

# EFFECT OF UNIAXIAL PRESSURE ON THE COEFFICIENT OF FREE CARRIER OPTICAL ABSORPTION IN MANY-VALLEY SEMICONDUCTORS (*n*-Ge)

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The effect of a uniaxial elastic deformation on the polarization dependence of the coefficient of free carrier optical absorption is considered with regard for the carrier scattering by acoustic phonons and impurity atoms. The optical absorption coefficient for *n*-Ge is numerically calculated both in the quantum and in the classical frequency range.

## 1. Introduction

The phenomenon of free carrier optical absorption in semiconductors has been thoroughly studied for a long time (see, e.g., [1, 2]). In the absence of heating, electrons in many-valley semiconductors are uniformly distributed over valleys, and the free carrier absorption coefficient does not depend on the polarization. However, as was shown in [3], the redistribution of carriers between valleys can result in the polarization dependence of the optical absorption coefficient. This redistribution can take place as a result of the nonuniform heating of carriers by either a constant or a high-frequency electric field or pressure. It is known that a uniaxial elastic deformation (UED) in many-valley semiconductors results in a change of the carrier spectrum and the dependence of the shift of the energy minima of conduction band carriers on the UED direction with respect to the crystallographic axes of a crystal. Different shifts of the minima along various crystallographic directions can give rise to the redistribution of carriers between valleys and to the appearance of the polarization dependence of the absorption coefficient on the UED parameters. For example, work [2] was devoted to the study of the effect of a uniaxial pressure on the absorption coefficient in *n*-Ge in the quantum frequency region. The aim of this work is to investigate a more general form of the polarization dependence of the free carrier absorption coefficient on the UED magnitude and direction by the example of *n*-Ge. The effects related to the carrier redistribution between valleys will be taken into account, whereas those related

to the variation of the carrier spectrum are insignificant [4,5] and will be neglected.

## 2. Problem Statement

For the sake of simplicity, we consider the case where the pressure or tension takes place along the crystallographic axis  $\langle 1,1,1 \rangle$  of an *n*-Ge crystal, whereas light falls normally to the  $\langle 1,1,1 \rangle$  axis, as is shown in Fig.1. At such an orientation, a UED results in a shift of the energy minimum of the  $\langle 1,1,1 \rangle$  valley by the magnitude  $\Delta_1 = -\Xi_u S_{44} X/3$ , while the energy minima of the  $\langle \bar{1},1,1 \rangle$ ,  $\langle 1,\bar{1},1 \rangle$ , and  $\langle 1,1,\bar{1} \rangle$  valleys shift by the magnitude  $\Delta_2 = \Xi_u S_{44} X/9$  [4]. Here,  $\Xi_u$  is the deformation potential constant,  $S_{44} = 1/c_{44}$ ,  $c_{44}$  is the elasticity constant, and  $X$  is the pressure. In the absence of the pressure, electrons are uniformly distributed over valleys with the concentrations  $n_1 = n_2 = n_3 = n_4 = n_0/4$ , where  $n_1$  denotes the electron concentration in the  $\langle 1,1,1 \rangle$  valley,  $n_2$ ,  $n_3$ , and  $n_4$  are the electron concentrations in the  $\langle \bar{1},1,1 \rangle$ ,  $\langle 1,\bar{1},1 \rangle$ , and  $\langle 1,1,\bar{1} \rangle$  valleys, respectively, and  $n_0$  is the total electron concentration. In what follows, we suppose that the optical emission intensity is low, i.e. the electron temperatures in different valleys are equal to each other and to the lattice temperature  $T$ . Under these conditions, the ratio between the electron concentrations in valleys under pressure takes the form

$$\frac{n_2}{n_1} = \exp\left(\frac{\Delta_1 - \Delta_2}{\theta}\right), \quad (1)$$

where  $\theta = kT$  is the electron temperature in energy units, and  $k$  is the Boltzmann constant.

