EFFECT OF EXTERNAL NOISE ON THE RELAXATION PROCESS IN BISTABLE TUNNELING SYSTEMS

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We consider the effects of intensity fluctuations of an incident electron flow incoming a double-barrier tunneling structure near an instability point. A simplified Langevin equation with multiplicative Gaussian white noise is used to describe noise effects in the system near a resonance under conditions of coherent tunneling. Numerically simulating this equation, we obtained the dependences of the mean first passage time on the noise intensity and a deviation of the average intensity of the incident electron flow from the critical value in the deterministic case. The numerical results satisfactorily agree with the theoretical results of Colet et al. The relaxation time has a maximum value in the absence of noise and decreases with increase in the noise intensity. Noise favors transitions at those incident intensities, for which the transition in the deterministic case was impossible.

1. Introduction

One of the important aspects in the study of nonlinear nonequilibrium systems lies in the allowance for the effect of external noises on their dynamic behavior. An external noise can be caused by fluctuations of the environment or can appear due to the action of a random force. From the practical point of view, it is important that parameters of an external noise can be controlled.

The action of external noises affects the nonequilibrium systems in a nontrivial way and results in the dynamics different from a purely deterministic motion [1, 2]. It is most pronounced at unstable points, where a system passes from one stationary state to another (bifurcation points). In the neighborhood of these points, external fluctuations can change the lifetime of a state, which is observed as a shift of the bifurcation point. Such a behavior is characteristic of bistable systems. The effect of noise on the behavior of bistable systems in the neighborhood of unstable points was considered in the literature for a number of physical systems, in particular optical [3–5], laser [6, 7, 9, 10], and tunneling ones [8, 21], some biological systems [11, 12, 14], and others.

Investigating the dynamic behavior of bistable systems, it is important to analyze the effect of noise on the process of relaxation from one stationary state to another. In this case, one can consider either the evolution of the probability density of some dynamic variable [4, 12] or its time correlation function [7, 10, 14] and the relaxation time related to such function. However, the relaxation of a system from one state to another is most often investigated, by using the first passage time (FPT), i.e. the time, by which a random process reaches the boundary separating one stable state from another. In the literature, one can find the studies of the transitions from an unstable state to a stable one (for example, in such systems as lasers near the generation threshold [10, 13]), as well as the transitions realized through marginal points presented, for instance, by the end points of a hysteresis cycle [15, 16].

In the given work, we consider the influence of external noises on the dynamics of a system describing the resonance electron tunneling in double-barrier nanostructures. Double-barrier tunnel structures are of great importance for the use in various electron solid-state devices [17]. They are characterized by the presence of a negative differential conductivity and a hysteresis loop in volt-ampere characteristics. The hysteresis behavior is explained by the intrinsic bistability arising due to the influence of the electrostatic potential formed by electrons accumulated in a quantum well on the tunnel current. Such a behavior was observed experimentally for the first time by Goldman et al. [18]. Theoretically, the phenomenon of intrinsic bistability in resonance tunneling structures was analyzed in many works, in particular, in [19, 20, 22].

The effect of external noises on tunnel processes is analyzed with the help of a model used in [22], whose short description is given in Section 2. We also present there the simplified Fokker–Planck model describing intensity fluctuations of an incident electron flow. In Section 3, we consider the effect of noise on the stationary behavior of a system. Its dynamic behavior under the action of noise is studied in Section 4. In Section 5, we apply the numerical modeling method to the calculation of the relaxation time depending on the noise intensity and a deviation of the incident flow intensity from the critical value. The results of our calculations are compared to the theoretical dependences obtained using the results of work [15].

2. Model of Nonlinear Resonance Tunneling

The process of electron tunneling through a double-barrier structure is studied under the assumption that the coherence length exceeds the dimensions of the system, i.e. it is supposed that the tunneling process is coherent. Such a process is described with the help of the model presented in [22]. It considers an electron flow incoming from the left on a tunnel structure consisting of two identical potential barriers of width a separated by a potential well of width b. The Coulomb interaction between the incoming electron wave and electrons accumulated in the potential well was considered in the single-electron approximation. It was supposed that the single-electron functions in the regions beyond the barriers ($x < 0,\ \ x > 2a + b$) depend on the coordinate and time in the following way:

