USING STRING THEORY TO DESCRIBE MONOPOLE SCATTERING

J.K. BARRETT

University of Iceland
(Raunvisindastofnun, Taeknigardi, Dunhaga 5, 107 Reykjavik, Iceland; e-mail: jessica@raunvis.hi.is)

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We explain how it is possible to describe the scattering of magnetic monopoles using the D-branes of string theory. We use this description to calculate the energy radiated during monopole scattering.

1. Introduction

String theory is currently the best candidate we have for a 'theory of everything'. However there are still many gaps in our understanding of string theory, and it has not yet been possible to derive a realistic description of the world we observe from string theory. But there have been many ways in which string theory has given us new perspectives on some of our existing knowledge. We will consider one such example here; the D-brane realization of the ADHMN construction. The ADHMN construction was a mathematical tool which was developed in the early 1980s to aid the construction of magnetic monopole solutions. It was realized in the 1990s that, in string theory, a D1-brane stretched between two D3-branes embodies the ADHMN construction and gives it a physical realization.

We describe here some material taken from [1] and [2], in which we used the D-brane configuration described above to study magnetic monopoles. Our aim was to calculate the energy radiated during monopole scattering. This calculation has already been studied in [3] with the result

$$E_{\rm rad} \sim 1.35\, m_{\rm mon} v_{-\infty}^5 \ , \quad \frac{E_{\rm rad}}{E_{\rm tot}} \sim 1.35\, v_{-\infty}^3 \ , \tag{1}$$

where $E_{\rm rad}$ is the energy radiated, $E_{\rm tot}$ is the total energy in the system, $m_{\rm mon}$ is the mass of each monopole, and $v_{-\infty}$ is the asymptotic velocity of each monopole. We hoped to confirm this result using an alternative perspective provided by the D-brane configuration.

The layout will be as follows. In Section 2, we will briefly review some facts about magnetic monopoles, and we will review the role played by D-branes in string theory in Section 3. In Section 4, we will discuss the D1-D3 brane description of magnetic monopoles, and, in Section 5, we will describe our calculations of the energy radiated during monopole scattering. We will conclude in Section 6.

2. Magnetic Monopoles

In this section, we review some of the theory concerning magnetic monopoles. The material from this section can be found in more details in [4] and [5].

2.1. Dirac monopole

Let us consider Maxwell's equations of electromagnetism,

$$\nabla \cdot \mathbf{E} = \rho_e \ , \quad \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}_e \ , \tag{2}$$

and

$$\nabla \cdot \mathbf{B} = 0 \ , \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \ . \tag{3}$$

The electric source terms ρ_e and $\mathbf{j}_e$ are present in Eqs. (2), because we have observed electric monopoles in the nature. On the other hand, magnetic monopoles have never been observed, and so the corresponding magnetic equations (3) contain no source terms. However, it is interesting to think about which consequences would be if magnetic monopoles did exist. Equation (3) would then become

$$\nabla \cdot \mathbf{B} = \rho_m \ , \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{j}_m \ . \tag{4}$$

It was shown by Dirac in the 1930s that the presence of a single magnetic monopole in the Universe is sufficient to guarantee the conservation of electric charge in the form

$$eg = 2\pi n \ , \quad n \in \mathbb{Z} \ , \tag{5}$$

where e is the electric charge, and g is the magnetic charge. This is a nice explanation of charge quantization, which makes the existence of magnetic monopoles an attractive proposition.

2.2. 't Hooft-Polyakov monopole

Let us move on from the theory of electromagnetism to the Yang–Mills–Higgs theory. The Yang–Mills–Higgs action is

$$S_{YM} = \frac{1}{g_{\rm YM}^2} \int d^4x \left\{ -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \right.$$

$$\left. + \frac{1}{2} D^\mu \Phi^a D_\mu \Phi^a - V(\Phi) \right\} , \tag{6}$$

where g_{YM} is the Yang–Mills coupling constant, the field $F_{\mu\nu}$ is a gauge field, and the field Φ is a scalar field called the Higgs field. Both $F_{\mu\nu}$ and Φ belong to the adjoint representation of the gauge group we take to be SU(2). So, we write

$$F_{\mu\nu} = F^a_{\mu\nu} T^a \ , \quad \Phi = \Phi^a T^a \ , \tag{7}$$

where T^a are the generators of SU(2). The gauge field is defined in terms of a gauge potential A_μ as follows:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon^{abc} A^b_\mu A^c_\nu \ , \tag{8}$$

and the covariant derivative is defined to be

$$D_\mu \Phi^a = \partial_\mu \Phi^a - \epsilon^{abc} A^b_\mu \Phi^c \ . \tag{9}$$

