

# EQUATION OF STATE FOR THE QUARK GLUON PLASMA WITH THE NEGATIVE BAG CONSTANT

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We propose a new analytical form of the quark-gluon plasma equation of state (EoS). This new EoS satisfies all qualitative features observed in the lattice QCD calculations and gives a good quantitative description of the lattice results in the SU(3) gluodynamics. The energy density for the suggested EoS looks similar to that in the bag model, but requires a negative value of the bag constant.

The transition from a confined hadron-resonance phase to a deconfined phase, the quark-gluon plasma (QGP), is expected at high temperatures and/or baryonic chemical potentials. For several decades, the equation of state in the bag model (BM) has been used to describe the QGP (see, e.g., [1]). In the simplest form, i.e. for non-interacting massless constituents and zero values of all conserved charges, the BM EoS reads:

$$\varepsilon(T) = \sigma_{\text{SB}} T^4 + B, \quad (1)$$

$$p(T) = \frac{\sigma_{\text{SB}}}{3} T^4 - B, \quad (2)$$

where the energy density  $\varepsilon$  and the pressure  $p$  have a simple dependence on the temperature  $T$  modified by adding a positive constant  $B$  (“vacuum pressure”). The Stefan–Boltzmann (SB) constant  $\sigma_{\text{SB}}$  in Eq. (1) is equal to

$$\sigma_{\text{SB}} = \frac{\pi^2}{30} \left( d_{\text{B}} + \frac{7}{8} d_{\text{F}} \right), \quad (3)$$

where  $d_{\text{B}}$  and  $d_{\text{F}}$  are the degeneracy factors for the bosons (gluons) and fermions (quarks and antiquarks), respectively. The zero value of the baryonic chemical potential in Eq. (1) is a valid approximation for the QGP created in nucleus-nucleus collisions at the BNL RHIC and even better for future experiments at the CERN LHC. Note that also most lattice QCD calculations for the QGP EoS correspond to zero or very small values

of the baryonic chemical potential. Equation (1) is assumed to be valid at  $T > T_{\text{c}}$ , where the critical temperature  $T_{\text{c}}$  corresponds to a first-order phase transition in the pure SU(3) gluodynamics or to a smooth crossover in the full QCD.

The main features of the QCD deconfined matter EoS can be illustrated by the Monte Carlo (MC) lattice results [2] for the SU(3) gluodynamics presented in Fig. 1.

They can be qualitatively summarized as follows:

1. The pressure  $p(T)$  rapidly increases at  $T \gtrsim T_{\text{c}}$ , while, at high  $T$ , the system reaches the ideal massless gas behavior  $p \cong \varepsilon/3$ .
2. However, the constant  $\sigma \cong \varepsilon/T^4 \cong 3p/T^4$  observed at high  $T$  is about 10 ÷ 20% smaller than the value of  $\sigma_{\text{SB}}$  in Eq. (3).
3. At high  $T$ , both  $\varepsilon/T^4$  and  $3p/T^4$  approach their limiting value  $\sigma$  from below.

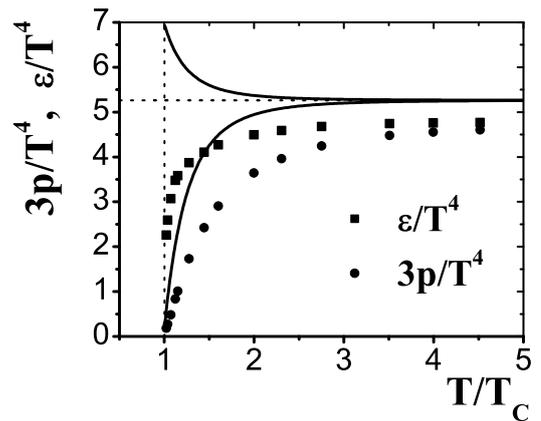


Fig. 1. The MC lattice results within the SU(3) gluodynamics [2] for the energy density (squares) and the pressure (circles) at  $T > T_{\text{c}}$ . The size of symbols corresponds to the error-bars reported in [2]. The dashed horizontal line corresponds to the SB constant  $\sigma_{\text{SB}} = 8\pi^2/15$ . The solid lines show the BM EoS (1) with  $d = 16$  and  $B = 1.7T_{\text{c}}^4$  for  $\varepsilon/T^4$  (upper line) and  $3p/T^4$  (lower line)

