VARIATIVITY OF MODE-MODE COUPLINGS IN A THREE-COMPONENT TODA LATTICE

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The alternative extension of the one-component Toda lattice into a three-component integrable nonlinear lattice is proposed. The main idea consists in replacing the elements of the standard Toda spectral operator by 3×3 submatrices given by the linear combinations of three basic matrices associated with the third-order Abel group. Specifically, this procedure assumes the splitting of the previously unique spectral parameter into two linearly independent parts. The zero-curvature representation of the extended system is found, and one of its possible parametrizations (namely the parametrization extracting the global Toda field supplemented by two satellite fields) is explicitly presented. The Lagrangian formulation for a nonlinear lattice corresponding to the chosen parametrization is obtained. In addition, the generic variativity of admissible parametrizations allow us to generate a broad class of integrable lattice models distinguished by the choice of mode-mode couplings.

1. Introduction

The one-component dynamical Toda system on an infinite one-dimensional lattice [1–3] is known to be reformulated either within the Lax representation in terms of auxiliary infinite Jacobi matrices without any spectral parameter [4, 5] or within the zero-curvature representation in terms of auxiliary 2×2 matrices with the spectral parameter $\lambda = z + 1/z$ [6]. Each of such recastings serves as the hallmark of integrability for the original nonlinear system.

It is remarkable that both the Lax and zero-curvature approaches allow the direct extensions [7] and [8] applicable to the same generalized matrix-valued Toda system [7, 8]. It is reasonable to render the above extensions as the trivial ones inasmuch as, e.g. in the framework of the extended zero-curvature scheme, we are obliged to generalize the spectral parameter via the sheer multiplication by the identity submatrix [8].

Nevertheless, there exists another sort of extensions distinguished by the separation of the initially single spectral parameter into two linearly independent parts. We will use this possibility to seek the zero-curvature representation for some specific three-component semidiscrete nonlinear evolutionary system and will try to reformulate this system in terms of clear dynamical variables.

2. Zero-curvature Representation

Although the formally written matrix-valued equation

$$\dot{L}(n|z) = A(n+1|z)L(n|z) - L(n|z)A(n|z) \qquad (1)$$

provides the common auxiliary tool for a variety of already known integrable semidiscrete nonlinear systems, its creative aspect allowing one to seek new types of the spectral $L(n|z)$ and evolution $A(n|z)$ operators and thereby to discover earlier unknown systems appears to be indisputable. Here and later on, the cell number n spans all integers from minus to plus infinity, whereas the overdot designates the derivative with respect to the dimensionless time τ.

Relying upon the above-mentioned creativity of the zero-curvature equation (1), we now postulate a spectral operator $L(n|z)$ in the block-matrix form

$$L(n|z) = \begin{pmatrix} L_{11}(n|z) & L_{12}(n|z) \\ L_{21}(n|z) & L_{22}(n|z) \end{pmatrix} \qquad (2)$$

with the submatrices $L_{jk}(n|z)$ given by the expressions

$$L_{11}(n|z) = [z + f_{11}(n)]\, S + g_{11}(n)T +$$

$$+\left[h_{11}(n)+1/z\right]U \tag{3}$$

$$L_{12}(n|z)=f_{12}(n)S+g_{12}(n)T+h_{12}(n)U \tag{4}$$

$$L_{21}(n|z)=f_{21}(n)S+g_{21}(n)T+h_{21}(n)U, \tag{5}$$

$$L_{22}(n|z)=0T. \tag{6}$$

Here, the functions $f_{jk}(n)$, $g_{jk}(n)$, $h_{jk}(n)$ have to be properly parametrized, once we would intend to introduce the genuine field variables. The submatrices S, T, U are assumed to be elements of the third-order Abel group characterized by the multiplication rules

$$SS = TU=UT=U, \tag{7}$$
$$TT = US=SU=T, \tag{8}$$
$$UU = ST=TS=S. \tag{9}$$

Thus, we have preferred from the very beginning to split the quantities z and $1/z$ into two linearly independent parts of some matrix-valued spectral parameter $zS+z^{-1}U$ in contrast to the trivial opportunity with the spectral parameter being unified as $(z+1/z)T$.

