ON THE POSSIBILITY TO USE TRANSITION RADIATION OF ELECTRON BUNCH WITH TRIANGULAR DENSITY PROFILE FOR DIAGNOSTICS OF INHOMOGENEOUS PLASMA

I.O. ANISIMOV, P.V. PARASHCHENKO, YU.M. TOLOCHKEVYCH

Taras Shevchenko National University of Kyiv, Radio Physics Faculty
(5 Build., 2, Academician Glushkov Ave., Kyiv 03127, Ukraine)

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The possibility of the diagnostics of plasma inhomogeneities via the transition radiation of an electron bunch with an initially triangular profile of the current density is considered. The computer simulation is executed to study the dynamics of a triangular bunch in the homogeneous plasma. Analytic calculations of the transition radiation are performed in the given current approximation, because the deformation of such a bunch by excited wake waves is shown to be negligible. The possibility of applications of this method of inhomogeneous plasma diagnostics is confirmed, and the range of its applicability is specified.

1. Introduction

A method of inhomogeneous plasma diagnostics via transition radiation has been proposed in [1]. For the simplest case of isotropic planarly stratified plasma, this method allows one to obtain information about the density profile from the angular dependence of the transition radiation of a modulated electron beam or bunch. Note that the usual method of microwave interferometry gives only information about the plasma density for homogeneous plasma objects [2]. Information about the density profile of an inhomogeneous plasma can be obtained, in principle, with the use of wideband radars (see, e.g. [3]).

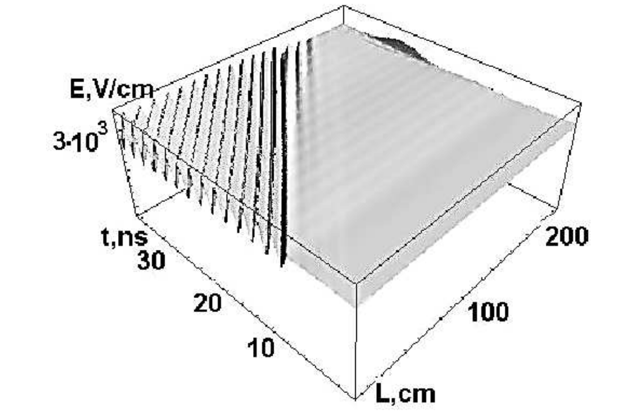
Although so far the method of plasma inhomogeneities diagnostics via the transition radiation has not been confirmed experimentally, theoretical calculations and computer simulations for detailed research were carried out. Particularly, the possibility of the diagnostics of plasma inhomogeneities via the HF (non-resonant) transition radiation of a modulated electron beam [1] or short electron bunch [4, 5] and the resonant transition radiation of such a bunch [6] was studied. The use of electron bunches is not quite suitable, because the bunch motion in plasma leads to the excitation of wake waves. These waves perturb the motion of the electron bunch and consequently its initial density profile [7]. An initially long enough bunch begins to decay into microbunches. Therefore, the calculation of the transition radiation field of such a system becomes too complex, and the solution of the inverse problem is impossible. The radiation of a short bunch can be used, but it is extremely low. So, the registration of the radiation field becomes substantially more complicated.

In what follows, we will consider the bunch with an initially triangular profile of the current density [8]. Simulation results [9] confirm that such a bunch with the sloping rising edge and the step falling edge excites wake waves mainly with its falling edge. So, the deformation of such a bunch will be significantly less than that of the bunch with an initially rectangular density profile. Therefore, the calculation of the transition radiation for a triangular bunch can be carried out in the given current approximation.