The potential $V(\Phi)$ is chosen such that the vacuum expectation value of Φ is non-zero. This means that the gauge bosons A^a_μ acquire masses, and we say that the gauge group is broken. We will take

$$V(\Phi) = \frac{\lambda}{4} \left(\Phi^a \Phi^a - v^2 \right)^2 \ , \tag{10}$$

so that the vacuum expectation value of Φ^2 will be v^2.

We will look for Yang–Mills–Higgs solutions which have finite energy. In particular, this means that we require $V(\Phi) = 0$ on the sphere at spatial infinity, which we denote S^2_∞. Let M be the set of Φ^a which satisfy $V(\Phi) = 0$. Then

$$M = \{\Phi : \Phi^a \Phi^a = v^2\} \tag{11}$$

which has the topology of a two-sphere. Therefore, the Higgs field configuration at spatial infinity defines a map from one two-sphere to another one,

$$\Phi : S^2_\infty \to M_H \ . \tag{12}$$

This map has an associated topological quantity called a winding number, $n \in \mathbb{Z}$ (it can be thought of as the number of times the first sphere 'wraps around' the second).

It turns out that, for a solution with $n \neq 0$ to have finite energy, there must be a non-zero gauge field. The required gauge field has the form of a magnetic field (where we define $B_i = \epsilon_{ijk} F_{jk}$ to be the magnetic field) with magnetic charge given by

$$g = \frac{4\pi n}{g_{\rm YM}} \ . \tag{13}$$

Equation (13) is once again Dirac's quantization condition, with $g_{\rm YM} = e/2$.

2.3. BPS limit

The BPS limit of the theory described in the previous section is the limit $\lambda \to 0$, where the limit is taken while maintaining the boundary condition on the Higgs field,

$$\Phi^2 \to v^2 \quad \text{as} \quad r \to \infty \ . \tag{14}$$

Consider a static configuration with the electric field, $E_i = F_{0i}$, set to zero. In the BPS limit, it can be shown that the energy of such a configuration with a given magnetic charge g is minimized if and only if the configuration satisfies the Bogomol'nyi equation

$$\mathbf{B}^a = \mathbf{D}\Phi^a \ . \tag{15}$$

The energy of the configuration is then given by

$$E = \frac{|vg|}{g^2_{\rm YM}} \ . \tag{16}$$

2.4. ADHMN construction

The ADHMN construction is a technique which can be used to obtain magnetic monopole solutions of Eq. (15). Instead of working with the fields A_μ and Φ, we work with Nahm data which consist of the matrices T_i. There is a one-to-one map between A_μ and Φ and the T_i which is called the Nahm transformation,

$$A_\mu\ ,\Phi\ \longleftrightarrow\ T_i\ . \tag{17}$$

To be more precise, T_i are $n \times n$ anti-Hermitian matrices, where n is the winding number of the corresponding monopole solution. They depend on the real parameter $\xi \in [0, 2]$ and satisfy the criteria

1. Nahm's equations

$$\frac{dT_i}{d\xi} = \frac{i}{2}\epsilon_{ijk}[T_j, T_k]\ . \tag{18}$$

2. T_i have simple poles at $\xi = 0$ and $\xi = 2$.

3. The matrix residues at the poles form an irreducible n-dimensional representation of SU(2).

Having obtained Nahm data which satisfy the above criteria, we can use the Nahm transformation to obtain the fields A_μ and Φ of the corresponding monopole solution.

3. D-Branes in String Theory

In this section, we will review some of the basic facts concerning D-branes in string theory. See [6, 7] and [8] for more details.

It was realized in the mid-1990s that string theory is not only a theory of strings; to obtain a consistent theory, we must include higher-dimensional objects which are called D-branes. The definition of a Dp-brane is a surface with p spacelike dimensions and one timelike dimension, on which the ends of open strings are constrained to lie.

D-branes have an alternative description as soliton (i.e., static and energy-minimizing) solutions of ten-dimensional supergravity which is the low energy limit of string theory.