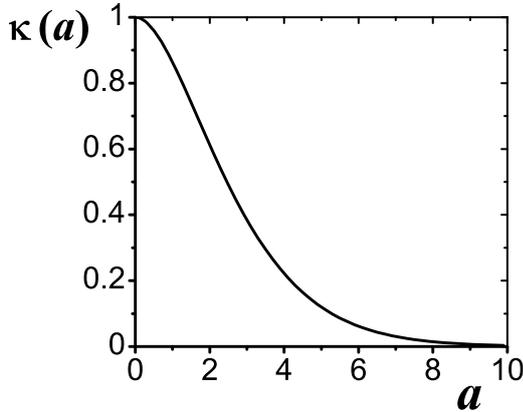


Fig. 2. The suppression factor  $\kappa(a)$  from Eq. (9) as a function of the parameter  $a$

Note that these properties of the gluon plasma EoS are also valid in the full QCD.

The phenomenological success of the BM EoS (1) is due to the fact that it satisfies the first property: it gives  $p \cong \varepsilon/3$  at high  $T$  and shows an abrupt drop of  $p(T)$  near  $T_c$ . However, the BM EoS is in a contradiction with the second and third features listed above (see Fig. 1). The BM EoS (1) gives no suppression of the SB constant. Note that we restrict our consideration to the present lattice results available at a finite temperature interval  $T_c < T < 4.5T_c$  and do not discuss the possible asymptotic behavior at  $T \rightarrow \infty$ . The BM energy density  $\varepsilon(T)/T^4$  approaches its SB limit from above. This contradicts the MC lattice results. Despite these evident problems, the BM EoS (1) is still one of the most popular models for phenomenological applications due to its simplicity.

Here, we suggest a new analytical parametrization for the QGP EoS. It satisfies all three properties listed above, gives a good quantitative description of the MC lattice results for the SU(3) gluodynamics, and is almost as simple as Eq. (1).

As the first step, we consider the suppression of the  $\sigma_{\text{SB}}$  constant. For this purpose, the quasiparticle approach of [3] (see also recent papers [4] and references therein) will be used. The system of interacting gluons is treated as a gas of non-interacting quasiparticles with gluon quantum numbers, but with mass  $m(T)$  which depends on  $T$ . The particle energy  $\omega$  and the momentum  $k$  are assumed to be connected as  $\omega = [k^2 + m^2(T)]^{1/2}$ . The energy density and the pressure take then the form

$$\varepsilon(T) = \frac{d}{2\pi^2} \int_0^\infty k^2 dk \frac{\omega}{\exp(\omega/T) - 1} + B^*(T) \equiv$$

$$\equiv \varepsilon_0(T, \omega) + B^*(T), \quad (4)$$

$$p(T) = \frac{d}{6\pi^2} \int_0^\infty k^2 dk \frac{k^2}{\omega \exp(\omega/T) - 1} - B^*(T) \equiv$$

$$\equiv p_0(T, \omega) - B^*(T), \quad (5)$$

where the degeneracy factor  $d = 2(N_c^2 - 1)$  equals 16 for the SU(3) gluodynamics. The temperature-dependent function  $B^*(T)$  in Eq. (4) was introduced for the first time in [3]. It results from the thermodynamical relation

$$T \frac{dp}{dT} - p(T) = \varepsilon(T) \quad (6)$$

which leads to the equation for the function  $B^*(T)$ :

$$\frac{dB^*}{dT} = - \frac{\Delta_0(T, \omega)}{m} \frac{dm}{dT}. \quad (7)$$

Here,  $\Delta_0 \equiv \varepsilon_0 - 3p_0$ , and  $\varepsilon_0, p_0$  defined by Eqs. (4) and (5) are the ideal gas expressions for massive bosons. If the function  $m(T)$  is known, one can calculate  $B^*(T)$  from Eq. (7) up to an arbitrary integration constant  $B$ . The linear relation  $m = aT$  with  $a = \text{const} \geq 0$  used for all  $T \geq T_c$  guarantees the high temperature behavior of  $\varepsilon(T)$  and  $p(T)$  in agreement with the MC lattice results. For  $m = aT$ , the function  $B^*(T)$  derived from Eq. (7) equals  $B^*(T) = B - \Delta_0(T, \omega)/4$ . One obtains the energy density (4) and pressure (5),

$$\varepsilon(T) = \sigma T^4 + B, \quad p(T) = \frac{\sigma}{3} T^4 - B, \quad (8)$$

where the modified SB constant  $\sigma$  equals

$$\sigma = \frac{3d}{2\pi^2} \sum_{n=1}^\infty \left[ \frac{a^2}{n^2} K_2(na) + \frac{a^3}{4n} K_1(na) \right] \equiv \kappa(a) \sigma_{\text{SB}}. \quad (9)$$

In Eq. (9),  $K_1$  and  $K_2$  are the modified Bessel functions. The constant  $\sigma$  in Eq. (8) includes the suppression factor  $\kappa(a)$  which is defined by Eq. (9) and presented in Fig. 2.