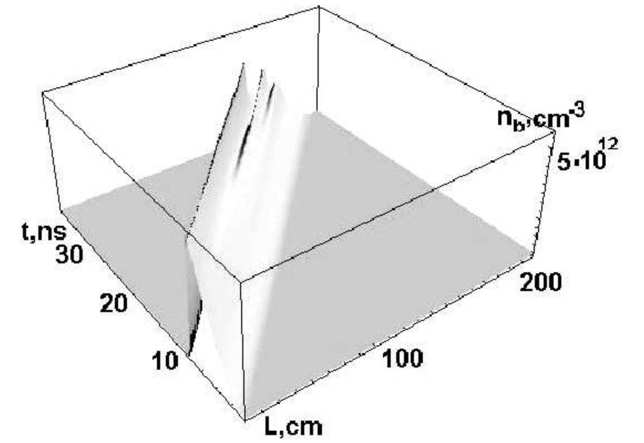
2. Bunch Dynamics

The dynamics of the electron bunch with an initially triangular density profile in the background plasma was studied via computer simulation.

The homogeneous warm plasma was treated. The following parameters were used: the system length was 200 cm, background plasma density was 10^9 cm^{-3}, electron plasma temperature was 2×10^5 K, and ion plasma



a



b

Fig. 1. Space-time distributions of the wake field (a) and the bunch density (b). The bunch velocity is 3×10^9 cm/s, the bunch duration is $T = 3T_{\rm Langm} = 1 \times 10^{-8}$ s, the bunch density is 7×10^6 cm^{-3}, and the plasma density is 10^9 cm^{-3}

temperature was 4×10^4 K. The cold electron bunch with an initially triangular density profile was injected into plasma with an initial velocity in the range $(1 \div 5) \cdot 10^9$ cm/s.

A 1D model was applied. The simulation using the PIC method was carried out using a modified PDP1 package [10].

The simulation results for the wake wave excitation are presented in Fig. 1,a (electric field) and Fig. 1,b (electron bunch density). One can see that the wake wave at the initial stage is excited only by the step falling edge of an electron bunch, and the electric field almost vanishes behind the rising front. But this electric field increases along the bunch trajectory and influences the bunch electrons. This results in the focusing of electrons into microbunches. One can see from Fig. 1 that the initial bunch forms three microbunches far from an injector, because the duration of the initial bunch equals three periods of background plasma Langmuir oscillations.

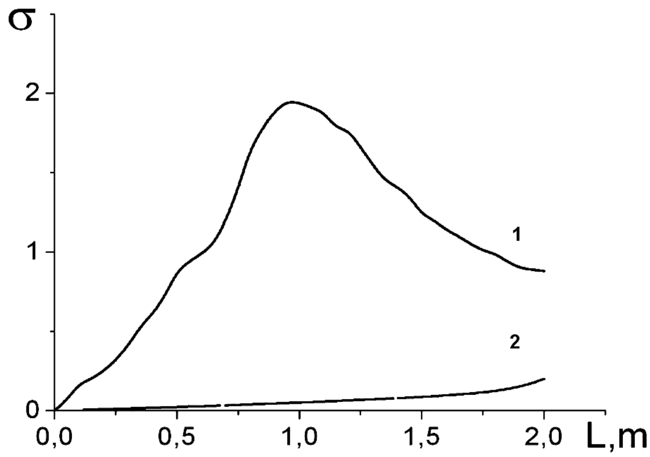


Fig. 2. Spatial dependence of the deformation index for a rectangular bunch (*1*) and a triangular bunch (*2*). The initial bunch density, velocity, and duration are 1×10^7 cm^{-3}, 3×10^9 cm/s, and 3.3×10^{-8} s, respectively, the plasma density is 10^9 cm^{-3}

To characterize the bunch deformation during its motion in plasma, it is convenient to introduce some integral characteristic which describes this process, namely the deformation index [7]:

$$\sigma = \int\limits_{-\infty}^{+\infty} \left[n_{0b}(x) - n_b(x)\right]^2 \, dx \Big/ \int\limits_{-\infty}^{+\infty} n_{0b}^2(x)\, dx, \qquad (1)$$

where $n_{0b}(x)$ and $n_b(x)$ are the bunch density profiles at the start and at the given time point, respectively. If the density distribution of the electron bunch remains constant, the deformation index is equal to zero. A perturbation of this parameter grows with the bunch density. Figure 2 presents the spatial dependences of deformation indices for two bunches with initially triangular and rectangular density profiles and with the same initial velocities and maximal densities.