$$\Psi_{\rm in}(x,t) = [D_0(t)e^{ikx} + R(t)e^{-ikx}]e^{-iw_0t},$$

$$\Psi_{\rm out}(x,t) = D(t)e^{i(kx-w_0t)}.$$

Here, $k = \sqrt{2m^*E/\hbar^2}$ denotes the wave vector of an incident electron, E is the electron energy, m^* is its effective mass, and $w_0 = E/\hbar = \hbar k^2/2m^*$. The quantities D_0 and $R(t)$ stand for the amplitudes of the incident and reflected electron wave functions, respectively. The solution for the amplitude $D(t)$ of the wave function of an electron going out of the tunnel system was obtained for the most interesting case – resonance tunneling. In this approximation, one can introduce a small parameter, namely a deviation of the wave vector k from the resonance value k_r : $\xi = k - k_r$, $|\xi|/k \ll 1$. In the limiting cases of high and narrow barriers, the following differential equation for the complex amplitude of the outgoing wave was obtained:

$$\frac{dD}{d\tau} = -D + iL\xi D - iL\kappa|D|^2D + D_0F_0, \tag{1}$$

where L is the reciprocal half-width of the resonance level in the k-space ($L = \xi_{1/2}^{-1}$) linked with the half-width in the energy space by the relation $\delta E_{1/2} = 2\hbar\nu = \hbar k_r/(m^*L)$, $\kappa = \kappa_0 L/b$ is the nonlinearity parameter (the explicit form of the coefficient κ_0 can be found in [22]), $F_0 = -\exp[-2ik(a+b)]$, and $\tau = \nu t$ is the dimensionless time. The presence of the nonlinear term is due to the electrostatic potential formed by the charge accumulation in the quantum well under the resonance conditions. Representing the complex amplitude D in terms of its real amplitude and phase, $D = |D(\tau)|\exp i[\eta(\tau) - 2k(a+b)]$, we obtained the following system of differential equations:

$$\begin{cases} \frac{dT}{d\tau} = 2[\sqrt{TT_0}|\cos\eta| - T], \\ \frac{d\eta}{d\tau} = z - T - \sqrt{T_0/T}\sin\eta, \ -\frac{\pi}{2} \le \eta \le \frac{\pi}{2}. \end{cases} \tag{2}$$

In order to simplify the analysis, we introduced the following dimensionless variables: $T(\tau) = \kappa L|D(\tau)|^2$ proportional to the intensity of the electron flow that passed through the system, $T_0(\tau) = F_0|D_0(\tau)|^2$ proportional to the intensity of the electron flow coming to the system, and $z = L\xi$ proportional to the deviation of the electron wave vector from the resonance value. In what follows for the sake of brevity, T and T_0 will mean, respectively, the intensities of the outgoing and incident electron flows.

Equations (2) imply that the intensities T and T_0 in the stationary case are linked by the functional dependence

$$T_0 = T[1 + (z-T)^2] = F(T). \tag{3}$$

This equation has three roots at $z > \sqrt{3}$. In Fig. 1, the dotted line presents the intensity T of the electron flow that passed through the system as a function of the intensity T_0 of the incoming flow determined for the stationary case by expression (3) at $z = 3.5$.

The stability analysis of system (2) performed in [23] has shown that the roots of its characteristic equation are determined by the relation

$$\lambda_{1,2} = -1 \pm \sqrt{1 - \frac{\partial F(T)}{\partial T}}.$$

Thus, in the case where $\frac{\partial F(T)}{\partial T} < 1$, the roots will be real, but with different signs, i.e. they will determine an unstable state of the saddle type. These states are marked in Fig. 1 as T_2. At $\frac{\partial F(T)}{\partial T} > 1$, the roots will be complex. They are related to stable states of the focus type which are located at the lower and upper branches of the curve (T_1 and T_3 states, respectively). The T_1 and T_3 states correspond to the modes with low- and high-efficiency tunneling, respectively.