Therefore, the assumption about a linearly  $T$ -dependent mass,  $m = aT$ , leads to EoS (8) similar to the bag model EoS, but with the suppressed SB constant (9). For  $a \rightarrow 0$ ,  $\kappa \rightarrow 1$  follows, and Eq. (8) coincides with Eq. (1). The modified SB constant  $\sigma = 4.73 < \sigma_{\text{SB}}$  allows to fit the high-temperature behavior of  $\varepsilon(T)$  and  $p(T)$ . This requires  $\kappa(a) \cong 0.90$  and  $a \cong 0.84$ .

At the second step which is the main point of our model construction, we include the linear in  $T$  contribution to the QGP pressure. If the function  $\varepsilon(T)$  is known, Eq. (6) is a first-order differential equation for the function  $p(T)$ . The general solution of this equation includes an arbitrary integration constant, which results in the linear in temperature term in the function  $p(T)$ . This was discussed for the first time in [5]. Thus, for  $\varepsilon(T)$  in the form of Eq. (8), the general solution of Eq. (6) for  $p(T)$  can be written as follows:

$$\varepsilon(T) = \sigma T^4 + B, \quad (10)$$

$$p(T) = \frac{\sigma}{3} T^4 - B - A T. \quad (11)$$

A sum of the first and second terms in the expression for  $p(T)$  is a particular solution of the inhomogeneous differential equation (6) with  $\varepsilon(T)$  given by (10), whereas the last term in  $p(T)$  corresponds to a general solution of the homogeneous equation  $T dp/dT - p = 0$ . Therefore, the thermodynamical relation (6) between the pressure and the energy density admits the linear in  $T$  contribution to  $p(T)$  which is fully invisible in the  $\varepsilon(T)$  function.

Equations (10), (11) define our model suggestion for the QGP EoS. For brevity, we call it the  $A$ -bag model ( $A$ -BM). The formula for  $\varepsilon(T)$  looks formally the same as in Eq. (1). However, the pressure function  $p(T)$  in the  $A$ -BM (11) contains one more parameter  $A$  comparing to the original BM EoS (1). The model parameters,  $\sigma = 4.73$  and  $B = -2.37 T_c^4$  are found from fitting the MC lattice results [2] for the energy density function  $\varepsilon(T)$ . The third  $A$ -BM parameter  $A = 3.94 T_c^3$  is fixed by fitting the pressure function  $p(T)$ . One finds a good description of the MC lattice results for  $\varepsilon(T)$  and  $p(T)$  within the  $A$ -BM EoS (10), (11) for all  $^1 T > T_c$  as shown in Fig. 3. The parameter  $\sigma$  in Eqs. (10), (11) regulates the high temperature behavior of  $\varepsilon/T^4 \cong 3p/T^4 \cong \sigma$ . As  $A > 0$ , the linear in  $T$  term gives a negative contribution to  $p(T)$  and guarantees both the correct high temperature asymptotic behavior of  $p(T)$  and its strong drop at  $T$  near  $T_c$ . The bag parameter  $B$  in Eqs. (10), (11) is found to be *negative*, in contrast to the *positive* bag constant  $B$  in the standard BM EoS (1). Thus, according to the  $A$ -BM (10),  $\varepsilon/T^4$  approaches its high temperature limit  $\sigma$  from below. This is in agreement with the MC lattice results.

