SUPERSYMMETRY REPRESENTATION OF BOSE–EINSTEIN CONDENSATION OF FERMION PAIRS

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We consider supersymmetry field theory with supercomponents being the square root of the Bose-condensate density, the amplitude of its fluctuations, and Grassmannian fields related to the density of Fermi particles. The fermion number is conserved in degenerated Fermi–Bose mixtures with unbroken supersymmetry when the system is invariant with respect to the inversion of the time arrow. We show that the supersymmetry breaking allows one to obtain field equations describing time–space dependences for real Bose–Fermi mixtures. The solution of these equations reveals that the cooled system with homogeneously distributed fermions arrives spontaneously at strong inhomogeneous fluctuations at a critical temperature, while, with the following temperature decrease, an inhomogeneously distributed Bose-condensate appears at a lower temperature dependent on the fermion density.

1. Introduction

Supersymmetry is one of the most beautiful productive conceptions of contemporary physics which has been proposed to describe the microworld (see works [1] and [2] reviewing the supersymmetry applications to the quantum mechanics, the survey [3] deals with disorder metals, and [4–8] consider the theory of superstrings). Based on the idea proposed in work [9] and developed in [10] and [11], it has been shown that the supersymmetry may be used effectively to present fluctuative fields determining the picture of a phase transition with both symmetry and ergodicity breaking in nonequilibrium condensed matter [12, 13] and random heteropolymers [14]. Along this line, the Martin–Siggia–Rose method [15] has been used as a basis permitting to combine stochastic fields into a supersymmetry construction [16]. One can perceive that the supersymmetry approach is developed along two lines: the former is based on the quantum operator representation [1–3], the latter considers the evolution of a supersymmetric field [4–12, 14–16].

Historically, the priority in the supersymmetry development belongs to researchers of the Former Soviet Union [17–20]. Since western researchers have not known anything about works done in the Soviet Union, their rediscovery of the supersymmetry has been made independently later on [21]. In 1971, Gol'fand and Lichtman [17] have found supersymmetry multiplets related to a massive photon and a photino, a charged Dirac spinor and two charged scalars, being particles with spin 0; they have observed as well the vanishing of the vacuum energy within a supersymmetric algebraic representation. One year later, Volkov and Akulov [18] stimulated by Heisenberg's ideas have tried to associate a massless fermion appearing as a result of the spontaneous supersymmetry breaking with a neutrino. Then, Volkov and Soroka [19] have gauged the super-Poincaré group that has led to the supergravity yielding a superpartner of a graviton with spin 3/2. To this end, a formalism related has been provided previously by Berezin and Katz [20]. In 1973, Wess and Zumino [21] have built the supersymmetry string theory basing on Veneziano's and Nambu's models of strong interactions.

Contemporary supersymmetry string theory is a unique theory expected to give a unified description of all interactions in nature [4–8]. However, none of the superpartners of any known elementary particles has been found in experiments so far. Therefore, it is very important to study the supersymmetry breaking. Such an opportunity is opened with experimental progress in atomic mixtures of ultracold Bose and Fermi atoms [22]. Theoretically, an ultracold superstring model was constructed [23], as well as an exactly solvable model of one-dimensional Bose–Fermi mixture was investigated [24]. According to [25], the supersymmetry is always broken either spontaneously or by a chemical potential difference between bosons and fermions. This article is devoted to the consideration of the Bose–Fermi mixture within the field-theoretic supersymmetry approach [16].

The outline of the paper is as follows. In Section 2, we adduce main field-theoretic statements based on the generating functional method introduced by Martin, Siggia, and Rose. Making use of this method allows us to write down both the supersymmetric Lagrangian of the problem and the related Euler equations. Section 3 is devoted to the derivation of the field equations for the superfield components being the most probable values of the square root of the Bose-condensate density, the amplitude of its fluctuations, and the most probable Grassmannian fields giving the density of the Fermi particles. Combining the above-mentioned equations, we show that the fermion number is conserved in supersymmetrically degenerated systems, whose superspace is invariant with respect to a rotation. According to Section 4, the supersymmetry invariance is broken by means of the fixation of such a rotation that results in the loss of the invariance of Grassmannian components with respect to the time inversion due to the Bose–Einstein condensation. We show that the supersymmetry breaking allows us to obtain the dependence of the Bose-condensate density on the density of Fermi particles. Section 5 is devoted to the discussion of the obtained results, and Appendix contains details of calculations at the derivation of the field equations.