We assume that the phase η of the wave function does not change in the tunneling process. In this case, the system of equations (2) is reduced to the differential equation

$$\frac{dT}{d\tau} = \sqrt{TT_0} - T. \tag{4}$$

As the dynamics of the process is considered near the stationary states, the expression on the right-hand side of Eq. (4) can be expanded in a Taylor series in the neighborhood of the stationary point. Confining ourselves to two first terms in the expansion and taking expression (3) into account, we obtain the equation

$$\frac{dT}{d\tau} = -T + \frac{T_0}{1 + (z - T)^2}. \tag{5}$$

Solving numerically Eq. (5) at z=3.5 at a successive slow variation of the parameter T_0 in the direct and reverse directions, we obtained a hysteresis dependence of T on T_0 shown by the solid curve in Fig. 1. The transition from the lower stationary state to the upper one is realized at a certain value of the intensity T_{0K} corresponding to the end point of the hysteresis loop (the so-called marginal point). This transition takes place at $\frac{\partial F(T)}{\partial T} = 0$. The roots of this equation,

$$T_{K,k} = \frac{1}{3}(2z \mp \sqrt{z^2 - 3}), \tag{6}$$

determine the magnitude of the outgoing flow at the time moments of the transitions from the lower state to the upper one and *vice versa*, respectively. At $z = 3.5$, the critical value of the parameter T_{0K} corresponding to the transition is equal to 7.593. This value of z will be used in all further calculations.

In what follows, we consider the behavior of the given system with regard for effects caused by fluctuations, by confining ourselves only to fluctuations of the incident flow intensity. The effect of external noises in the tunnel system will be investigated, by using an approach based on the Fokker–Planck equation. If the difference in the temporary scales between the amplitude and phase fluctuations for the chosen set of parameters is rather large, the Fokker–Planck model for amplitude fluctuations is a good approximation.

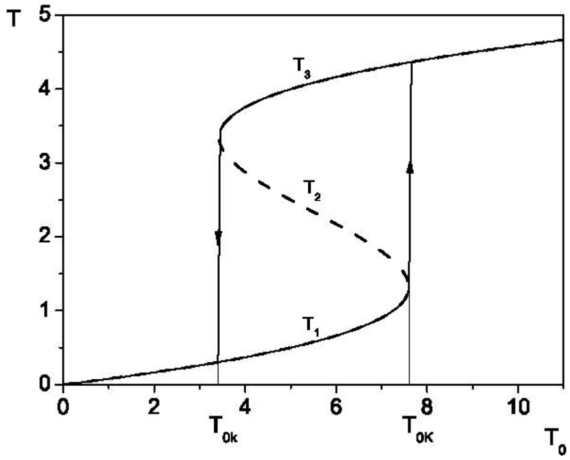

Fig. 1. Dependence of T on T_0 in the stationary case at $z = 3.5$

3. Effect of Noise on the Behavior of the System in the Stationary Case

Let us consider the effect of fluctuations of the incident flow intensity T_0 on the behavior of the system. The intensity T_0 will be considered as a stochastic quantity, $T_0 = \langle T_0 \rangle + p(\tau)$, where $\langle T_0 \rangle$ is the mean value of the intensity, and $p(\tau) = \sqrt{2q}\xi(\tau)$ is its noise component. The quantity $\xi(\tau)$ is a Gaussian white noise with zero average, zero correlation $\langle \xi(\tau)\xi(\tau') \rangle = 0$, and an intensity equal to 1, and q is the noise intensity. With regard for fluctuations of the incident flow intensity, Eq. (5) results in the following stochastic differential equation:

$$\dot{T} = f(T) + g(T)p(\tau). \tag{7}$$

where

$$f(T) = -T + \frac{\langle T_0 \rangle}{1 + (z - T)^2},$$

$$g(T) = \frac{1}{1 + (z - T)^2}.$$

The noise term in Eq. (7) is of multiplicative character, i.e. it depends on the state of the system at a given time moment.

The process $T(\tau)$ can be investigated with the help of the probability density $P(T, \tau)$ that represents the solution of the Fokker–Planck equation in the Stratonovich representation [26]:

$$\frac{\partial P(T, \tau)}{\partial \tau} = -\frac{\partial}{\partial T} K_1 P(T, \tau) + \frac{1}{2} \frac{\partial^2}{\partial T^2} K_2 P(T, \tau), \tag{8}$$