Following the mnemonic rule correlating the admissible forms of the spectral $L(n|z)$ and evolution $A(n|z)$ operators (as it emerges from the zero-curvature representation of the lowest system in the Toda hierarchy [6, 8]), we chose the evolution operator $A(n|z)$ for the future system to be

$$A(n|z)=\begin{pmatrix} A_{11}(n|z) & A_{12}(n|z) \\ A_{21}(n|z) & A_{22}(n|z)\end{pmatrix}, \tag{10}$$

where the submatrices $A_{jk}(n|z)$ are given by the ansätze

$$A_{11}(n|z)=0T, \tag{11}$$

$$A_{12}(n|z)=a_{12}(n)S+b_{12}(n)T+c_{12}(n)U, \tag{12}$$

$$A_{21}(n|z)=a_{21}(n)S+b_{21}(n)T+c_{21}(n)U, \tag{13}$$

$$A_{22}(n|z)=\left[a^{+}_{22}(n)z+a_{22}(n)\right]S+b_{22}(n)T+$$

$$+\left[c_{22}(n)+c^{-}_{22}(n)/z\right]U. \tag{14}$$

The direct substitution of formulas (2)–(6) and (10)–(14) for the spectral $L(n|z)$ and evolution $A(n|z)$ operators into the zero-curvature equation (1) proves the selection of ansätze (11)–(14) as quite manageable provided the constraint

$$L_{12}(n|z)L_{21}(n|z)=-T \tag{15}$$

is imposed.

As a result, we are able both to concretize the trial quantities $a^{+}_{22}(n)$, $a_{jk}(n)$, $b_{jk}(n)$, $c_{jk}(n)$, $c^{-}_{22}(n)$ in terms of the prototype field functions $f_{jk}(n)$, $g_{jk}(n)$, $h_{jk}(n)$ and to isolate the set of nonlinear evolution equations. Precisely without any loss of generality, we have