According to [6], the coefficient of optical absorption by free electrons in *n*-Ge in the case of impurity-atom scattering is presented as

$$K_p = \frac{(2\pi)^{\frac{3}{2}} e^6 N \sqrt{m_{\text{par}}}}{(\varepsilon_0 \varepsilon)^{\frac{5}{2}} c (m_{\text{par}} - m_{\text{per}})^2 \hbar \omega^3} \times$$

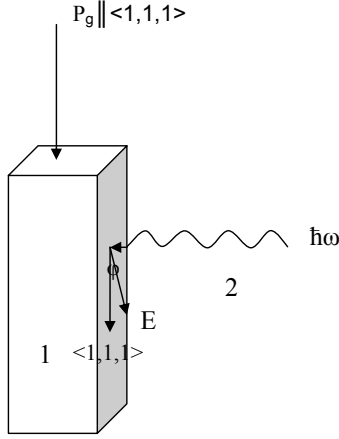


Fig. 1. 1 – *n*-Ge crystal, 2 – incident electromagnetic wave with polarization *E*

$$\begin{aligned} & \times \sum_{i=1}^4 \frac{n_i}{\sqrt{\theta_i}} \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta_i}\right) \right) \times \\ & \times \int_0^{\infty} \frac{dx e^{-x} (\Psi(q_{\max}) + \Psi(q_{\min}))}{\sqrt{x \left(x + \frac{\hbar\omega}{\theta_i}\right)}}, \end{aligned} \quad (2)$$

where  $\varepsilon$  is the dielectric permittivity of *n*-Ge,  $\varepsilon_0$  is the dielectric permittivity of vacuum,  $c$  is the velocity of light,  $\hbar$  is the Planck constant,  $m_{\text{par}}$  and  $m_{\text{per}}$  are the longitudinal and transverse electron masses, respectively,  $n_i$  and  $\theta_i$  are the electron concentration and the temperature in the *i*-th valley, respectively,  $\omega$  is the light frequency, and  $N$  is the concentration of impurity atoms. In addition, we have

$$\begin{aligned} \Psi(q^*) &= \frac{1}{b^2} + \frac{1-b^2}{b^3} \arctan \frac{1}{b} + (\mathbf{q}_0 \mathbf{l}_i)^2 \times \\ & \times \left[ -\frac{1}{b^2} - \frac{1-b^2}{b^3} \arctan \frac{1}{b} + \right. \\ & \left. + 2 \frac{m_{\text{per}}}{m_{\text{par}}} \left( -\frac{1}{1+b^2} + \frac{1}{b} \arctan \frac{1}{b} \right) \right], \end{aligned} \quad (3)$$

$$b^2 = \frac{m_{\text{per}}}{m_{\text{par}} - m_{\text{per}}} \left( 1 + \frac{1}{(r_d q^*)^2} \right) \quad (4)$$

$$q_{\max} = \frac{\sqrt{2 m_{\text{per}} \theta}}{\hbar} \left\{ \sqrt{x} + \sqrt{x + \frac{\hbar\omega}{\theta}} \right\},$$

$$q_{\min} = \frac{\sqrt{2 m_{\text{per}} \theta}}{\hbar} \left\{ -\sqrt{x} + \sqrt{x + \frac{\hbar\omega}{\theta}} \right\}, \quad (5)$$

where  $r_d$  is the shielding radius,  $\mathbf{q}_0$  is the unit vector characterizing the wave polarization, and  $\mathbf{l}_i$  is the unit vector directed along the axis of revolution of the ellipsoid of the *i*-th valley. Formula (2) describes the optical absorption by electrons in the general case of impurity-atom scattering. However, it can be significantly simplified in various limiting cases. For instance, for the quantum absorption ( $\hbar\omega \gg \theta$ ), one obtains

$$\begin{aligned} K_p &= \frac{4 \sqrt{2} \pi^2 e^6 N \sqrt{m_{\text{par}}}}{(\varepsilon_0 \varepsilon)^{5/2} c (m_{\text{par}} - m_{\text{per}})^2 \omega^2 (\hbar\omega)^{3/2}} \times \\ & \times \sum_{i=1}^4 n_i \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) \left( \frac{1}{b_0^2} + \frac{1-b_0^2}{b_0^3} \arctan \frac{1}{b_0} + \right. \\ & \left. + (\mathbf{q}_0 \mathbf{l}_i)^2 \left[ -\frac{1}{b_0^2} - \frac{1-b_0^2}{b_0^3} \arctan \frac{1}{b_0} + \right. \right. \\ & \left. \left. + 2 \frac{m_{\text{per}}}{m_{\text{par}}} \left( -\frac{1}{1+b_0^2} + \frac{1}{b_0} \arctan \frac{1}{b_0} \right) \right] \right), \end{aligned} \quad (6)$$

where  $b_0^2 = \frac{m_{\text{per}}}{m_{\text{par}} - m_{\text{per}}}$ .