2. Main Field-Theoretic Statements

We consider an attractive Fermi system characterized by the pair of conjugate wave functions $\psi(\mathbf{r},t)$, $\overline{\psi}(\mathbf{r},t)$ and the Bose condensate with the density $n(\mathbf{r},t)$, where $\mathbf{r}$ and t are the coordinate and time, respectively. The behavior of the Bose subsystem is presented by the fluctuating order parameter

$$x(\mathbf{r},t) := \sqrt{n(\mathbf{r},t)}\mathrm{e}^{\mathrm{i}\phi(\mathbf{r},t)} \tag{1}$$

with $\phi(\mathbf{r},t)$ being a condensate phase. Within the framework of the standard field-theoretic scheme [16], the system evolution is described by the Langevin equation

$$\dot{x}(\mathbf{r},t) - D\nabla^2 x = -\gamma\,\frac{\partial F}{\partial x} + \zeta(\mathbf{r},t). \tag{2}$$

Here, a dot stands for the time derivative, $\nabla \equiv \partial/\partial\mathbf{r}$, D and γ are the diffusion and kinetic coefficients, respectively, $F(x)$ is the specific free energy of the condensate, and $\zeta(\mathbf{r},t)$ is a stochastic addition defined by the white noise conditions

$$\langle\zeta(\mathbf{r},t)\rangle = 0, \qquad \langle\zeta(\mathbf{r},t)\zeta(\mathbf{0},0)\rangle = \gamma T\delta(\mathbf{r})\delta(t), \tag{3}$$

where the broken brackets notice the averaging over system states scattered with dispersion T, being the temperature measured in energy units. With the introduction of the scales $t_s \equiv (\gamma T)^2/D^3$, $r_s \equiv \gamma T/D$, $F_s \equiv D^3/\gamma^3T^2$, and $\zeta_s \equiv D^3/(\gamma T)^2$ for time t, coordinate $\mathbf{r}$, specific free energy F and stochastic force ζ, respectively, the equation of motion (2) takes the simple form

$$\dot{x}(\mathbf{r},t) = -\frac{\delta\mathcal{F}}{\delta x} + \zeta(\mathbf{r},t), \tag{4}$$

where the short notation of a variational derivative

$$\frac{\delta\mathcal{F}}{\delta x} \equiv \frac{\delta\mathcal{F}\{x(\mathbf{r},t)\}}{\delta x(\mathbf{r},t)} = \frac{\partial F(x)}{\partial x} - \nabla^2 x \tag{5}$$

is used for the Ginzburg–Landau model

$$\mathcal{F}\{x\} \equiv \int\left[F(x) + \frac{1}{2}(\nabla x)^2\right]\mathrm{d}\mathbf{r}. \tag{6}$$

Along the standard line [16], our approach is stated on the generating functional

$$Z\{u(\mathbf{r},t)\} = \int Z\{x(\mathbf{r},t)\}\exp\left(\int ux\mathrm{d}\mathbf{r}\mathrm{d}t\right)\mathrm{D}x(\mathbf{r},t), \tag{7}$$

being determined by the partition functional

$$Z\{x(\mathbf{r},t)\} := \left\langle\prod_{(\mathbf{r},t)}\delta\left\{\dot{x} + \frac{\delta F}{\delta x} - \zeta\right\}\det\left|\frac{\delta\zeta}{\delta x}\right|\right\rangle. \tag{8}$$

Here, the argument of the δ-functional takes the equation of motion (4) into account, and the determinant is the Jacobian of the transition from the noise field $\zeta(\mathbf{r},t)$ to the order parameter $x(\mathbf{r},t)$. Over the distribution of the latter, the continuous integration in definition (7) is carried out.

Within the simple case of the Itô calculus, the Jacobian determinant equals one, and expression (8) arrives at the pair of Bose fields only [26]. A much more interesting situation is generated by the Stratonovich calculus, when the Jacobian

$$\det\left|\frac{\delta\zeta}{\delta x}\right| = \int \exp\left(\overline{\psi}\frac{\delta\zeta}{\delta x}\psi\right) d^2\psi, \quad d^2\psi = \mathrm{d}\psi\ d\overline{\psi} \tag{9}$$

is presented by Grassmannian conjugate fields $\psi(\mathbf{r},t)$ and $\overline{\psi}(\mathbf{r},t)$ which satisfy the conditions

$$\overline{\psi}\psi + \psi\overline{\psi} = 0, \quad \int \mathrm{d}\psi = 0;$$

$$\int \psi\ \mathrm{d}\psi = 1, \quad \int \mathrm{d}\overline{\psi} = 0, \quad \int \overline{\psi}\ \mathrm{d}\overline{\psi} = 1. \tag{10}$$

