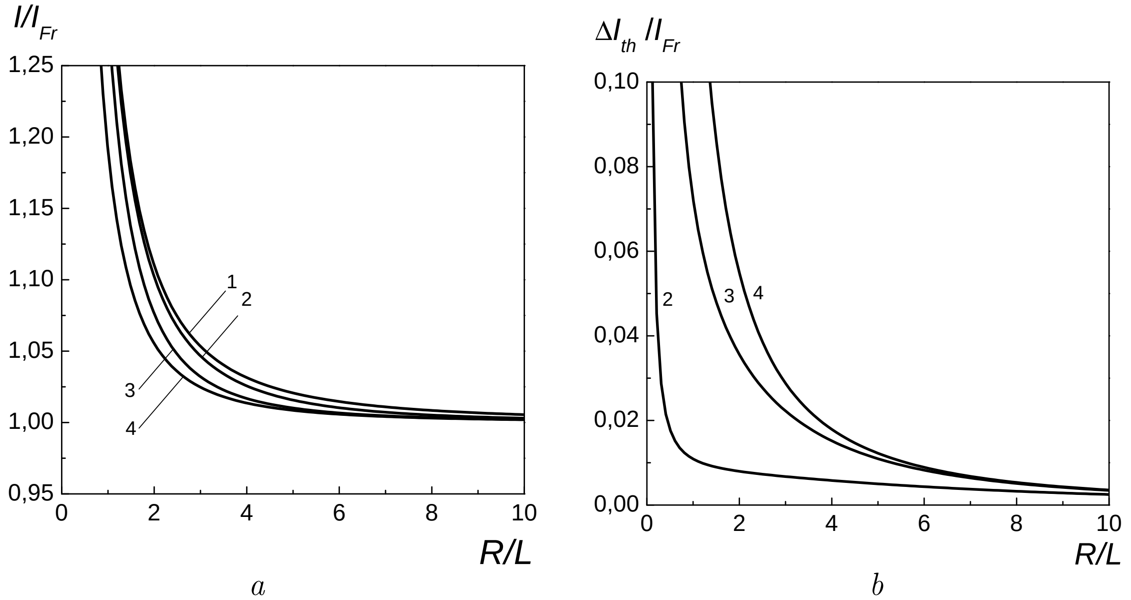


Fig. 5. (a) LIFT threshold $I_{\rm th}/I_{\rm Fr}$ at growing (curve *1*) and falling (curves *2* to *4*) light beam intensities and (b) the width of the hysteresis loop $\Delta I_{\rm th}/I_{\rm Fr}$ as functions of the transverse size R/L of light beams confined in two dimensions with $I(r) = I_0\Theta(R-r)$. $\varepsilon_a/\varepsilon_\| = 0.5$ (*2*), 0.6 (*3*), and 0.9 (*4*)

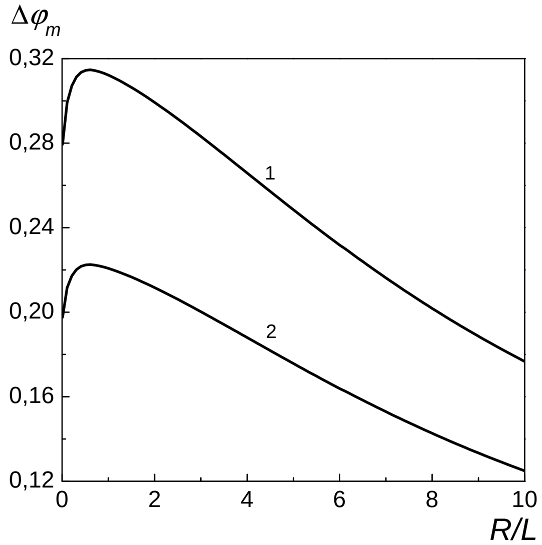


Fig. 6. Dependences of the jumps $\Delta\varphi_m$ of the maximal director deviation angle on the ratio R/L for light beams confined in two dimensions at growing (curve *1*) and falling (curve *2*) light beam intensities. $\varepsilon_a/\varepsilon_\| = 0.5$

and τ; the corresponding expression for the latter is not given here, because it is very cumbersome.