$$a_{12}(n)=-f_{12}(n), \tag{16}$$

$$b_{12}(n)=-g_{12}(n), \tag{17}$$

$$c_{12}(n)=-h_{12}(n), \tag{18}$$

$$a^{+}_{22}(n)=1, \tag{19}$$

$$a_{22}(n)=b_{22}(n)=c_{22}(n)=0, \tag{20}$$

$$c^{-}_{22}(n)=1, \tag{21}$$

$$a_{21}(n)=-f_{21}(n-1), \tag{22}$$

$$b_{21}(n)=-g_{21}(n-1), \tag{23}$$

$$c_{21}(n)=-h_{21}(n-1) \tag{24}$$

and

$$\dot f_{11}(n)=f_{12}(n)g_{21}(n-1)-f_{12}(n+1)g_{21}(n)+$$

$$+g_{12}(n)f_{21}(n-1)-g_{12}(n+1)f_{21}(n)+$$

$$+h_{12}(n)h_{21}(n-1)-h_{12}(n+1)h_{21}(n), \tag{25}$$

$$\dot g_{11}(n)=f_{12}(n)h_{21}(n-1)-f_{12}(n+1)h_{21}(n)+$$

$$+g_{12}(n)g_{21}(n-1)-g_{12}(n+1)g_{21}(n)+$$

$$+h_{12}(n)f_{21}(n-1)-h_{12}(n+1)f_{21}(n), \tag{26}$$

$$\dot h_{11}(n)=f_{12}(n)f_{21}(n-1)-f_{12}(n+1)f_{21}(n)+$$

$$+g_{12}(n)h_{21}(n-1)-g_{12}(n+1)h_{21}(n)+$$

$$+h_{12}(n)g_{21}(n-1)-h_{12}(n+1)g_{21}(n), \qquad (27)$$

$$2f_{11}(n)=f_{12}(n)\dot{g}_{21}(n)-\dot{f}_{12}(n)g_{21}(n)+$$

$$+g_{12}(n)\dot{f}_{21}(n)-\dot{g}_{12}(n)f_{21}(n)+$$

$$+h_{12}(n)\dot{h}_{21}(n)-\dot{h}_{12}(n)h_{21}(n), \qquad (28)$$

$$2g_{11}(n)=f_{12}(n)\dot{h}_{21}(n)-\dot{f}_{12}(n)h_{21}(n)+$$

$$+g_{12}(n)\dot{g}_{21}(n)-\dot{g}_{12}(n)g_{21}(n)+$$

$$+h_{12}(n)\dot{f}_{21}(n)-\dot{h}_{12}(n)f_{21}(n), \qquad (29)$$

$$2h_{11}(n)=f_{12}(n)\dot{f}_{21}(n)-\dot{f}_{12}(n)f_{21}(n)+$$

$$+g_{12}(n)\dot{h}_{21}(n)-\dot{g}_{12}(n)h_{21}(n)+$$

$$+h_{12}(n)\dot{g}_{21}(n)-\dot{h}_{12}(n)g_{21}(n). \qquad (30)$$

According to the general terminology [6], the zero-curvature equation (1) with the spectral $L(n|z)$ and evolution $A(n|z)$ operators specified by expressions (2)–(6) and (10)–(14), (16)–(24) should be referred to as the zero-curvature representation for the system governed by the evolutionary equations (25)–(30) subject to the compatibility condition (15) being fulfilled.

3. Variativity in the Parametrization of Field Variables

Now let us switch over to the problem of converting the prototype field functions $f_{jk}(n)$, $g_{jk}(n)$, $h_{jk}(n)$ into the true field variables.

First of all, we can readily conclude that constraint (15) having been imposed onto the functions $f_{jk}(n)$, $g_{jk}(n)$, $h_{jk}(n)$ reduces the number of independent functions, thus fixing implicitly the so necessary balance with the number of evolution equations. Therefore, it is reasonable to put it as the starting point in establishing the balance explicitly via some appropriate functional parametrization converting the constraint equation (15) into identity.

Condition (15) rewritten in its component-wise form yields

$$F_{12}(n)G_{21}(n)+G_{12}(n)F_{21}(n)+H_{12}(n)H_{21}(n)=0, \qquad (31)$$

$$F_{12}(n)H_{21}(n)+G_{12}(n)G_{21}(n)+H_{12}(n)F_{21}(n)=$$

$$=D_{12}(n)D_{21}(n), \qquad (32)$$

$$F_{12}(n)F_{21}(n)+G_{12}(n)H_{21}(n)+H_{12}(n)G_{21}(n)=0, \qquad (33)$$

where definitions (4) and (5) for the submatrices $L_{12}(n|z)$ and $L_{21}(n|z)$ have been used and the substitutions

$$f_{12}(n)=iF_{12}(n)/D_{12}(n), \qquad (34)$$

$$g_{12}(n)=iG_{12}(n)/D_{12}(n), \qquad (35)$$

$$h_{12}(n)=iH_{12}(n)/D_{12}(n) \qquad (36)$$

and

$$f_{21}(n)=iF_{21}(n)/D_{21}(n), \qquad (37)$$

$$g_{21}(n)=iG_{21}(n)/D_{21}(n), \qquad (38)$$

$$h_{21}(n)=iH_{21}(n)/D_{21}(n) \qquad (39)$$

have been made. From the first (31) and the third (33) parts of the constraint equations (31)–(33), we evidently have

$$G_{12}(n)=\frac{F_{12}(n)F_{21}(n)F_{12}(n)-H_{12}(n)H_{21}(n)H_{12}(n)}{F_{21}(n)H_{12}(n)-H_{21}(n)F_{12}(n)}, \qquad (40)$$

$$G_{21}(n)=\frac{H_{21}(n)H_{12}(n)H_{21}(n)-F_{21}(n)F_{12}(n)F_{21}(n)}{H_{12}(n)F_{21}(n)-F_{12}(n)H_{21}(n)}. \qquad (41)$$