In the spectrum of open string theory, there is a massless vector which has the U(1) gauge symmetry. We can promote this to the U(N) gauge symmetry by endowing the ends of the open string with non-dynamical charges (these are called Chan–Paton factors), as in the diagram in Fig. 1, where $a, b = 1, \ldots N$.

The number of massless vectors is now N^2, because there are N choices for the charge of each end of the string. These correspond to the gauge bosons of the U(N) theory. Let us consider Chan–Paton factors in the presence of D-branes. Suppose we have N parallel D-branes, as in Fig. 2.

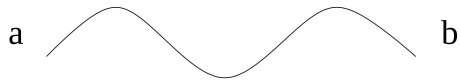

Fig. 1. A fundamental string with Chan–Paton factors at the ends

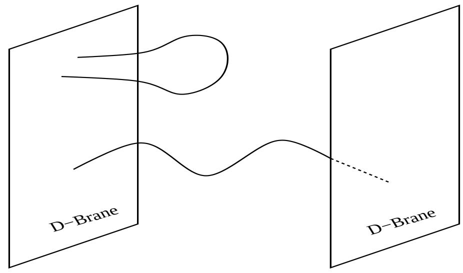

Fig. 2. Parallel D-branes with strings stretching between them

Then we can imagine the Chan–Paton factors at each end of the string might correspond, in some sense, to the D-brane on which the end of the string lies. (Note that we are discussing the oriented string theory, in which a string which begins on brane 1 and ends on brane 2 is distinct from a string which begins on brane 2 and ends on brane 1). So, for N parallel D-branes, the gauge theory is U(N). When the D-branes are separated, as in Fig. 2, the strings stretching between D-branes are massive, and the gauge group is U(N) broken down to $U(1)^N$ (as in Section 2 2.2, where the breaking of the gauge group by the Higgs vacuum expectation value leads to gauge bosons acquiring masses). When the D-branes are coincident, then all the open string vector states are massless, and the gauge group is the unbroken U(N).

We will denote the gauge potential, which is confined to the world volume of the D-brane since it is an open string field, by A_μ with the corresponding field strength $F_{\mu\nu}$, where $\mu, \nu = 0, \ldots, p$ denote D-brane directions. There are other fields which exist on the brane's surface we will denote by Φ^I, $I = p + 1, \ldots, 9$ (the existence of these fields can be shown using T-duality - see one of the references on D-branes mentioned above for details). The quantities Φ^I also belong to the adjoint representation of the gauge group, and they roughly correspond to the position of the D-brane(s) in the dimensions which are transverse to the D-brane's surface (recall that string theory contains ten dimensions in total). We can label $x^0, \ldots, x^p$ as the brane's directions. Then $X^I = \alpha'\Phi^I$ tell us about the position of the brane(s) in the x^I-

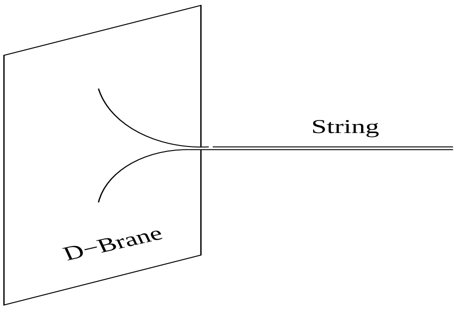

Fig. 3. A BIon spike solution

directions (more precisely, in the case of N branes, the N eigenvalues of Φ^I represent the positions! of the N branes in the x^I direction).

The action for a single D-brane in a flat background (i.e. in a space with a flat metric, and all other closed string fields set to zero) is the Born–Infeld action

$$S \sim -\int d^{p+1}x\Big(-\det\Big(\eta_{\mu\nu} + (\alpha')^2\partial_\mu\Phi^I\partial_\nu\Phi_I + $$

$$+\alpha' F_{\mu\nu}\Big)\Big)^{1/2}, \qquad (19)$$

where the integral is taken over the D-brane directions. The non-Abelian extension of this action, at least up to order $(\alpha')^2$, is given by

$$S \sim -\int d^{p+1}x \ \mathrm{STr}\Big(-\det\Big(\eta_{\mu\nu} +$$

$$+(\alpha')^2 D_\mu\Phi^I Q^{-1}_{IJ} D_\nu \Phi^J + \alpha' F_{\mu\nu}\Big)\Big)^{1/2}, \qquad (20)$$

where D_μ denotes the covariant derivative and

$$Q_{IJ} = \delta_{IJ} + i\alpha'[\Phi_I, \Phi_K]E_{KJ} . \qquad (21)$$

In (20), STr denotes a symmetrized trace, which indicates that we should symmetrize over all orderings of $F_{\mu\nu}$, $D_\mu\Phi^I$, and $[\Phi^I, \Phi^J]$, when we take the trace (this avoids ordering ambiguities in calculating the determinant of a matrix, whose entries are non-Abelian objects).