where

$$K_1 = f(T) + g(T)g\prime(T)q =$$

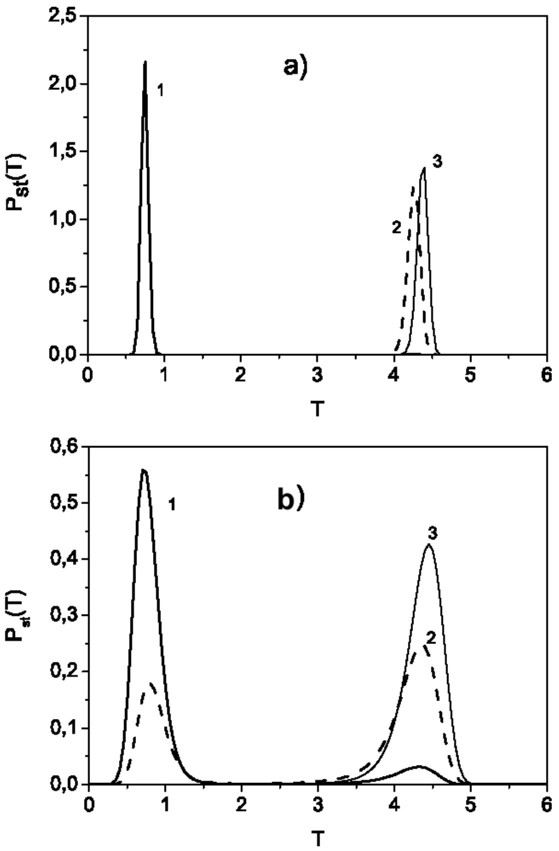

Fig. 2. Form of the stationary probability density at *a*) $q = 0.04$; *b*) $q = 0.5$. Curves *1* are obtained at $\langle T_0\rangle$=6.4, curves *2* – at $\langle T_0\rangle$ =6.7, curves *3* — at $\langle T_0\rangle$=7.7

$$-T + \frac{\langle T_0\rangle}{1+(z-T)^2} + \frac{2(z-T)q}{[1+(z-T)^2]^2},$$

$$K_2 = qg(T)^2 = \frac{q}{[1+(z-T)^2]^2},$$

and $g\prime(T)$ is the derivative with respect to T.

Equating the time derivative in Eq. (8) to zero, one obtains the equation for the stationary probability distribution $P_s(T)$, whose solution has the form [1]

$$P_s(T) = \frac{N_0}{K_2} \exp \int\limits_0^\infty \frac{K_1}{K_2} dT. \tag{9}$$

The normalization factor N_0 is determined by the numerical integration of $P_s(T)$ in the range from 0 to ∞ representing the natural limits of the process.

Figure 2,*a* shows the shape of the stationary probability density at a low noise level ($q = 0.04$) in the cases where the system is far from the transition point (curve *1*) and successively approaches it (curves *2* and *3*). Under low-intensity noise conditions, the extrema of the probability density coincide with the stationary solutions (T_1 and T_3). An increase of the noise intensity up to $q = 0.5$ (see Fig. 2,*b*) in the case of a large distance from the transition point (curve *1*) results in the appearance of one peak in the probability distribution corresponding to $T = T_1$.

As the system approaches the transition point (curve *2*), the probability distribution breaks into two peaks in the neighborhood of $T = T_1$ and $T = T_3$. At $\langle T_0\rangle > T_{0K}$ (curve *3*), $P_s(T)$ has again one peak with a maximum at $T = T_3$. Thus, an increase of the noise intensity (see Fig. 2,*b*) results not only in the broadening of the probability peaks, but also in the appearance of the bimodality in the probability distribution (curve *2*).

4. Time Behavior of the System Under Noise

The time behavior of the system is determined by the temporal evolution of the probability distribution. The latter was obtained by numerically integrating the Fokker–Planck equation (8). The initial condition was chosen in the form of the delta-function $P(T, 0) = \delta(T)$ and approximated by the rectangular function $\varepsilon/[(\pi(T^2 + \varepsilon^2)]$ with $\varepsilon = 0.001$. Figure 3 presents the time variation of $P(T, \tau)$ in the case where the parameter T_0 is less than T_{0K}, and the noise intensity equals $q = 1$. In this figure, it is easy to distinguish different time scales in the development of the process. First, the system relaxes from the initial state to the stationary state T_1 during the short relative time $\tau_1 \approx 2$. After $\tau_2 \approx 7.5$, the probability distribution becomes double-peaked. The transition to the state T_3 is realized during the relative time $\tau_3 \approx 12$.

In the next section, we consider the effect of noise fluctuations on the dynamics of relaxation processes near the marginal point T_{0K}.