Then, inserting these expressions into the second part (32), we come to the single constraint equation

$$\left[F^3_{12}(n)-H^3_{12}(n)\right]\left[H^3_{21}(n)-F^3_{21}(n)\right]=$$

$$=\left[F_{12}(n)H_{21}(n)-H_{12}(n)F_{21}(n)\right]^2 D_{12}(n)D_{21}(n) \qquad (42)$$

allowing us to simplify the entire procedure of parametrization.

Indeed, the above equation (42) prompts us the following six formulas

$$F_{12}(n) = \exp[+q(n) + q_+(n)], \tag{43}$$

$$G_{12}(n) = \frac{\sinh[q_-(n) - 2q_+(n)]}{\sinh[q_-(n) + q_+(n)]} \exp[+q(n)], \tag{44}$$

$$H_{12}(n) = \exp[+q(n) - q_+(n)] \tag{45}$$

and

$$F_{21}(n) = \exp[-q(n) - q_-(n)], \tag{46}$$

$$G_{21}(n) = \frac{\sinh[q_+(n) - 2q_-(n)]}{\sinh[q_+(n) + q_-(n)]} \exp[-q(n)], \tag{47}$$

$$H_{21}(n) = \exp[-q(n) + q_-(n)] \tag{48}$$

ensuring the explicit parametrizations for the numerators $iF_{12}(n)$, $iG_{12}(n)$, $iH_{12}(n)$ and $iF_{21}(n)$, $iG_{21}(n)$, $iH_{21}(n)$ of the adopted substitutions (34)–(36) and (37)–(39). As for the denominators $D_{12}(n)$ and $D_{21}(n)$, their parametrizations

$$D_{12}(n) = \frac{\sinh[3q_+(n)]}{\sinh[q_+(n) + q_-(n)]} \frac{V[q_+(n), +q(n), q_-(n)]}{V[q_-(n), -q(n), q_+(n)]}, \tag{49}$$

$$D_{21}(n) = \frac{\sinh[3q_-(n)]}{\sinh[q_-(n) + q_+(n)]} \frac{V[q_-(n), -q(n), q_+(n)]}{V[q_+(n), +q(n), q_-(n)]} \tag{50}$$

are essentially determined by the arbitrary coupling function $V[q_+(n), +q(n), q_-(n)]$, where the arguments $q_+(n)$, $q(n)$, $q_-(n)$ must be treated as true field variables.

Speaking broadly, the representability of $D_{12}(n)$ and $D_{21}(n)$ in the so general forms (49) and (50) allows us to establish a wide class of nonlinear lattice models, where the integrability remains preserved, while the function $V[q_+(n), +q(n), q_-(n)]$ might variate either qualitatively or quantitatively. Here, the term "quantitatively" indicates the changes admissible by variations of the mode-mode coupling parameters introducible within qualitatively the same coupling function $V[q_+(n), +q(n), q_-(n)]$, whereas the term "qualitatively" assumes the total replacement of one coupling function by basically another one. Thus, besides the essentially discrete possibility to control the mere type of mode-mode mixing, we gain a good chance to tune the strengths of mode-mode interactions in an essentially continuous manner. Both of these perspectives appear to be useful for the physical applications.

According to the main findings of the present section, the set of independent field variables consistent both with constraint (15) and the original version (25)–(30) of evolution equations can be taken at least in two distinct modifications $q_+(n)$, $q(n)$, $q_-(n)$, $f_{11}(n)$, $g_{11}(n)$, $h_{11}(n)$ or $q_+(n)$, $q(n)$, $q_-(n)$, $\dot{q}_+(n)$, $\dot{q}(n)$, $\dot{q}_-(n)$.