Figure 3,a presents the dependences of the maximum amplitude of the wake wave field excited by a triangular electron bunch on the initial bunch density. The character of this dependence is close to linear. The dependence of the maximal wake field on the bunch duration is plotted in Fig. 3,b. The maximum field decreases slightly, as the bunch duration increases.

Simulation results confirm that the electron bunch with an initially triangular density profile is much less perturbed during its motion in plasma in comparison

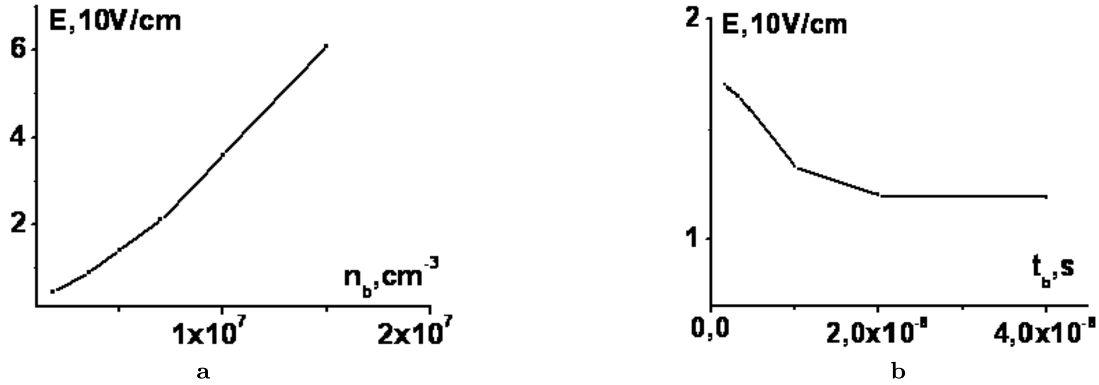


Fig. 3. Dependences of the maximal wake field on the initial bunch density (*a*) and the initial bunch duration (*b*). The bunch velocity is 3×10^9 cm/s, and the background plasma density is 10^9 cm^{-3}

with the bunch with an initially rectangular profile [7]. The minimum deformation takes place in the case where the triangular bunch length is less than the wake wave length. Therefore, the calculation of the transition radiation for a triangular bunch can be carried out in the given current approximation.

3. Model Description

We will consider a cold isotropic plasma. Let its density depend only on the z coordinate, and $n(z \to \pm\infty) \to 0$. Let the axially symmetric radially bounded electron bunch with a given current density move along the plasma density gradient with the velocity v_0:

$$j(r,t,z) = en(z,t)v_0\Phi(r). \qquad (2)$$

Here, $\Phi(r)$ is a function that characterizes the current density distribution in the bunch cross-section ($\Phi(r \to \infty) \to 0$), and $n(z,t)$ is the triangular density profile along the z axis:

$$n(z,t) = \begin{cases} n_0(1 - \frac{\zeta}{L}), & 0 < \zeta < L, \\ 0, & \zeta \le 0, \zeta \ge L, \end{cases} \qquad (3)$$

$$\zeta = z - vt.$$

Calculations are performed in the given current approximation. The high-frequency (non-resonant) component will be found out, because the solution of the inverse problem in this case is much simpler.

4. Analytical Calculation

The magnetic field excited by current (2) in plasma satisfies the wave equation

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi^0)\right] + \frac{\partial^2 H_\varphi^0}{\partial z^2} - \frac{1}{\varepsilon(z)}\frac{\partial \varepsilon(z)}{\partial z}\frac{\partial H_\varphi^0}{\partial z} +$$

$$+k_0^2\varepsilon(z)H_\varphi^0 = \frac{4\pi}{c}\frac{\partial j}{\partial r}, \qquad (4)$$

$$\varepsilon(z) = 1 - \frac{4\pi n(z)e^2}{m\omega(\omega + \imath\nu)}. \qquad (5)$$