An important characteristic of the EoS is the so-called interaction measure,  $(\varepsilon - 3p)/T^4$ , which shows the deviation from the system of noninteracting massless par-

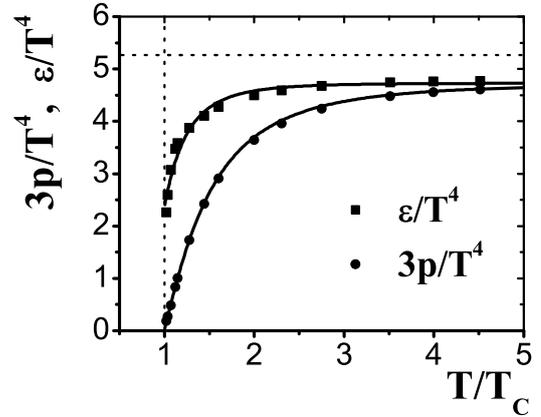


Fig. 3. The MC lattice results and the dotted horizontal line are the same as in Fig. 1. The solid lines correspond to the  $A$ -BM EoS (10), (11) with  $\sigma = 4.73$ ,  $A = 3.94 T_c^3$ , and  $B = -2.37 T_c^4$  for  $\varepsilon/T^4$  (upper line) and  $3p/T^4$  (lower line)

ticles. For the  $A$ -BM EoS (10), (11), the interaction measure reads

$$\frac{\varepsilon - 3p}{T^4} = \frac{3A}{T^3} + \frac{4B}{T^4}. \quad (12)$$

The MC lattice results [2] demonstrate a prominent maximum of the function  $(\varepsilon - 3p)/T^4$  at  $T_{\max} \cong 1.1 T_c$ . The maximum of  $(\varepsilon - 3p)/T^4$  is described in the  $A$ -BM. This happens due to different signs of the  $A$ - and  $B$ -terms ( $A > 0$ ,  $B < 0$ ) on the r.h.s. of Eq. (12). Note that such a maximum is not reproduced by the so-called fuzzy bag model [6]. In that model, there are  $T^2$  contributions to both  $p(T)$  and  $\varepsilon(T)$ ,

$$\varepsilon(T) = \sigma T^4 - C T^2 + B, \quad (13)$$

$$p(T) = \frac{\sigma}{3} T^4 - C T^2 - B, \quad (14)$$

and a comparison with the MC lattice results [2] gives  $C > 0$  and  $B > 0$ . In that case,  $(\varepsilon - 3p)/T^4 = 2C/T^2 + 4B/T^4$  corresponds to a monotonous decreasing function of  $T$ , as both terms are positive. A comparison of the EoS (10), (11) and (13), (14) will be discussed in more details in [7]. An extension of the  $A$ -BM to the  $SU(N_c)$  gluodynamics with  $N_c > 3$  [8], the quark degrees of freedom, and non-zero baryonic chemical potentials can be done along the same scheme and will be considered elsewhere.

In summary, we have suggested a new EoS for the deconfined matter – the  $A$ -BM (10), (11). It satisfies all qualitative features of the MC lattice results at  $T > T_c$  and gives a good quantitative description of the lattice results [2] for the  $SU(3)$  gluodynamics, see Fig. 3. The

<sup>1</sup> To be precise, note that we consider the MC lattice points with  $T \geq 1.02 T_c$  to avoid the uncertainties of  $\varepsilon(T)$  at  $T = T_c$ .

expression for  $\varepsilon(T)$  in the  $A$ -BM (10) looks similar to that in BM (1). However, the pressure function  $p(T)$  in  $A$ -BM (11) contains a new linear in  $T$  negative term which does not contribute to  $\varepsilon(T)$ . The presence of this negative pressure term leads to a principal difference between the bag term  $B$  in the BM and that in the  $A$ -BM. The bag parameter in the  $A$ -BM is found to be *negative*, in contrast to the *positive* bag constant  $B$  in the BM EoS (1).  $A$ -BM (10), (11) gives a simple analytical parametrization of the QGP EoS. This opens new possibilities for its applications in the hydrodynamic description of the QGP.

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#### РІВНЯННЯ СТАНУ КВАРК-ГЛЮОННОЇ ПЛАЗМИ З ВІД'ЄМНОЮ КОНСТАНТОЮ КВАРК-ГЛЮОННОГО МІШКА

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#### Резюме

Ми пропонуємо нову аналітичну форму рівняння стану кварк-глюонної плазми. Це нове рівняння стану задовольняє усім якісним особливостям, знайденим у КХД розрахунках на ґратці, та дає хороше кількісне узгодження з результатами розрахунків на ґратці в SU(3) глюодинаміці. Густина енергії для запропонованого рівняння стану схожа на звичайну модель мішка, але вимагає від'ємного значення константи мішка.