4. Magnetic Monopoles as Solutions of the Born–Infeld Action

It has been known for some time that the end of a fundamental string attached to a D-brane acts as a source for the electric field on the brane, and the end of a D-string (i.e. a D1-brane) attached to a D-brane acts as a source for the magnetic field. It was shown in [9] that the Born–Infeld action for a D-brane has a solution which corresponds to a fundamental string attached to the D-brane. (called the BIon spike solution, because the tension of the string pulls the brane into the form of a spike – see Fig. 3).

The case of a D-string attached to a D3-brane was studied in [10] and [11], and we shall review calculations from these papers in more details here. In particular, there are two ways of studying this object – we can use the Born–Infeld action for the D3-brane or the Born–Infeld action for the D-string.

Consider the Born–Infeld action for a D3-brane with the magnetic field on the brane, B_i, excited, and with a single transverse field, Φ, excited. If we look for a static solution which minimizes the energy (i.e. a soliton solution), we find that the solution obeys the usual BPS equations for a magnetic monopole

$$B_i = D_i\Phi . \qquad (22)$$

The simplest solution is

$$\Phi(r) = \frac{n}{2r}, \quad \mathbf{B}(\mathbf{r}) = \mp\frac{n}{2r^3}\mathbf{r}, \qquad (23)$$

where r is the radial coordinate in the D-brane's world volume. Note that this solution for $\Phi(r)$ indicates that the D3-brane has been pulled into an infinitely long spike in the direction corresponding to the field Φ. It can be shown that this solution represents n semiinfinite D-strings attached to the D3-brane at the origin.

On the other hand, this configuration can also be studied using the non-Abelian Born–Infeld action for n D-strings. Exciting three transverse scalars in the action, say Φ_1, Φ_2, and Φ_3, and looking for a soliton solution, lead to the BPS equations

$$\partial_\sigma\Phi^i + \frac{1}{2i}\epsilon_{ijk}[\Phi^j, \Phi^k] = 0 , \qquad (24)$$

where σ is the D-strings' spatial direction. Note that Eqs. (24) are identical to Nahm's equations (18) from the ADHMN construction of a magnetic monopole from Section 2 2.4. The solution corresponding to n semiinfinite D-strings 'funnelling out' into a D3-brane is

$$\Phi^i = \pm\frac{\alpha^i}{2\sigma} , \qquad (25)$$

where α^i are an $n\times n$ representation of the SU(2) algebra.

In [1], we discussed the solution to Nahm's equations which describes two D-strings stretched between two D3-branes. First, we define a new string coordinate $\xi =$

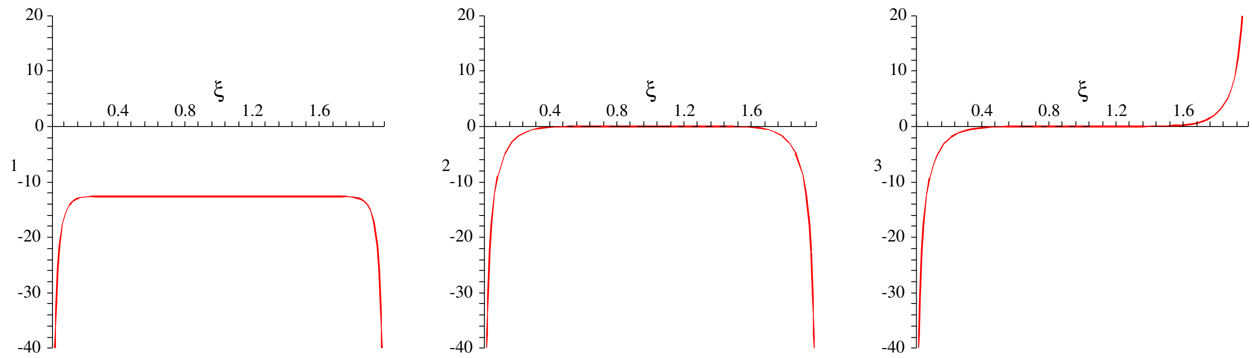

Fig. 4. Graphs of $f_1(\xi,k)$, $f_2(\xi,k)$, and $f_3(\xi,k)$ with $k=0.999999999$