Then, after the generalized Laplace transformation of the δ-functional in Eq. (8), we obtain the supersymmetry Lagrangian

$$\mathcal{L}(x,p,\psi,\overline{\psi}) = \left(p\dot{x} - \frac{p^2}{2} + \frac{\delta F}{\delta x}p\right) -$$

$$-\overline{\psi}\left(\frac{\partial}{\partial t} + \frac{\delta^2 F}{\delta x^2}\right)\psi \tag{11}$$

with a ghost field $p(\mathbf{r},t)$. Introducing the four-component superfield

$$\Phi := x + \overline{\theta}\psi + \overline{\psi}\theta + \overline{\theta}\theta p, \tag{12}$$

it is easy to convince ourselves that expression (11) can be written in the canonical supersymmetric form

$$\mathcal{L} = \int \Lambda d^2\theta, \quad d^2\theta \equiv d\theta d\overline{\theta},$$

$$\Lambda(\Phi) \equiv \frac{1}{2}(\overline{\mathcal{D}}\Phi)\,(\mathcal{D}\Phi) + F(\Phi), \tag{13}$$

where θ and $\overline{\theta}$ are Grassmannian conjugate coordinates defined by the properties

$$\overline{\theta}\theta + \theta\overline{\theta} = 0, \quad \int \mathrm{d}\theta = 0;$$

$$\int \theta d\theta = 1, \quad \int d\overline{\theta} = 0, \quad \int \overline{\theta} d\overline{\theta} = 1, \tag{14}$$

being similar to Eqs. (10). The supersymmetry generators in Eq. (13) are as follows:

$$\mathcal{D} := \frac{\partial}{\partial\overline{\theta}} - 2\theta\frac{\partial}{\partial t}, \qquad \overline{\mathcal{D}} := \frac{\partial}{\partial\theta}. \tag{15}$$

3. Supersymmetry Field Equations

A variation of the action related to the supersymmetric Lagrangian (13) over superfield (12) leads to the supersymmetry Euler equation

$$\frac{1}{2}[\overline{\mathcal{D}},\mathcal{D}]\Phi = \frac{\delta\mathcal{F}}{\delta\Phi}, \tag{16}$$

where the square brackets mean the commutation. As is shown in Appendix, projections of Eq. (16) onto the superspace axes 1, $\overline{\theta}$, θ and $\overline{\theta}\theta$ arrive at the explicit form of the equations of motion:

$$\dot{\eta} - \nabla^2\eta = -\frac{\partial F}{\partial\eta} + \varphi, \tag{17}$$

$$\dot{\varphi} + \nabla^2\varphi = \frac{\partial^2 F}{\partial\eta^2}\ \varphi - \frac{\partial^3 F}{\partial\eta^3}\ \overline{\Psi}\Psi, \tag{18}$$

$$\dot{\Psi} - \nabla^2\Psi = -\frac{\partial^2 F}{\partial\eta^2}\ \Psi, \tag{19}$$

$$-\dot{\overline{\Psi}} - \nabla^2\overline{\Psi} = -\frac{\partial^2 F}{\partial\eta^2}\ \overline{\Psi}. \tag{20}$$

Since minimal action relates to the most probable realizations of the superfield (12), solutions of equations (17) — (20) determine the most probable components $x^{(\max)} \equiv \eta$, $p^{(\max)} \equiv \varphi$, $\psi^{(\max)} \equiv \Psi$, $\overline{\psi}^{(\max)} \equiv \overline{\Psi}$. The first of these equations takes the form of the Langevin equation (4). This indicates that a ghost field $\varphi \equiv p^{(max)}$ represents the most probable realization of the fluctuation amplitude ζ. A specific peculiarity of the field φ is that the gradient term in the governing equation (18) has inverse sign, so that the inhomogeneity in the space distribution of the most probable fluctuation increases in the course of the time until a non-linearity stabilizes its amplitude.