5. Noise Effect on the Relaxation Time Close to the Marginal Point

The process of transition from a state with low-intensity tunneling to that with high-intensity one corresponds to a deviation of the intensity T of the electron flow going out of the system from the initial magnitude T_1 to that exceeding the limiting value T_{tr}, at which the system passes to the state T_3. We introduce the parameter $\beta = \langle T_0\rangle - T_{0K}$ representing a small deviation of the operating intensity of the incident flow from the critical value T_{0K}. It is known [24] that, in the deterministic case determined as $\bar{t} = 1/\varphi\prime(x)$, where $\varphi(x)$ is the system potential, the relaxation time infinitely increases when approaching the marginal point. This phenomenon is called the critical slowing-down. Beyond the marginal zone, the dynamics of the system obeys deterministic laws, and noise is of minor importance. However, in the neighborhood of the marginal point, the dynamics is mainly determined by noise. Therefore, almost all its action on the system is concentrated at this point. The system evolves in two different modes depending on the difference $\beta = \langle T_0 \rangle - T_{0K}$. If $\beta < 0$, the transition can take place only under the action of noise due to the mechanism of its activation through the potential barrier.

In the case where a system passes from an unstable state to a stable one, the relaxation time is most often obtained, by determining the mean first passage time (MFPT). One calculates the time interval t_1, during which a stochastic process starting from some initial value x_0 reaches a certain critical magnitude x_F. This time interval represents a stochastic quantity, whose average value $\langle t_1 \rangle$ is the MFPT.

The MFPT will be calculated using the method proposed in [15] for a system in the neighborhood of the marginal point. In its vicinity, the potential related to the model has a horizontal component. This means that almost all the action of noise on the system is concentrated at this point. Then, investigating the noise effect, one can confine oneself to only several terms in the expansion of $f(T)$ in the neighborhood of the marginal point T_K:

$$f(T) = f(T_K) + f'(T - T_K) + \frac{f''(T_K)}{2!}(T - T_K)^2 +$$

$$+0(T - T_K)^3. \tag{10}$$

Let us introduce the change of variables: $\beta = \langle T_0 \rangle - T_{0K}$ and $x = T - T_K$. Then, with regard for the expressions for T_{0K} (3) and T_K (6), we obtain

$$f(x) = -\alpha x^2 - \gamma\beta, \tag{11}$$

where the constants α and γ can be estimated from expression (10), by using relation (6). Thus, the dynamics of the system near the marginal point under the action of noise can be determined by the Langevin equation

$$\dot{x} = \varphi'(x) + g(T)p(\tau) \approx \varphi'(x) + \gamma p(\tau), \tag{12}$$

where the potential of the system $\varphi(x) = -\alpha x^3 - \gamma\beta x$, and the function $g(T)$ is taken at the point T_K, $(g(T_K) = \gamma)$.

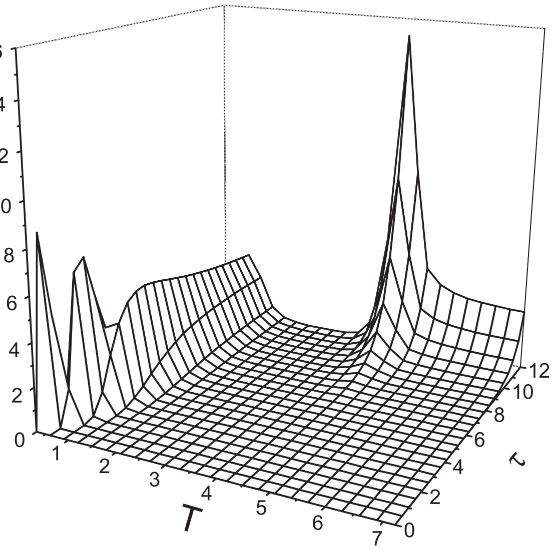

Fig. 3. Time evolution of $P(T, \tau)$ at $\langle T_0 \rangle = 7.5$ and $q = 1$

The stochastic equation (12) can be converted to the Fokker–Planck equation

$$\frac{\partial P}{\partial \tau} = \frac{\partial}{\partial x}[\varphi'(x)P] + \frac{\partial^2}{\partial x^2}(\sigma P), \tag{13}$$

where $\sigma = \gamma q$, whose stationary solution has the form

$$P_{st}(x) = N \exp[-\frac{\varphi(x)}{\sigma}]. \tag{14}$$

From the standard theory of stochastic processes [26], it follows that the mean first passage time $\langle t_1 \rangle$ satisfies the equation

$$-1 = -\frac{d\varphi(x)}{dx}\frac{d\langle t_1 \rangle}{dx} + \sigma\frac{d^2\langle t_1 \rangle}{dx^2}. \tag{15}$$

Solving (15) for $\langle t_1 \rangle$, we obtain

$$\langle t_1 \rangle = \frac{1}{\sigma}\int\limits_{x_0}^{x_F} dx_1 e^{\varphi(x_1)/\sigma} \int\limits_{-\infty}^{x_1} dx_2 e^{-\varphi(x_2)/\sigma}, \tag{16}$$

where x_0 is the initial value, and x_F is the final one.