$2\sigma/L$, where L is the distance between the branes in the σ coordinate. Then the distance between the branes in the ξ coordinate is 2. We make the ansatz

$$\Phi^i = \frac{2}{L} f_i(\xi,t)\,\sigma_i, \quad \text{no summation over } i, \tag{26}$$

where σ_i are the Pauli matrices. Then Nahm's equations (24) reduce to

$$f_1' - f_2 f_3 = 0\ , \quad f_2' - f_3 f_1 = 0\ , \quad f_3' - f_1 f_2 = 0\ , \tag{27}$$

where $'$ denotes the differentiation with respect to ξ. The appropriate solution to these equations which were first derived in [12] are

$$f_1(\xi,k) = \frac{-K(k)}{\mathrm{sn}(K(k)\xi,k)}\ , \tag{28}$$

$$f_2(\xi,k) = \frac{-K(k)\mathrm{dn}(K(k)\xi,k)}{\mathrm{sn}(K(k)\xi,k)}\ , \tag{29}$$

$$f_3(\xi,k) = \frac{-K(k)\mathrm{cn}(K(k)\xi,k)}{\mathrm{sn}(K(k)\xi,k)}\ , \tag{30}$$

where $K(k)$ is the complete elliptic integral of the first kind and $\mathrm{sn}(\xi,k)$, $\mathrm{cn}(\xi,k)$, and $\mathrm{dn}(\xi,k)$ are the Jacobian elliptic functions; and k is a parameter of the solutions with $0 \le k < 1$. Note that the f_i all have poles at $\xi=0$ and $\xi=2$, which correspond to the D-strings 'funnelling out' into D3-branes.

Let us consider the effect of the parameter k on solutions (28)–(30). The graphs in Fig. 4 are of the f_i with $k=0.999999999$.

In the limit $k\to 1$, we can approximate as follows:

$$f_1(\xi,k) \sim K(k)\ , \quad f_2(\xi,k) \sim f_3(\xi,k) \sim 0\ , \tag{31}$$

where the approximation is valid, except near the poles at $\xi = 0$ and $\xi = 2$. Recall from Section 3 that the eigenvalues of Φ_1 are the positions of the D-strings in the x_1-direction. Equation (31) then implies that the positions of the D-strings in the x_1-direction are $\pm K(k)$. So we see that the value of the parameter k tells us about the position of the D-strings – the closer k is to 1, the further apart the D-strings are along the x_1-axis. The configuration with $k=0$ is also of interest, since $f_1(\xi,k=0) = f_2(\xi,k=0)$. So this configuration is axially symmetric in the x_1-x_2 plane; it corresponds to a known two-monopole solution which has the shape of a 'doughnut'.

5. Energy Radiated During Monopole Scattering

5.1. Description of monopole scattering

Let us consider taking a static D-string configuration with k close to 1, and giving the D-strings a nudge, so that they are moving slowly toward each other. The energy of the D-strings is very small because we have taken the initial configuration to be (28)–(30), which minimizes the potential energy, and the kinetic energy is small because the velocity is small. Therefore, as this configuration evolves in time, it must always be close to the static solutions – it does not have enough energy to move far from them. So we can approximate the D-strings' motion by allowing the configuration only to depend on time through $k(t)$ in (28)–(30); at any point in time, the configuration still has the form of the static solution with small velocity. This is Manton's moduli space approximation of [13] – the motion is described by a geodesic in the moduli space, whose coordinate is k. Since the D-strings are moving toward one another initially, we must have $\dot{k}<0$. As $k\to 0$, the D-strings continue moving toward one another, until they are coincident at $k = 0$. At this point, f_1 and f_2 swap roles (we have shown numerically that this is the case). So the D-strings have scattered at 90° – it is known that two monopoles scatter at 90°, so this was to be expected.

In order to calculate the energy radiated during scattering, we need to go beyond the moduli space approximation described above. We write the full solution to the equations of motion as follows:

$$\varphi_i(\xi, t) = f_i(\xi, k(t)) + \epsilon_i(\xi, t) , \qquad (32)$$

where the f_i are the static solutions which are the zero modes, and the ϵ_i are perturbations which are small for the reasons described above – they are the non-zero modes. We showed in [2] that the zero modes and the non-zero modes decouple after the scattering, so that the energy can no longer be transferred between them. The energy in non-zero modes after the scattering is the energy that has been radiated during the scattering.