Another feature consists in the Grassmannian conjugation of Eqs. (19) and (20) which coincide with each other if the time arrow is inverted in one of them. Thus, one can conclude that the pair of conjugate fields Ψ and $\overline{\Psi}$ describes the evolution of a Fermi particle and a Fermi antiparticle, for which the time runs in opposite directions. Combining Eqs. (19) and (20) yields the continuity equation

$$\dot{\rho} + \nabla\mathbf{j} = 0 \tag{21}$$

for the fermion density

$$\rho := \overline{\Psi}\Psi \tag{22}$$

and the relevant current

$$\mathbf{j} := \nabla\overline{\Psi}\Psi - \overline{\Psi}\nabla\Psi. \tag{23}$$

Equation (21) expresses the conservation law of the number of Fermi particles in a supersymmetric system, whose state space spanned onto axes 1, $\overline{\theta}$, θ, and $\overline{\theta}\theta$ is invariant with respect to a direction choice.

4. Supersymmetry Breaking

To break the above invariance, we will follow the Bogolyubov method of quasiaverages, according to which taking off the system degeneration is provided by switching an infinitesimal source of the type of a slight magnetic field in magnets [27]. In our case, the role of such a field is played by the conjugate Grassmannian components $\overline{\theta}\Psi$ and $\overline{\Psi}\theta$ which are related to the forward and backward directions of the time arrow. Formally, we should replace superfield (12) by the transformed field

$$\widetilde{\Phi} := \mathrm{e}^{-\overline{\theta}\Psi}\Phi\mathrm{e}^{\overline{\theta}\Psi}. \tag{24}$$

Writing this superfield in the explicit form

$$\widetilde{\Phi} = x + (1-x)\,\overline{\theta}\Psi + (1+x)\,\overline{\Psi}\theta + (p + x\rho)\,\overline{\theta}\theta, \tag{25}$$

it is easy to see that transformation (24) squeezes the axis $\overline{\theta}\Psi$ and stretches the axis $\overline{\Psi}\theta$ by the value x being the order parameter of the Bose condensate, while the axis $\overline{\theta}\theta$ is stretched by the value $x\rho$ proportional to both order parameter and density of Fermi particles $\rho = \overline{\Psi}\Psi$. In any case, the above transformation breaks the supersymmetry, so that the Euler equation (16) is reduced to the components (see Appendix)

$$\frac{\partial\eta}{\partial t} - \nabla^2\eta = -\frac{\partial F}{\partial\eta} + \varphi + \eta\rho, \tag{26}$$

$$\frac{\partial}{\partial t}(\varphi + \eta\rho) + \nabla^2\varphi =$$

$$= \frac{\partial^2 F}{\partial\eta^2}(\varphi + \eta\rho) - \frac{\partial^3 F}{\partial\eta^3}\left(1-\eta^2\right)\rho, \tag{27}$$

$$\Psi\frac{\partial}{\partial t}\ln\left[(1-\eta)\,\Psi\right] - \nabla^2\Psi = -\frac{\partial^2 F}{\partial\eta^2}\Psi, \tag{28}$$

$$-\overline{\Psi}\frac{\partial}{\partial t}\ln\left[(1+\eta)\,\overline{\Psi}\right] - \nabla^2\overline{\Psi} = -\frac{\partial^2 F}{\partial\eta^2}\overline{\Psi}. \tag{29}$$

In contrast to Eqs. (19) and (20), the pair of equations (28) and (29) becomes non-invariant with respect to the time inversion due to the Bose–Einstein condensate appearance ($\eta \neq 0$). Combining Eqs. (28) and (29), we obtain

$$\rho\frac{\partial}{\partial t}\ln\left[\left(1-\eta^2\right)\rho\right] + \nabla\mathbf{j} = 0. \tag{30}$$

At the steady-state condensation ($\mathbf{j}$ = const), Eq.(30) yields the relation

$$n = 1 - \frac{\rho_c}{\rho}, \tag{31}$$

where the integration constant ρ_c plays the role of a critical density of fermions, and we take definition (1) into account, according to which $\eta^2 = n$. Dependence (31) means that the density n of the Bose-condensate increases steadily from $n = 0$ to $n = 1$ with growth of the density ρ of Fermi particles above a critical value ρ_c.