The asymptotic calculation of expression (16) performed in [15] yielded the approximated formula for the determination of the MFPT close to the marginal point at the transition from $\beta < 0$ to $\beta > 0$:

$$\langle t_1 \rangle = \Phi(k)(\alpha^2\sigma)^{-1/3} + C(x_0, R) +$$

$$+0(\sigma/\alpha R^3, \beta/\alpha x_0^2), \tag{17}$$

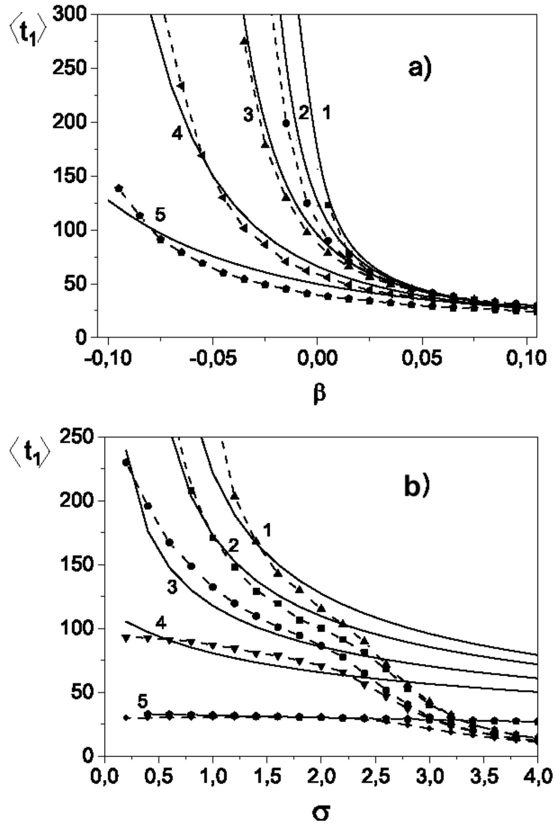

Fig. 4. *a*) Dependence of $\langle t_1 \rangle$ on the parameter β at different noise intensities (*1* – σ= 0.01, *2* – σ = 0.15, *3* – σ= 0.4, *4* – σ = 1.0, *5* – σ = 2.5); *b*) Dependence of $\langle t_1 \rangle$ on the noise intensity σ at different β (*1* – β =-0.007, *2* – β =-0.002, *3* – β =0, *4* – β= 0.01, *5* – β= 0.1)

where $C(x_0, R)$ is the deterministic time, $R = x_F - x_0$,

$$\Phi(k) = \sum_{n=0}^{\infty} \frac{B_n}{n!}(-k)^n, \tag{18}$$

$$B_n = \frac{1}{3}\sqrt{\frac{\pi}{3}}2^{\frac{2n+1}{2}}\Gamma(\frac{2n+1}{6}),$$

$$k = (\beta/\alpha)(\alpha/\sigma)^{2/3}.$$

The structure of series (18) explains the different behaviors of $\langle t_1 \rangle$ at $\beta < 0$ and $\beta > 0$. At $\beta < 0$, all terms in the expansion are positive, and therefore one obtains larger values of $\langle t_1 \rangle$. At $\beta > 0$, the terms in (18) change their signs, and $\langle t_1 \rangle$ becomes small.

Using this formula, we obtained the dependences of the MFPT on the quantities β and σ. They are depicted in Fig. 4 (*a* and *b*) by the solid lines. In our calculations, we assumed $x_0 = -2$, $x_F = 2$, and $R = 4$. The constants α and γ were chosen equal to $\alpha = 0.02$ and $\gamma = 0.174$ in order that the calculated curves coincide with the results of numerical simulations.