In Fig. 5,a, the results of numerical calculations concerning the dependences of the LIFT threshold at growing, $I_{\rm th}$, and falling, $I'_{\rm th}$, intensities of incident light beam on the transverse beam size are depicted for various values of the ratio $\varepsilon_a/\varepsilon_\|$. Figure 5,$b$ demonstrates that, unlike the case of light beams confined in one dimension, the growth of the transverse beam size is accompanied by a monotonous decrease of the hysteresis loop width from infinity, for infinitesimally narrow beams, to zero, for unconfined ones. The growth of the ratio $\varepsilon_a/\varepsilon_\|$ gives rise to a monotonous growth of the hysteresis loop width.

The dependences of the jump $\Delta\varphi_m$ of the maximal director deviation angle at the LIST on the transverse light beam dimension, which are exhibited in Fig. 6, are nonmonotonous: as the ratio R/L grows, the $\Delta\varphi_m$-magnitude first increases to a certain maximum and then monotonously vanishes in the limiting case $R/L \to \infty$. In contrast to the case of light beams confined in one dimension, the dependences of the jump $\Delta\varphi_m$ and the hysteresis loop width $\Delta\varphi_m$, in this case, on the transverse size of a light beam are qualitatively different.

Note that, for light beams confined in two dimensions, the LIFT is accompanied by a hysteresis in those NLC cells, for which $\frac{\varepsilon_a}{\varepsilon_\|} > \frac{4}{9}$, as was in the case of an unconfined uniform light beam [8].

Hence, the shape of the incident light beam and the finiteness of its transverse dimensions affect not only the LIFT threshold, but also the conditions for the hysteresis to be observed reliably. The ranges of the parameters k and m, where the LIFT hysteresis exists, become narrower as the width of a light beam confined in one dimension decreases, being practically independent of the intensity distribution function. The shape of a light beam confined in one dimension substantially affects the threshold magnitude and the hysteresis loop width. The calculations showed that the dependence of the hysteresis loop width on the transverse beam size can be both nonmonotonous (for a beam with $I(y) = I_0\Theta(a - |y|)$) and monotonous (for a beam with $I(y) = I_0 \cosh^{-2}(y/a)$). In the former case, the maximal width of the hysteresis loop is reached at $a/L \approx 3$, being, by magnitude, almost the same as in the latter case at $a/L \gtrsim 20$. The dependences of jumps of the maximal director deviation angle on the transverse size of light beams confined in one dimension qualitatively reproduce the dependences of the hysteresis loop width, and their maximal values practically do not depend on the beam shape.

In the light beams confined in two dimensions, the width of the LIFT hysteresis loop is maximal for infinitesimally narrow beams and monotonously tends to zero as the transverse beam size grows.

The hysteresis loop width grows with increase in the nematic liquid crystal anisotropy, irrespective of the beam shape and the confinement.

The authors express their sincere gratitude to I.P. Pinkevich for his useful remarks, while discussing the results obtained.

APPENDIX

Taking the explicit forms of the functions $f(y) = \cosh^{-2}(y/a)$ and $A(y,z)$ into account (see Eq. (12)) and substituting the solution $B(y,z)$,

$$B(y,z) = b_1(y)\sin\frac{\pi z}{L} + b_3(y)\sin\frac{3\pi z}{L}\,, \tag{D1.1}$$

into Eq. (8), we obtain two independent differential equations to find the unknown functions $b_1(y)$ and $b_3(y)$. Introducing the variable $\xi = \tanh(y/a)$ and making the substitution $b_n(\xi) = (1-\xi^2)^{n\varepsilon/2}\omega_n(\xi)$, $(n = 1,3)$, those equations can be expressed as follows:

$$\hat{L}_1\omega_1(\xi) = -\frac{\varepsilon^2}{4}\left[2k(1-\xi^2)^{\varepsilon-1} + 3\alpha\rho(1-\xi^2)^{\varepsilon} + 4\sigma\right]\,, \tag{D1.2}$$

$$\hat{L}_3\omega_3(\xi) - 2\varepsilon(4\varepsilon+1)\omega_3(\xi) = \frac{\varepsilon^2}{4}\left[2k(1-\xi^2)^{-1} + \alpha\rho\right]\,, \tag{D1.3}$$

where

$$\hat{L}_n = (1-\xi^2)\frac{d^2}{d\xi^2} - 2(n\varepsilon+1)\xi\frac{d}{d\xi}\,.$$