We now substitute the solution in the form of a Fourier integral in (4):

$$H_\varphi^0(r,z,t) = \int_{-\infty}^{+\infty} H_\varphi(r,z,\omega)\exp(\imath\omega t)\,d\omega, \qquad (6)$$

$$j(r,z,t) = \int_{-\infty}^{+\infty} j(r,z,\kappa)\exp(\imath\omega t)\,d\omega, \qquad \kappa = \frac{\omega}{v_0}. \qquad (7)$$

For the chosen bunch profile,

$$j(r,z,\kappa) = -\frac{en_0v_0}{2\pi}\Phi(r)n(\kappa)\exp(-\imath\kappa z), \qquad (8)$$

$$n(\kappa) = \frac{1}{\kappa^2 L}\left[1 + \imath\kappa L - \exp(\imath\kappa L)\right]. \qquad (9)$$

Substituting (6)–(7) in (4), we obtain the equation for the high-frequency component of the magnetic field:

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi)\right]+\frac{\partial^2 H_\varphi}{\partial z^2}-\frac{1}{\varepsilon(z)}\frac{\partial\varepsilon(z)}{\partial z}\frac{\partial H_\varphi}{\partial z}+$$

$$+k_0^2\varepsilon(z)H_\varphi=\frac{2en_0v_0n(\kappa)}{c}\exp(-\imath\kappa z)\frac{\partial}{\partial r}[\Phi(r)]. \quad (10)$$

A solution of this equation can be presented as the Fourier–Bessel integral over the radius:

$$H_\varphi(r,z)=\int\limits_0^\infty h_\varphi(k_r,z)J_1(k_r r)k_r\,dk_r. \quad (11)$$

Substituting (11) in (10), we get the equation for h_φ:

$$\frac{d^2h_\varphi}{dz^2}-\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\frac{dh_\varphi}{dz}+(\varepsilon k_0^2-k_r^2)h_\varphi=$$

$$=\frac{2ev_0n_0}{c}n(\kappa)k_r\Phi(k_r)\exp(-\imath\kappa z), \quad (12)$$

$$\Phi(k_r)=\int\limits_0^\infty \Phi(r)J_0(k_r r)r\,dr. \quad (13)$$

Suppose that the radiation frequency ω is high enough to satisfy the condition

$$|\Delta(z)|\ll 1, \qquad \Delta(z)=1-\varepsilon(z). \quad (14)$$

Then Eq. (12) can be solved with the use of the method of successive approximations in the small parameter Δ:

$$h_\varphi=h^{(0)}+h^{(1)}+\cdots \quad (15)$$

Equations in the zero and first approximations read, respectively:

$$\frac{d^2h^{(0)}}{dz^2}+k_z^2h^{(0)}=\frac{2ev_0n_0}{c}n(\kappa)k_r\Phi(k_r)\exp(-\imath\kappa z),$$

$$k_z^2=k_0^2-k_r^2; \quad (16)$$

$$\frac{d^2h^{(1)}}{dz^2}+k_z^2h^{(1)}=\left(k_0^2\Delta+\imath\kappa\frac{d\Delta}{dz}\right)h^{(0)}. \quad (17)$$

The solution of (16) that describes the electron bunch field without plasma has the form

$$h^{(0)}=\frac{ev_0n_0n(\kappa)k_r\Phi(k_r)}{c(k_z^2-\kappa^2)}\exp(-\imath\kappa z). \quad (18)$$

The scattering of field (18) by plasma inhomogeneities forms the transition radiation. The solution of Eq. (17) corresponding to the lack of electromagnetic waves falling on the plasma inhomogeneity has the form [11]

$$h^{(1)}=f_+\int\limits_{-\infty}^{z}\frac{f_-F}{W}\,dz+f_-\int\limits_{z}^{+\infty}\frac{f_+F}{W}\,dz, \quad (19)$$

where $f_\pm=\exp(-\imath k_z z)$ are solutions of (17) without the right part, corresponding to electromagnetic waves running to the right and to the left along the z axis, $W=-\imath 2k_z$ is their Wronskian, and F is the right part of (17). The amplitudes of forward and backward radiation, respectively, can be written as

$$h_m^{(1)}(z\to\pm\infty)=\int\limits_{-\infty}^{+\infty}\frac{f_\pm F}{W}\,dz. \quad (20)$$