5. Discussion

A characteristic feature of our consideration consists in making use of the supersymmetry field theory that is based on the principle of minimal superaction

$$S\{\Phi(\mathbf{r},t)\} := \int \mathcal{L}[\Phi(\mathbf{r},t)]\mathrm{d}\mathbf{r}\mathrm{d}t,$$

whose values are related to the maximal probability

$$P\{\Phi(\mathbf{r},t)\} \propto \exp\left(-S\{\Phi(\mathbf{r},t)\}\right)$$

in the distribution of the system over superfields (12). As a result, the governing equations (26)–(29) determine the most probable Bose components η and φ and Fermi ones Ψ and $\overline{\Psi}$. Such a description differs crucially from the standard picture, where the observable values are determined in terms of averages over sets of quantum states.

According to Eqs. (19) and (20), Fermi–Bose mixtures with unbroken supersymmetry are invariant with respect to the inversion of the time arrow. To break this symmetry, we transform superfield (12) to form (24), whose explicit appearance (25) reveals the breaking of the above invariance due to the Bose–Einstein condensation. Because the time runs always forward in macroscopical systems, one needs to use system (26)–(29) to describe real Fermi–Bose mixtures.

To represent the system behavior, let us study first the simplest case of a steady-state homogeneous Fermi–Bose mixture. It is described by Eqs. (26), (27), and

(30) which are simplified to the forms

$$\varphi + \eta\rho = \frac{\partial F}{\partial \eta}, \tag{32}$$

$$\frac{\partial^2 F}{\partial \eta^2}(\varphi + \eta\rho) = \frac{\partial^3 F}{\partial \eta^3}\left(1-\eta^2\right)\rho, \tag{33}$$

$$\left(1-\eta^2\right)\rho = \rho_c. \tag{34}$$

To reveal the effect of external conditions, let us introduce the Landau free energy

$$F := -\frac{\varepsilon}{2}\eta^2 + \frac{1}{4}\eta^4, \quad \varepsilon \equiv \frac{T_c - T}{T_c} \tag{35}$$

with the parameter ε determining a moving off a critical temperature T_c. Then, the solution of Eqs. (32)–(34) gives the stationary order parameter

$$\eta_0^2 = \frac{2}{3}\varepsilon - \sqrt{\frac{1}{9}\varepsilon^2 + 2\rho_c}. \tag{36}$$

It takes a physically meaningful magnitude $\eta_0 \neq 0$, when the parameter ε exceeds the critical value

$$\varepsilon_c = \sqrt{6\rho_c} \tag{37}$$

fixed by the critical density of fermions ρ_c (together with the critical temperature T_c, the above density ρ_c represents a phenomenological parameter of the developed theory).

Expression (36) takes the familiar square-root form

$$\eta_0 \simeq \sqrt{\frac{\varepsilon - \varepsilon_c}{2}} \tag{38}$$

in a vicinity $\varepsilon - \varepsilon_c \ll \varepsilon_c$ of the critical curve (37). Here, Eqs. (32)–(34) give the stationary amplitude of fluctuations

$$\varphi_0 \simeq -\left(\varepsilon_c + \rho_c\right)\sqrt{\frac{\varepsilon - \varepsilon_c}{2}} \tag{39}$$

and the related density of fermions

$$\rho_0 \simeq \rho_c\left(1 + \frac{\varepsilon - \varepsilon_c}{2}\right). \tag{40}$$

According to Eqs. (38) and (40), the densities of both fermions and bosons grow linearly with decrease in the temperature near the critical curve (37), whereas the fluctuation amplitude (39) takes negative values varying in the square-root manner.

Let us consider finally an inhomogeneous steady-state system described by the equations

$$\nabla^2\eta = -\left(\varepsilon - \eta^2\right)\eta - (\varphi + \eta\rho), \tag{41}$$

$$\nabla^2\varphi = -\left(\varepsilon - 3\eta^2\right)(\varphi + \eta\rho) - 6\left(1-\eta^2\right)\eta\rho \tag{42}$$

following from Eqs. (32) and (33), where the Landau free energy (35) is used. The linearization of these equations with regard for Eq.(34) shows that a decrease in the temperature gives initially ($\varepsilon = 0$) the loss of homogeneity in the space distribution of the fluctuation amplitude, and then (at $\varepsilon = \varepsilon_c$) the order parameter distribution becomes inhomogeneous. What about the fermion distribution, it is supposed to be homogeneous due to the equilibrium condition $\mathbf{j} = 0$ in the continuity equation (30) (consideration of the more general steady-state condition $\mathbf{j} = \text{const} \neq 0$ is out of the scope of our study). From physical point of view, this means that, in a cooled Bose–Fermi mixture characterized by a homogeneous distribution of fermions, strong inhomogeneous fluctuations appear spontaneously at the critical temperature T_c, while the following temperature decrease gives an inhomogeneously distributed Bose-condensate at the point $T = \left(1 - \sqrt{6\rho_c}\right)T_c$ only.