The solution of Eqs. (D1.2) and (D1.3), which would be finite at $\xi = \pm1$ $(y \to \pm\infty)$, is sought as a series expansion in the eigenfunctions of the operators $\hat{L}_n$, namely,

$$\omega_1(\xi) = \sum_{m=0}^{\infty} C_m\, P_m^{(\varepsilon,\varepsilon)}(\xi)\,,\quad \omega_3(\xi) = \sum_{m=0}^{\infty} D_m\, P_m^{(3\varepsilon,3\varepsilon)}(\xi)\,, \tag{D1.4}$$

where

$$\hat{L}_n P_m^{(n\varepsilon,n\varepsilon)}(\xi) = -m(m+2n\varepsilon+1)P_m^{(n\varepsilon,n\varepsilon)}(\xi)\,, \qquad n = 1,3.$$

Here, $P_m^{(n\varepsilon,n\varepsilon)}$ are the Jacobi polynomials, and C_m and D_m are the expansion coefficients to be found.

Substituting expansions (D1.4) into Eqs. (D1.2) and (D1.3), using the orthogonality of Jacobi polynomials, and taking condition (6) for the function $B(y,z)$ into account, we obtain the coefficients C_m and D_m, and, hence, the expression for the parameter σ in the form

$$\sigma = -\frac{2^{2\varepsilon-2}\Gamma(2\varepsilon+2)\Gamma^2(2\varepsilon)}{\Gamma^2(\varepsilon+1)\Gamma(4\varepsilon)}\left[\frac{k}{2} + \frac{3\alpha\rho\varepsilon}{4\varepsilon+1}\right]\,, \tag{D1.5}$$

where $\Gamma(x)$ is the gamma function.

In a similar way, solving Eq. (9) with the explicitly given functions $A(y,z)$ and $B(y,z)$, we obtain the following expression for the parameter τ:

$$\tau = \frac{\Gamma(2\varepsilon+2)}{2^{2\varepsilon+3}\Gamma^2(\varepsilon+1)}\Bigg[\sum_{m=0}^{\infty}\Big(6kD_m w_m - 6kC_m v_m - 9\alpha\rho C_m u_m\Big) + k\frac{2^{6\varepsilon-2}\Gamma^2(3\varepsilon)}{\Gamma(6\varepsilon)} - 5\beta\rho\frac{2^{6\varepsilon}\Gamma^2(3\varepsilon+1)}{\Gamma(6\varepsilon+2)} - 3\alpha\sigma\frac{2^{4\varepsilon+1}\Gamma^2(2\varepsilon+1)}{\Gamma(4\varepsilon+2)} + 3\alpha\rho D_0\frac{2^{6\varepsilon+1}\Gamma^2(3\varepsilon+1)}{\Gamma(6\varepsilon+2)} - \sigma C_0\frac{2^{2\varepsilon+3}\Gamma^2(\varepsilon+1)}{\Gamma(2\varepsilon+2)}\Bigg], \tag{D1.6}$$

where

$$u_m = \int_{-1}^{1}(1-x^2)^{2\varepsilon}P_m^{(\varepsilon,\varepsilon)}(x)\,dx,$$

$$v_m = \int_{-1}^{1}(1-x^2)^{2\varepsilon-1}P_m^{(\varepsilon,\varepsilon)}(x)\,dx,$$

$$w_m = \int_{-1}^{1}(1-x^2)^{3\varepsilon-1}P_m^{(3\varepsilon,3\varepsilon)}(x)\,dx.$$

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Received 27.05.10.
Translated from Ukrainian by O.I. Voitenko

ВПЛИВ ОБМЕЖЕНОСТІ СВІТЛОВИХ ПУЧКІВ НА ГІСТЕРЕЗИС ПЕРЕХОДУ ФРЕДЕРІКСА В НЕМАТИЧНІЙ КОМІРЦІ

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Р е з ю м е

Розглянуто вплив форми і обмеженості поперечного розміру падаючого світлового пучка на гістерезис світлоіндукованого переходу Фредерікса в гомеотропно орієнтованій комірці нематичного рідкого кристала. Розглянуто випадки одно- і двовимірно обмежених світлових пучків. Чисельно знайдено значення порогів орієнтаційної нестійкості і стрибків кута відхилення директора при збільшенні і зменшенні інтенсивності падаючого світла залежно від поперечних розмірів світлового пучка. Визначено умови, за яких петля гістерезису стає максимально широкою.