5.2. Numerical solution of the equations of motion

In order to be able to solve the equations of motion numerically, we will take the limit $\alpha' \to 0$ which is the low energy limit of string theory. There are two ways to take this limit, as we discuss in [1] in detail. The distance between the D3-branes is given by

$$L = \alpha' v , \qquad (33)$$

where, in the D3-brane description, v is the expectation value of the field Φ. So when we take the limit $\alpha' \to 0$, we can either keep v fixed, with $L \to 0$, or we can keep L fixed, with $v \to \infty$. We will choose the second limit here, because it leads to a simpler low-energy action. A consequence of this is that the mass of the monopole/D-string, which is given by

$$m_{\rm mon} = \frac{v}{g_s} , \qquad (34)$$

is infinite. To keep our calculations finite, we will calculate the ratio of the energy radiated to the total energy (the total energy is conserved).

Taking the limit $\alpha' \to 0$, $v \to \infty$ of the Born–Infeld action results in the action

$$S_{\rm YM} \sim \int_{-\infty}^{\infty} dt \int_0^2 d\xi \left(\dot\varphi_1^2 + \dot\varphi_2^2 + \dot\varphi_3^2 - (\varphi_1' - \varphi_2\varphi_3)^2 - \right.$$

$$\left. - (\varphi_2' - \varphi_3\varphi_1)^2 - (\varphi_3' - \varphi_1\varphi_2)^2 \right) . \qquad (35)$$

The energy of a static solution is minimized when

$$\varphi_1' - \varphi_2\varphi_3 = 0 , \quad \varphi_2' - \varphi_3\varphi_1 = 0 , \quad \varphi_3' - \varphi_1\varphi_2 = 0 , \qquad (36)$$

which are identical in form to Eqs. (27) which were derived from the full Born–Infeld action. The equations of motion are

$$\ddot\varphi_1 - \varphi_1'' + \varphi_1(\varphi_2^2 + \varphi_3^2) = 0 , \qquad (37)$$

$$\ddot\varphi_2 - \varphi_2'' + \varphi_2(\varphi_3^2 + \varphi_1^2) = 0 , \qquad (38)$$

$$\ddot\varphi_3 - \varphi_3'' + \varphi_3(\varphi_1^2 + \varphi_2^2) = 0 . \qquad (39)$$

We solved the equations of motion numerically using the RK4 method. Our initial configuration was a static solution of the form (28)–(30) with $k = 0.9999999999$, so that the D-strings were a long way apart initially. We ran our numerical programs twice, once with $v_{\rm init} = 0.05$ and once with $v_{\rm init} = 0.1$.

5.3. Calculating the Energy Radiated

5.3.1. The energy in the φ_i

To begin with, we calculated the energy in the full solutions, the φ_i. The total energy is conserved, so we can use the order of the discrepancies in the total energies to get an idea of the numerical inaccuracy in our results. We found that, for $v_{\rm init} = 0.05$, the inaccuracy in the total energy was around 10^{-8}, and, for $v_{\rm init} = 0.1$, it was around 10^{-9}.

The potential energy density is given by

$$\text{P.E. density} = \frac{m_{\rm mon}}{2} \Big((\varphi_1' - \varphi_2\varphi_3)^2 + (\varphi_2' - \varphi_1\varphi_3)^2 +$$

$$+ (\varphi_3' - \varphi_1\varphi_2)^2 \Big) , \qquad (40)$$

where $m_{\rm mon}$ is the mass of a monopole, which we can neglect since we will always deal with ratios of energies. The equations for the zero modes f_i show that the potential energy is zero. Therefore, the only contribution to the potential energy in the φ_i comes from the non-zero modes ϵ_i. Figures 5 and 6 show the logarithmic plots of the potential energy in the φ_i. In both cases, the potential energy peaks at the point of scattering, around $t = 200$ for $v_{\rm init} = 0.05$, and $t = 100$ for $v_{\rm init} = 0.1$, and then falls off again. Note that the final potential energy is around the level of numerical inaccuracy in both cases, which suggests that no energy has been radiated. We will confirm this result by calculating the energy in the non-zero modes in the next section.