The function $\Delta(z)$ can be presented as a Fourier integral to calculate integral (20):

$$\Delta(z)=\int\limits_{-\infty}^{+\infty}\Delta(K)\exp(\imath Kz)\,dK. \quad (21)$$

Substituting (21) in (20) and changing the order of integration, we obtain

$$h_m^{(1)}(z\to\pm\infty)=2\pi\imath ev_0n_0n(\kappa)k_r\Phi(k_r)\times$$

$$\times\frac{\Delta(\kappa\mp k_z)(k_0^2-\kappa^2\pm k_z\kappa)}{ck_z(k_z^2-\kappa^2)}. \quad (22)$$

Solution (22) should be substituted to (11). For $k_r>k_0$, the solutions describe surface waves with an amplitude decreasing exponentially, as $|z|$ increases. Therefore, the integration should be carried out in the interval $0\le k_r\le k_0$ to obtain the radiation in the far zone.

It is suitable to turn from the variable k_r to the variable Θ defined by the relations

$$k_z=k_0\cos\Theta, \quad z\to+\infty,$$

$$k_z=-k_0\cos\Theta,\ z\to-\infty. \quad (23)$$

It is also suitable to use the spherical coordinate system $r = R\sin\theta$, $z = R\cos\theta$. Combining the integrals for forward and backward radiation, we get

$$H_\varphi(R,\theta,\omega) = \frac{\imath 2\pi k_0^2 e v_0 n_0 n(\kappa)}{c} \int\limits_0^\pi \frac{J_1(k_0 R\sin\theta\sin\Theta)}{(k_0^2\cos^2\Theta - \kappa^2)} \times$$

$$\times\Delta(\kappa - k_0\cos\Theta)\sin^2\Theta(k_0^2 - \kappa^2 + \kappa k_0\cos\Theta)\times$$

$$\times\Phi(k_0\sin\Theta)\exp(-\imath k_0 R\cos\theta\cos\Theta)\, d\Theta. \quad (24)$$

The far radiation zone corresponds to the case $R \to \infty$. Then integral (24) can be calculated using the stationary phase method. Replacing the Bessel functions by their asymptotics, we have

$$H_\varphi(R,\theta,\omega) = \frac{\imath 2\pi k_0^2 e v_0 n_0 n(\kappa)\Delta(\kappa - k_0\cos\theta)}{c(\kappa^2 - k_0^2\cos^2\theta)} \times$$

$$\times\Phi(k_0\sin\theta)(k_0^2 - \kappa^2 + \kappa k_0\cos\theta)\times$$

$$\times\sin\theta\frac{\exp(-\imath k_0 R)}{k_0 R}, \quad (25)$$

where $0 \le \theta \le \pi$, and $n(\kappa)$ is given by (9).

5. Discussion

The analysis of the HF magnetic field H_φ (25) in the far radiation zone shows that the spatial spectrum of plasma inhomogeneities in the range of K from $\kappa - k_0$ to $\kappa + k_0$ can be obtained from the measurements of the radiation field in the range of θ from 0 to π at a certain frequency ω_0.

For non-relativistic bunches, $\kappa = \omega/v_0$, $k_0 = \omega/c$, and $\kappa \gg k_0$, because $c \gg v_0$, so such a range will be relatively narrow and will give no sufficient information about the function $\varepsilon(z)$. However, for the relativistic bunch, one can obtain $\kappa \approx k_0$, so the function $\Delta(K)$ can be found out in the range from 0 to $2k_0$. If the characteristic size l of an inhomogeneity $\varepsilon(z)$ is significantly larger than the vacuum wavelength of the transition radiation,

$$\lambda = \frac{2\pi}{k_0} \le l, \quad (26)$$

the plasma density profile can be found from the spectrum $\Delta(K)$.