In the case of classical absorption ( $\hbar\omega \ll \theta$ ),

$$\begin{aligned} K_p &= \frac{(2\pi)^{3/2} e^6 N \sqrt{m_{\text{par}}}}{(\varepsilon_0 \varepsilon)^{5/2} c (m_{\text{par}} - m_{\text{per}})^2 \hbar\omega^3} \times \\ & \times \sum_{i=1}^4 n_i \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) \left( \frac{1}{b_0^2} + \frac{1-b_0^2}{b_0^3} \arctan \frac{1}{b_0} + \right. \\ & \left. + (\mathbf{q}_0 \mathbf{l}_i)^2 \left[ -\frac{1}{b_0^2} - \frac{1-b_0^2}{b_0^3} \arctan \frac{1}{b_0} + \right. \right. \\ & \left. \left. + 2 \frac{m_{\text{per}}}{m_{\text{par}}} \left( -\frac{1}{1+b_0^2} + \frac{1}{b_0} \arctan \frac{1}{b_0} \right) \right] \right) \ln \left( \frac{1}{C_1 x_{\min}} \right), \end{aligned} \quad (7)$$

where  $\ln[C_1]$  is the Euler constant and  $x_{\min} = \frac{\hbar^2}{8 m_{\text{per}} \theta r_d^2}$ .

The free carrier absorption coefficient in the case of acoustic-phonon scattering has a form [5]

$$K_a = \frac{16 \sqrt{\pi} e^2 \theta}{3 \sqrt{\varepsilon_0 \varepsilon} c \hbar \omega^3 m_{\text{par}}} \sum_{i=1}^4 n_i \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) \times$$

$$\times \left\{ \frac{1}{\tau_{\text{per}}} + (\mathbf{q}_0 \mathbf{l}_i)^2 \left( \frac{m_{\text{per}}}{m_{\text{par}} \tau_{\text{par}}} - \frac{1}{\tau_{\text{per}}} \right) \right\} \times$$

$$\times \left\{ a^3 \frac{d}{da} \left( \frac{K_1(a)}{a} \right) \right\}, \quad (8)$$

where  $a = \frac{\hbar\omega}{2\theta}$ ,  $K_1(a)$  is the modified Bessel function,  $\tau_{\text{par}}$  and  $\tau_{\text{per}}$  are the longitudinal and transverse times of the electron momentum relaxation via acoustic phonons. In the case of quantum absorption ( $\hbar\omega \gg \theta$ ), Eq. (8) takes the form

$$K_a = \frac{4\pi e^2}{3\sqrt{\varepsilon_0 \varepsilon} c \omega^2 m_{\text{per}}} \left( \frac{\hbar\omega}{\theta} \right)^{1/2} \times$$

$$\times \sum_{i=1}^4 n_i \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) \times$$

$$\times \left\{ \frac{1}{\tau_{\text{per}}} + (\mathbf{q}_0 \mathbf{l}_i)^2 \left( \frac{m_{\text{per}}}{m_{\text{par}} \tau_{\text{par}}} - \frac{1}{\tau_{\text{per}}} \right) \right\}, \quad (9)$$

while, in the case of classical absorption ( $\hbar\omega \ll \theta$ ), it turns into

$$K_a = \frac{32\sqrt{\pi} e^2 \theta}{3\sqrt{\varepsilon_0 \varepsilon} c \hbar \omega^3 m_{\text{per}}} \sum_{i=1}^4 n_i \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) \times$$

$$\times \left\{ \frac{1}{\tau_{\text{per}}} + (\mathbf{q}_0 \mathbf{l}_i)^2 \left( \frac{m_{\text{per}}}{m_{\text{par}} \tau_{\text{par}}} - \frac{1}{\tau_{\text{per}}} \right) \right\}. \quad (10)$$

### 3. Numerical Calculations and Discussion of the Results

The total absorption coefficient is  $K = K_a + K_p$ . The structure of formulas (2) and (8) allows one to represent the total absorption coefficient  $K$  as

$$K = \sum_{i=1}^4 \left[ R_i + (\mathbf{q}_0 \mathbf{l}_i)^2 Q_i \right] \frac{n_i}{n_0}, \quad (11)$$

where  $R_i$  is the part of the absorption coefficient of the  $i$ -th valley that does not depend on its orientation, and  $Q_i$  is the part of the absorption coefficient dependent on the mutual orientation of the  $i$ -th valley and the vector of light polarization. Summing over valleys in (11) under the above conditions, one obtains the general expression for the absorption coefficient as a function of the UED

$$K = R + \frac{4}{3} Q \frac{e^{-\frac{4\varepsilon_u S_{44} X}{9\theta}}}{1 + 3e^{-\frac{4\varepsilon_u S_{44} X}{9\theta}}} +$$

$$+ Q \frac{1 - e^{-\frac{4\varepsilon_u S_{44} X}{9\theta}}}{1 + 3e^{-\frac{4\varepsilon_u S_{44} X}{9\theta}}} \cos^2(\varphi). \quad (12)$$