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APPENDIX

With regard for definitions (12) and (15), we obtain l.h.s. of Eq.(16):

$$\frac{1}{2}[\overline{\mathcal{D}}, \mathcal{D}]\Phi = (\varphi - \dot\eta) - \overline\theta\dot\Psi + \dot{\overline\Psi}\theta + \overline\theta\theta\dot\varphi. \tag{A.1}$$

Being the supersymmetry variational derivative, the r.h.s. of this equation is written as a generalization of expression (5):

$$\frac{\delta\mathcal{F}}{\delta\Phi} = F'\left(\eta + \overline\theta\Psi + \overline\Psi\theta + \overline\theta\theta\varphi\right) -$$

$$- \left[\nabla^2\eta + \overline\theta\left(\nabla^2\Psi\right) + \left(\nabla^2\overline\Psi\right)\theta + \overline\theta\theta\left(\nabla^2\varphi\right)\right], \tag{A.2}$$

where the prime denotes the differentiation with respect to the related argument. According to the first rule (14), the expansion in powers of the addition $\overline\theta\Psi + \overline\Psi\theta + \overline\theta\theta\varphi$ gives

$$F'(\Phi) = F'\left[\eta + \left(\overline\theta\Psi + \overline\Psi\theta + \overline\theta\theta\varphi\right)\right] =$$

$$= F^{'}(\eta) + F^{''}(\eta)\left(\overline\theta\Psi + \overline\Psi\theta\right) + \left[F^{''}(\eta)\varphi - F^{'''}(\eta)\rho\right]\overline\theta\theta. \tag{A.3}$$

The comparison of multipliers standing before 1, $\overline\theta$, θ, and $\overline\theta\theta$ yields the system of equations (17)–(20).

In a more complicated case of the transformed superfield (25), expressions (A.1) and (A.3) take the form

$$\frac{1}{2}[\overline{\mathcal{D}},\mathcal{D}]\widetilde{\Phi}=[(\varphi-\dot{\eta})+\eta\rho]-\left[(1-\eta)\,\frac{\dot{\Psi}}{\Psi}-\dot{\eta}\right]\overline{\theta}\Psi+$$

$$+\left[(1+\eta)\,\frac{\dot{\overline{\Psi}}}{\overline{\Psi}}+\dot{\eta}\right]\overline{\Psi}\theta+\frac{\partial}{\partial t}\,(\varphi+\eta\rho)\,\overline{\theta}\theta, \qquad (A.4)$$

$$F'\left(\widetilde{\Phi}\right)=F^{'}(\eta)+F^{''}(\eta)\,(1-\eta)\,\overline{\theta}\Psi+F^{''}(\eta)\,(1+\eta)\,\overline{\Psi}\theta+$$

$$+\left[F^{''}(\eta)\,(\varphi+\eta\rho)-F^{'''}(\eta)\left(1-\eta^{2}\right)\rho\right]\overline{\theta}\theta. \qquad (A.5)$$

The comparison of the related supersymmetry terms gives system (26)–(29).

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СУПЕРСИМЕТРИЧНЕ
ПОДАННЯ БОЗЕ–ЕЙНШТЕЙНІВСЬКОЇ
КОНДЕНСАЦІЇ ФЕРМІОННИХ ПАР

О.І. Олемской, І.О. Шуда

Р е з ю м е

Розвинено суперсиметричну теорію, в межах якої компоненти поля зводяться до квадратного кореня густини бозеконденсату, амплітуди її флуктуацій і грасманових полів, що визначають густину фермі-частинок. Показано, що у вироджених фермі–бозевських сумішах з непорушеною суперсиметрією число ферміонів зберігається, а система інваріантна відносно обернення стріли часу. Знайдено польові рівняння, які описують просторово-часові залежності для фермі–бозевських сумішей з порушеною суперсиметрією. Розв'язок цих рівнянь показує, що охолодження системи з однорідно розподіленими ферміонами приводить до спонтанного наростання неоднорідних флуктуацій нижче критичної температури. Подальше зниження температури нижче порога, що визначається густиною ферміонів, зумовлює появу неоднорідно розподіленого бозеконденсату.