In the experiment, the radiation treated must not be very small. It is clear from (25) that this condition is always violated when $\theta \to 0$ and $\theta \to \pi$. For a relativistic beam ($\beta = v_0/c \le 0.62$), another zero of the function $H_\varphi(\theta)$ appears when $\theta = \arccos(\beta^{-1} - \beta)$. It should also be aware that the radiation of an ultra-relativistic bunch is focused in the range of θ near 0 and π and gives a very limited information about $\varepsilon(z)$. Thus, moderately relativistic bunches ($v_0/c \approx 0.7 \div 0.9$) are the most convenient for the plasma inhomogeneities' diagnostics.

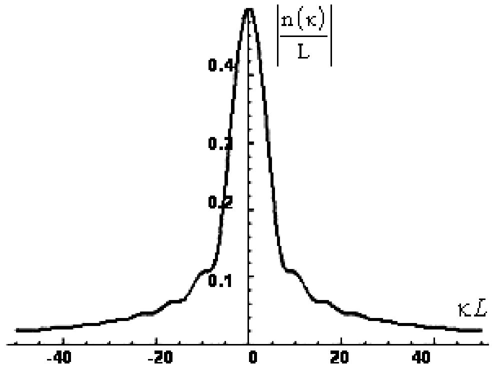


Fig. 4. Normalized spectral function $n(\kappa)$

For $\theta \to 0$, $\Phi(k_0\sin\theta) \to 0$ in all the cases. This function vanishes for $\sin\theta \ge 4/k_0 a$, where a is the bunch radius. Therefore, bunches with radius of the order of the radiated waves' length are preferable.

The normalized spectral function (9) is plotted in Fig. 4. The half-width of the spectrum range is $\Delta\kappa \approx 3.4/L$, where L is the bunch length. The diagnostics of a plasma inhomogeneity $\Delta(K)$ is possible only if the spectral width of the function $\Delta\kappa$ exceeds the width of the function $\Delta(\kappa - k_0\cos\theta)$, i.e. approximately $2\pi/l$, where l is the characteristic length of the plasma inhomogeneity. This condition can be presented in the form

$$l \gg L. \quad (27)$$

Therefore, selecting the appropriate bunch length, one can always satisfy condition (27).

The charged bunch moving in plasma excites wake waves, so the minimum deformation of the initial current density profile takes place for

$$L > \lambda_{\text{wake}} = \frac{2\pi v_0}{\omega_p}, \quad (28)$$

where $v_0 \approx c$ for relativistic bunches. Combining conditions (27)–(28), we get

$$l \gg L > \lambda_{\text{wake}}. \quad (29)$$

Therefore, the diagnostics of plasma inhomogeneities via the transition radiation in the analyzed case is possible only under condition (29).

6. Conclusions

One can obtain the spatial spectrum of $\varepsilon(z)$ in the range of wave numbers from $\kappa - k_0$ to $\kappa + k_0$ from the measurements of the high-frequency (non-resonant) transition radiation excited by the charged bunch with an initially triangular profile moving in the inhomogeneous plasma. The transition radiation frequency should significantly exceed the Langmuir frequency. The bunch should be moderately relativistic, and its radius must be less than the wavelength of the transition radiation. The bunch length must be large on the scale of the wake wave length and much less than the characteristic length of plasma inhomogeneities.

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ПРО МОЖЛИВІСТЬ ВИКОРИСТАННЯ ПЕРЕХІДНОГО ВИПРОМІНЮВАННЯ ЕЛЕКТРОННОГО ЗГУСТКУ З ТРИКУТНИМ ПРОФІЛЕМ КОНЦЕНТРАЦІЇ ДЛЯ ДІАГНОСТИКИ НЕОДНОРІДНОЇ ПЛАЗМИ

І.О. Анісімов, П.В. Паращенко, Ю.М. Толочкевич

Р е з ю м е

Розглянуто можливість діагностики плазмової неоднорідності за перехідним випромінюванням електронного згустку з трикутним профілем концентрації. Проведено комп'ютерне моделювання для дослідження динаміки трикутного згустку в однорідній плазмі. Розрахунок перехідного випромінювання виконано в наближенні заданого струму, оскільки для такого згустку вплив збуджених ним кільватерних хвиль на його динаміку є незначним. Підтверджена можливість застосування запропонованого методу діагностики неоднорідної плазми та визначені межі його застосовності.