Here,  $\phi$  is the angle between the polarization vector and the UED direction (coinciding with the  $\langle 1,1,1 \rangle$  axis),

$$R = \left[ \frac{(2\pi)^{3/2} e^6 N \sqrt{m_{\text{par}}}}{(\varepsilon_0 \varepsilon)^{5/2} c (m_{\text{par}} - m_{\text{per}})^2 \hbar \omega^3 \sqrt{\theta}} \times \right.$$

$$\times \left( \frac{1}{b_{\text{max}}^2} + \frac{1 - b_{\text{max}}^2}{b_{\text{max}}^3} \arctan \frac{1}{b_{\text{max}}} + \frac{1}{b_{\text{min}}^2} + \right.$$

$$\left. + \frac{1 - b_{\text{min}}^2}{b_{\text{min}}^3} \arctan \frac{1}{b_{\text{min}}} \right) + \frac{16\sqrt{\pi} e^2 \theta}{3\sqrt{\varepsilon_0 \varepsilon} c \hbar \omega^3 m_{\text{per}} \tau_{\text{per}}} \times$$

$$\times \left\{ a^3 \frac{d}{da} \left( \frac{K_1(a)}{a} \right) \right\} \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) n_0, \quad (13)$$

$$Q = \left[ \frac{(2\pi)^{3/2} e^6 N \sqrt{m_{\text{par}}}}{(\varepsilon_0 \varepsilon)^{5/2} c (m_{\text{par}} - m_{\text{per}})^2 \hbar \omega^3 \sqrt{\theta}} \times \right.$$

$$\times \left( -\frac{1}{b_{\text{max}}^2} - \frac{1 - b_{\text{max}}^2}{b_{\text{max}}^3} \arctan \frac{1}{b_{\text{max}}} - \frac{1}{b_{\text{min}}^2} - \right.$$

$$\left. - \frac{1 - b_{\text{min}}^2}{b_{\text{min}}^3} \arctan \frac{1}{b_{\text{min}}} + 2 \frac{m_{\text{per}}}{m_{\text{par}}} \left( -\frac{1}{1 + b_{\text{max}}^2} + \right. \right.$$

$$\left. \left. + \frac{1}{b_{\text{max}}} \arctan \frac{1}{b_{\text{max}}} - \frac{1}{1 + b_{\text{min}}^2} + \frac{1}{b_{\text{min}}} \arctan \frac{1}{b_{\text{min}}} \right) \right) +$$

$$+ \frac{16\sqrt{\pi} e^2 \theta}{3\sqrt{\varepsilon_0 \varepsilon} c \hbar \omega^3 m_{\text{per}} \tau_{\text{per}}} \left( \frac{m_{\text{per}} \tau_{\text{per}}}{m_{\text{par}} \tau_{\text{par}}} - 1 \right) \times$$

$$\times \left\{ a^3 \frac{d}{da} \left( \frac{K_1(a)}{a} \right) \right\} \left( 1 - \exp\left(-\frac{\hbar\omega}{\theta}\right) \right) n_0, \quad (14)$$

where  $b_{\text{max}} = b(q_{\text{max}})$ , and  $b_{\text{min}} = b(q_{\text{min}})$ .

One can see that, in the absence of pressure ( $X = 0$ ), the third term in (12) equals zero, and the dependence of the absorption coefficient on the emission polarization disappears. However, in the presence of pressure