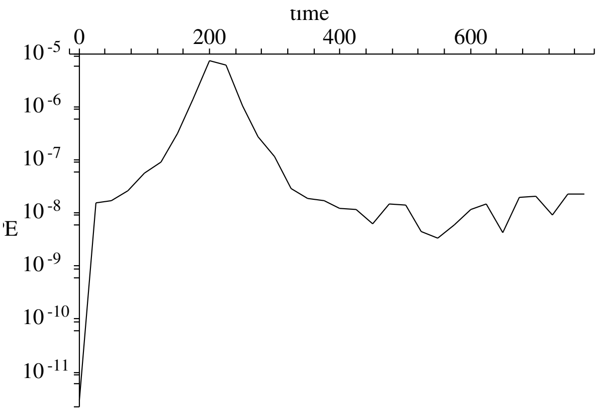

Fig. 5. Logarithmic plot of the potential energy in the φ_i with $v_{\rm init} = 0.05$

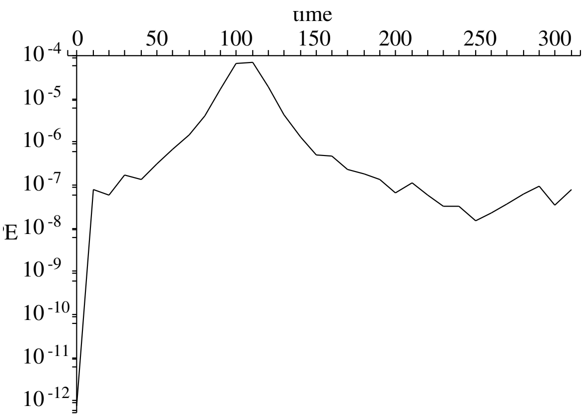

Fig. 6. Logarithmic plot of the potential energy in the φ_i with $v_{\rm init} = 0.1$

5.3.2. The energy in ϵ_i

We separated the zero modes from the non-zero modes in our numerical solutions using a technique which depended on approximation (31) for k close to 1 (see [2] for the details of our method). When the D-strings are far apart, the kinetic energy density for ϵ_i is given by

$$\text{K.E. density} = \frac{1}{2}(\dot\epsilon_1^2 + \dot\epsilon_2^2 + \dot\epsilon_3^2)\ , \tag{41}$$

and the potential energy density is

$$\text{P.E. density} = \frac{1}{2}(\epsilon_1'^2 + \epsilon_2'^2 + \epsilon_3'^2) +$$

$$+\frac{1}{\xi^2}\Big(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \epsilon_1\epsilon_2 + \epsilon_2\epsilon_3 + \epsilon_3\epsilon_1\Big) +$$

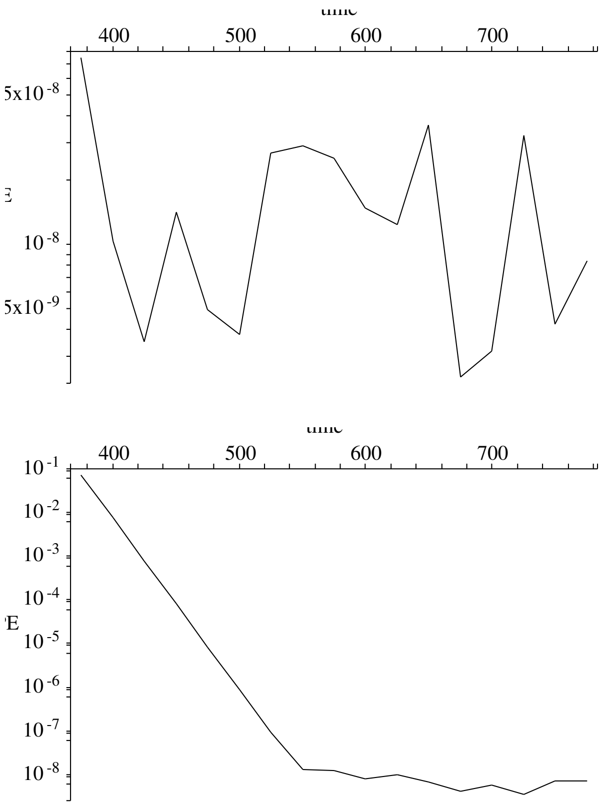

Fig. 7. Logarithmic plots of the kinetic and potential energies in ϵ_i with $v_{\rm init} = 0.05$

$$+\frac{1}{\xi}\Big(\epsilon_1(\epsilon_2' + \epsilon_3') + \epsilon_2(\epsilon_3' + \epsilon_1') + \epsilon_3(\epsilon_1' + \epsilon_2')\Big)\ , \tag{42}$$

where we have neglected all terms of order ϵ^3 and higher in the potential energy density (42).