The results of theoretical calculations were compared to the data obtained by the numerical modeling of Eq. (7). The relaxation time from the lower to the upper state was determined with the help of a procedure described in [25]. It was assumed that the bifurcation took place at the time moment when $T(\tau)$ exceeded the limiting value $T_{tr} = 3.5$ for the first time, which ensures a transition to the upper state. In order to determine the mean time, we calculated a large number of trajectories (as a rule, 3000) for each set of parameters with various noise realizations. The results of our numerical calculations depending on the parameter β at different noise intensities are presented in Fig.4,*a* by the dotted lines.

In the absence of noise, the time necessary for the system to reach a stationary state increases as far as it approaches the transition point (see curve *1*). In the subthreshold region $\beta < 0$, the relaxation time grows with increase in a deviation of the control parameter $\langle T_0 \rangle$ from the critical value. At equal deviations, an increase of the noise intensity results in a reduction of the relaxation time. In the region beyond the threshold $\beta > 0$, the relaxation time decreases with increase in the deviation and reaches a stationary value which is very small as compared with that at $\beta = 0$.

One can see from Fig. 4,*a* that the theoretical calculations with the use of the approximate formula (17) satisfactorily agree with the results of numerical modeling of Eq. (8). The comparison of the results of the numerical modeling and analytical calculations in the case of the MFPT dependences on the noise intensity σ is less acceptable (see Fig. 4,*b*), though the basic regularity (a decrease of the first passage time with increase in the noise intensity) is conserved. Some difference in the curves is most probably caused by the multiplicative character of noise in Eq. (7) used for numerical calculations, whereas noise in the Langevin equation (12) used to obtain the analytical expression was of additive character. Such an assumption is based on the results of works [7] and [14] dealing with the comparison of the effect of multiplicative and additive noises on the relaxation time of fluctuations in laser and biological systems, respectively. It was shown that, in the case of multiplicative noise, the relaxation time at small σ first grows with increase in σ, reaches a maximum, and then falls at high σ. In the case of additive noise, the behavior was opposite, namely the relaxation time tended to infinity at $\sigma \to 0$ and fell with increase in σ.

6. Conclusions

We considered a simplified stochastic model of tunnel process that allowed us to determine the basic regularities governing its response to the action of intensity fluctuations of an incident electron flow (characterized by white noise). In particular, it was found that an increase in the noise intensity results not only in the broadening of the probability peaks but also in the appearance of the bimodality in the probability distribution that was absent at low noise levels under the same conditions.

The mean first passage time as a function of the noise intensity was obtained both numerically and using the theoretical model proposed in [15]. It is shown that, in the subthreshold region, $\beta < 0$, the MFPT grows with increase in a deviation β of the control parameter $\langle T_0 \rangle$ from the critical value. The increase of the first passage time to values exceeding the value of critical slowing-down in the deterministic case practically means that the process of transition does not take place. At equal deviations, the MFPT falls with increase in the noise intensity. The obtained results imply that, with a rise in the deviation β in the subthreshold region, one must increase the noise intensity in order to realize a transition, i.e. the noise assists the transition at the values $\langle T_0 \rangle$, at which it is impossible in the deterministic case. This conclusion differs from that made in [12, 28], where it was stated that the bifurcation point shifted toward the growth of a control parameter with increase in the noise intensity.

It can be seen from the obtained results that, for the proposed model, the process of transition in the neighborhood of the critical point is mainly determined by additive noise. This conclusion confirms the result of work [27] about the minimal role of multiplicative noise in the determination of the first passage time.

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ВПЛИВ ЗОВНІШНЬОГО ШУМУ НА ПРОЦЕС РЕЛАКСАЦІЇ В БІСТАБІЛЬНИХ ТУНЕЛЬНИХ СИСТЕМАХ

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Р е з ю м е

Розглянуто вплив флуктуацій інтенсивності падаючого на двобар'єрну тунельну систему потоку електронів поблизу точки нестабільності. Для опису ефектів шуму в системі, що перебуває поблизу резонансу в умовах когерентності тунелювання, використано спрощене рівняння Ланжевена з мультиплікативним білим шумом. Методом чисельної симуляції цього рівняння отримано залежності середнього часу першого проходу від інтенсивності шуму й відхилення середнього значення інтенсивності падаючого потоку електронів від критичного в детерміністичному випадку. Результати чисельних розрахунків задовільно збіглися з теоретичними розрахунками роботи Коле і ін. Час релаксації був максимальним у відсутності шуму й спадав із ростом інтенсивності шуму. Для тих значень інтенсивності потоку, за яких перехід у детерміністичному випадку був неможливий, введення шуму сприяло переходу.