In the next section, we will consider a particular realization of the semidiscrete nonlinear integrable model corresponding to the choice

$$\frac{V[q_+(n), +q(n), q_-(n)]}{V[q_-(n), -q(n), q_+(n)]} = \frac{\sinh[q_-(n)]}{\sinh[q_+(n)]} \tag{51}$$

and assuming the description in terms of the variables $q_+(n)$, $q(n)$, $q_-(n)$, $\dot{q}_+(n)$, $\dot{q}(n)$, $\dot{q}_-(n)$.

4. Lagrangian Equations for the Model with the Global Toda Field

One can easily verify that the system governed by the evolution equations (25)–(30) under adopted constraints (31)–(33) possesses a global Toda mode $t(n)$ defined by the expressions

$$f_{12}(n) + g_{12}(n) + h_{12}(n) = i \exp[+t(n)] \tag{52}$$

and

$$f_{21}(n) + g_{21}(n) + h_{21}(n) = i \exp[-t(n)]. \tag{53}$$

The question arises whether there is any chance to adjust the coupling function $V[q_+(n), +q(n), q_-(n)]$ in such a way as to identify the field variable $q(n)$ with the purely Toda one $t(n)$. The answer turns out to be positive having been substantiated by the coupling ratio (51) presented in the previous section.

Moreover, despite the fact that the dynamics of the field $q(n)$ specified in such a manner is completely described by the separate Toda equation

$$\ddot{q}(n) = \exp[q(n+1) - q(n)] - \exp[q(n) - q(n-1)], \tag{54}$$

the modes $q_+(n)$ and $q_-(n)$ remain to be influenced by the Toda one $q(n)$. In so doing, the dynamics of the model as a whole can be embedded into the explicit Lagrangian form

$$\frac{d}{d\tau}\frac{\partial\mathcal{L}}{\partial\dot{q}_+(n)}=\frac{\partial\mathcal{L}}{\partial q_+(n)}, \tag{55}$$

$$\frac{d}{d\tau}\frac{\partial\mathcal{L}}{\partial\dot{q}(n)}=\frac{\partial\mathcal{L}}{\partial q(n)}, \tag{56}$$

$$\frac{d}{d\tau}\frac{\partial\mathcal{L}}{\partial\dot{q}_-(n)}=\frac{\partial\mathcal{L}}{\partial q_-(n)}\,. \tag{57}$$

Here, the model Lagrangian $\mathcal{L}$ is found to be

$$\begin{aligned}\mathcal{L}=&\sum_{m=-\infty}^{\infty}\left\{\frac{2\cosh^2[q_+(m)]+1}{\sinh^2[3q_+(m)]}\,\dot{q}_+^2(m)+\frac{\dot{q}^2(m)}{2}+\frac{2\cosh^2[q_-(m)]+1}{\sinh^2[3q_-(m)]}\,\dot{q}_-^2(m)\right\}+\\&+\sum_{m=-\infty}^{\infty}\frac{2\cosh[q_+(m)]}{\sinh[3q_+(m)]}\,\dot{q}_+(m)\dot{q}(m)-\sum_{m=-\infty}^{\infty}\frac{2\cosh[q_-(m)]}{\sinh[3q_-(m)]}\,\dot{q}_-(m)\dot{q}(m)-\\&-\sum_{m=-\infty}^{\infty}\frac{4\cosh[q_+(m)]\cosh[q_-(m)]+2\cosh[q_+(m)+q_-(m)]}{\sinh[3q_+(m)]\sinh[3q_-(m)]}\,\dot{q}_+(m)\dot{q}_-(m)-\\&-\sum_{m=-\infty}^{\infty}\frac{\sinh[q_+(m)]\exp[+q(m)-q(m-1)]\sinh[q_-(m-1)]}{\sinh[3q_+(m)]\sinh[q_-(m)]\sinh[3q_-(m-1)]\sinh[q_+(m-1)]}\times\\&\times\Big\{2\sinh[q_+(m)+q_-(m)]\cosh[q_+(m)+q_-(m-1)]\sinh[q_+(m-1)+q_-(m-1)]+\\&+\sinh[2q_+(m)-q_-(m)]\sinh[2q_-(m-1)-q_+(m-1)]\Big\}.\end{aligned} \tag{58}$$