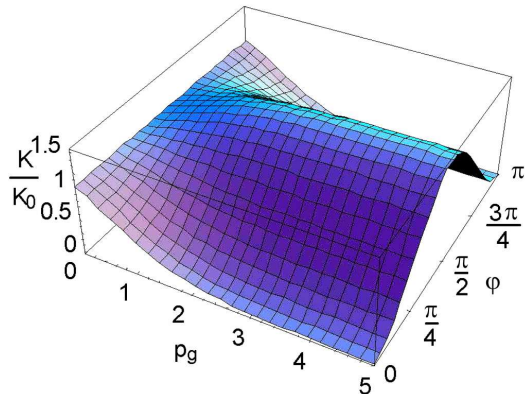


Fig. 2. Relative absorption coefficient  $\frac{K}{K_0}$  in  $n$ -Ge as a function of the pressure  $p_g$  and the polarization angle  $\phi$  at  $\omega = 10^{14} \text{ s}^{-1}$ ,  $n_0 = 2.5 \times 10^{15} \text{ cm}^{-3}$ ,  $T = 77 \text{ K}$

( $X \neq 0$ ), the magnitude of the absorption coefficient changes (due to a variation of the second term in (12)), and there appears a dependence of the absorption coefficient on the light polarization. Numerical calculations of the free carrier absorption coefficient in many-valley semiconductors were performed for various concentrations of free carriers and impurity atoms in the temperature range  $T = 40\text{--}200 \text{ K}$  for the far infrared spectral region. For convenience, the obtained results were normalized to the value of the absorption coefficient in the absence of pressure,  $K_0$ . The results of numerical calculations of the free carrier absorption coefficient as a function of the pressure and the angle of polarization are presented in Fig. 2. Figure 3 shows the results of numerical calculations in the cases where the vectors of light polarization are directed in parallel or normally to the axis  $\langle 1,1,1 \rangle$  ( $\phi=0$  and  $\phi = \pi/2$ , respectively). One can see from Fig. 3 that the pressure considerably changes the free carrier absorption coefficient. In particular, the absorption coefficient grows by 40% for a wave polarized in parallel to the axis  $\langle 1,1,1 \rangle$  and decreases almost fivefold for a wave polarized normally to it.

Thus, it is demonstrated that a UED in many-valley semiconductors results in the appearance of the dependence of the free carrier absorption coefficient on the direction of the polarization of incident radiation. In the case where the UED is directed along the  $\langle 1,1,1 \rangle$  axis, the free carrier absorption coefficient can vary by several times, depending on the orientation of the light polarization vector and the pressure.

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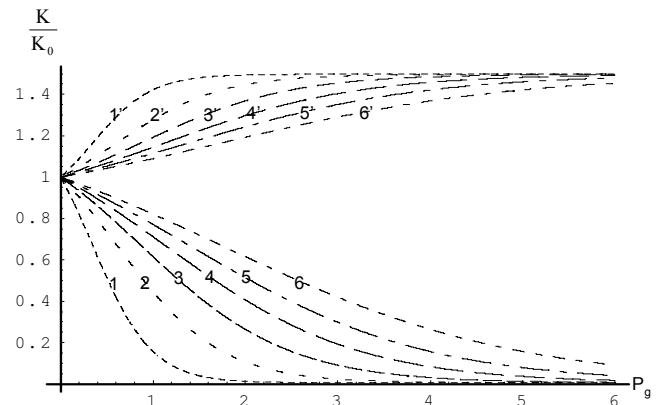


Fig. 3. Relative absorption coefficient  $\frac{K}{K_0}$  in  $n$ -Ge as a function of  $p_g$  at  $\omega = 10^{14} \text{ s}^{-1}$ ,  $n_0 = 2.5 \times 10^{15} \text{ cm}^{-3}$ ;  $T = 40 \text{ K}$  (1, 1'),  $T = 70 \text{ K}$  (2, 2'),  $T = 100 \text{ K}$  (3, 3'),  $T = 130 \text{ K}$  (4, 4'),  $T = 160 \text{ K}$  (5, 5'),  $T = 200 \text{ K}$  (6, 6');  $\phi = 0$  (1-6),  $\phi = \pi/2$  (1'-6')

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#### ВПЛИВ ОДНОВІСНОГО ТИСКУ НА КОЕФІЦІЄНТ ПОГЛИНАННЯ СВІТЛА ВІЛЬНИМИ НОСІЯМИ В БАГАТОДОЛИННИХ НАПІВПРОВІДНИКАХ ( $n$ -Ge)

О.Є. Левшин

#### Резюме

Розглянуто вплив одновісної пружної деформації на поляризаційну залежність коефіцієнта поглинання світла вільними носіями з урахуванням розсіяння носіїв на акустичних фонах та домішках. Проведено чисельні розрахунки коефіцієнта поглинання світла на прикладі  $n$ -Ge як у квантовій, так і класичній області частот.