The graphs in Figs. 7 and 8 show the potential and kinetic energies calculated for $v_{\rm init} = 0.05$ and $v_{\rm init} = 0.1$, respectively. In Fig. 7, for $v_{\rm init} = 0.05$, the energy in ϵ_i after the scattering, in the region where the approximation is valid, is $\sim 10^{-8}$, which agrees with the results discussed in Section 5.3. In Fig. 8, for $v_{\rm init} = 0.1$, the energy radiated is $\sim 10^{-7}$, which also agrees with the results of Section 5.3.

6. Conclusions

We have reviewed how string theory can provide a physical description of the ADHMN construction of magnetic monopoles. We used the string theory description to calculate the energy radiated during the monopole scat-

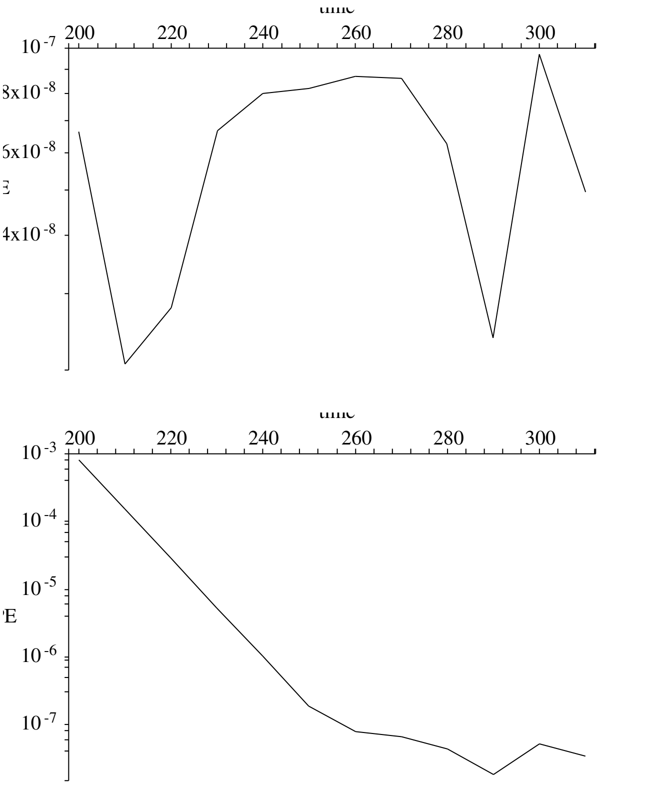

Fig. 8. Logarithmic plots of the kinetic and potential energies in ϵ_i with $v_{\rm init} = 0.1$

tering. The results we obtained were

$$\frac{E_{\rm rad}}{E_{\rm tot}^{\rm init}} \sim 10^{-8} \quad \text{for } v_{-\infty} = 0.05\,,$$
$$\frac{E_{\rm rad}}{E_{\rm tot}^{\rm init}} \sim 10^{-7} \quad \text{for } v_{-\infty} = 0.1\ . \qquad (43)$$

Since our results are approximately of the same order as the numerical inaccuracy in our calculations, they are consistent with those there being no energy radiated during the monopole scattering.

Our results contrast with those of [3], whose prediction (1) gives

$$\frac{E_{\rm rad}}{E_{\rm tot}} \sim 10^{-4} \qquad \text{for } v_{-\infty} = 0.05\,,$$
$$\frac{E_{\rm rad}}{E_{\rm tot}} \sim 10^{-3} \qquad \text{for } v_{-\infty} = 0.1\ . \qquad (44)$$

In the future, we should seek to understand this discrepancy.

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ЗАСТОСУВАННЯ ТЕОРІЇ СТРУН ДЛЯ ОПИСУ РОЗСІЯННЯ МОНОПОЛІВ

Дж.К. Барретт

Р е з ю м е

Пояснено, як можна описати розсіяння магнітних монополів, використовуючи D-брани в теорії струн. Знайдено енергію, що випромінюється при розсіюванні монополів.