Conversely, considering the Lagrangian equations (55)–(57) as a set of linear algebraic equations with respect to the accelerations $\ddot{q}_+(n)$, $\ddot{q}(n)$, $\ddot{q}_-(n)$, we inevitably recognize the pure Toda equation (54) in one of the solutions. Thus, taking the global character of the Toda mode $q(n)$ into account, the modes $q_+(n)$ and $q_-(n)$ should be referred to as the satellite ones.

5. Conclusion

Summarizing the results of this article, we underline their two aspects.

On the one hand, we have proposed an alternative matrix extension of the Toda system characterized by a two-component spectral parameter. In the framework of the three-component Abel basis for submatrices of the spectral and evolution operators, we have obtained the evolution equations for the generalized nonlinear lattice system and found its zero-curvature representation.

On the other hand, we have managed to introduce the field variables for the basic integrable system in a rather flexible fashion allowing us to construct a class of semidiscrete integrable nonlinear models, where, seemingly, the same field variables are able to acquire a new physical sense from model to model owing to practically inexhaustible possibilities for implementing the mode-mode couplings via two principally distinct types of changes. As an example, we have demonstrated the model with two satellite modes essentially affected by one self-sufficient global Toda mode. We have shown that this particular model allows the explicit Lagrangian formulation in spite of very complicated intermode couplings both in the kinetic and potential parts of the requested Lagrangian function.

It is worth noting that the unparametrized evolution equations (25)–(30) could be derived also from the general matrix Toda lattice [7–9], by using the proper reduction within its 3×3 matrix realization. However, such an approach does not remove the key question how to rebuff the additional constraint akin to the compatibility constraint (15) adopted in the present paper. The most constructive step in this direction is to propose an appropriate parametrization reducing the number of field variables and converting the constraint equations into identities, as it has been demonstrated in Section 3.

In general, the successful parametrization of a new or the already known integrable system is capable to provide the adequate treatment of a particular physical problem [10, 11]. Concerning the multicomponent Toda system, its proper parametrization may shed a new light on the problem of the compression pulse propagation in granular chains [12–17]. As an example, we can point out the evident parallels between the purely theoretical activity on the development of perturbation theory for the Toda lattice with an impurity [18] and the experimental observations of the mechanical energy localization in a strongly nonlinear Hertzian chain with a mass defect [19].

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ВАРІАТИВНІСТЬ МІЖМОДОВИХ ЗВ'ЯЗКІВ
В ТРИКОМПОНЕНТНІЙ МОДЕЛІ ТОДИ

О.О. Вахненко

Р е з ю м е

Запропоновано альтернативне розширення однокомпонентної нелінійної ґратчастої моделі Тоди до трикомпонентної. Головна ідея полягає в заміні елементів стандартного тодівського спектрального оператора на 3×3 матриці, які задані лінійними комбінаціями трьох основних матриць, що утворюють комутативну групу третього порядку. Така процедура вимагає розщеплення раніше єдиного спектрального параметра на дві лінійно незалежні частини. Знайдено представлення нульової кривизни для розширеної ґратчастої системи та явним чином подано одну з її можливих параметризацій, що виділяє суто тодівську моду та дві сателітні моди. В рамках вказаної параметризації одержано лагранжівське формулювання моделі. З огляду на природжену гнучкість допустимих параметризацій окреслено широкий клас інтегровних ґратчастих моделей з найрізноманітнішими варіантами вибору міжмодових зв'язків.