CAN LAYERED-STRUCTURE EFFECTS BE OBSERVED, IF THE FERMI SURFACE IS CLOSED?

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By analyzing the longitudinal conductivity in a quantizing magnetic field directed perpendicularly to the crystal lattice layers, it has been demonstrated that the layered-structure effects can be observed not only in crystals with highly open Fermi surfaces, as was conventionally believed earlier, but also in crystals with closed ones. The calculations were carried out in the constant-relaxation-time approximation. In weak magnetic fields, layered-structure effects manifest themselves as a phase retardation of Shubnikov–de Haas oscillations and a certain increase of the relative contribution made by the latter. In the range of high magnetic fields, there exists an optimal interval, in which the layered-structure effects reveal themselves in the form of a sharp non-monotonous dependence of conductivity on the magnetic field. In addition, it has been shown that the layered-structure effects result in a decrease of the proportionality factor between the magnetoresistance and the magnetic induction in the longitudinal Kapitsa effect. The longitudinal conductivity of layered crystals in ultra-quantum magnetic fields has also been analyzed. It is shown that the following dependences of the magnetoresistance on the magnetic field can be obtained, depending on the model used for the filling of the single Landau subband and on whether the longitudinal conductivity is considered to be of either the drift or diffusion type: $\rho_{zz} \propto TB^2$, $\rho_{zz} \propto B^3$, and $\rho_{zz} \propto B^4$.

1. Introduction

The model of band spectrum for charge carriers in layered crystals was proposed by R.F. Fivaz as early as in 1967 [1]. Since then, a lot of works devoted to theoretical and experimental researches of various physical characteristics of those crystals have been published. However, it has been considered till now that the layered-structure effects in electronic processes that take place in those crystals are pronounced only if the corresponding Fermi surface (FS) is open, i.e. the FS occupies the whole first Brillouin zone and, when being periodically continued, forms a connected surface [2–5]. On the contrast, crystals with closed FSes—i.e. FSes which occupy only some part of the first Brillouin zone and turn out unconnected at the periodic continuation–are not classified as layered ones by the majority of researchers, even if the motion of charge carriers along the superlattice axis (i.e. perpendicularly to the layers) is described by the strong-coupling law rather than the effective mass one. At the same time, in the works published earlier [6–9], the analysis of the diamagnetic susceptibility of an electron gas showed that the layered-structure effects can manifest themselves, if the FS is closed too, provided that the level of the filling of a miniband is high. This work aimed at studying the influence of layered-structure effects on the longitudinal conductivity of layered crystals with closed FSes in a strong quantizing magnetic field. The problem concerned is examined in the approximation of Ohm's law applicability to the case where a strong quantizing magnetic field and an electric field are parallel to each other and perpendicular to the layer planes.

2. Calculation of Longitudinal Electroconductivity in the Layered Crystal and Discussion of the Results Obtained

To calculate the longitudinal conductivity of a layered crystal in a quantizing magnetic field directed perpendicularly to the planes of layers, the following law for the charge carrier dispersion was used:

$$\varepsilon(n,x) = \mu^* B(2n+1) + W(x). \quad (1)$$

Here, $\mu^* = \mu_{\rm B} m_0/m^*$, $\mu_{\rm B}$ is the Bohr magneton, m_0 the free electron mass, m^* the effective mass of electron in the layer plane, B the magnetic field induction, n the number of a Landau level, $W(x)$ the dispersion law for

charge carriers along the superlattice axis, $x = ak_z$, k_z is the quasimomentum component along the superlattice axis, and a the distance between translationally equivalent layers.

To study the influence of layered-structure effects on the longitudinal electroconductivity, the latter was calculated in two cases, namely, in the case of the strong-coupling approximation,

$$W(x) = \Delta(1 - \cos x), \tag{2}$$

where Δ is the miniband halfwidth which governs the electron motion between layers; and in the case where the right-hand side in formula (2) is expanded into a series in x up to the quadratic term, which corresponds to the effective mass approximation. In both cases, the dependence of the chemical potential in the electron gas on the magnetic field induction was taken into account. To simplify calculations, the relaxation time of charge carriers was assumed to be constant. With the same purpose in view, the influence of the Dingle factor on longitudinal conductivity oscillations (Shubnikov–de Haas oscillations) was not considered in detail, although it can be substantial at the electron scattering by impurities and defects in the crystal lattice [5, 10].

In work [11], a detailed derivation was given for the equation of chemical potential in the electron gas in a quantizing magnetic field, as well as the formulas for components of the longitudinal electroconductivity, in the case of an arbitrary function $W(x)$ and provided that the relaxation time τ of charge carriers is either a constant or a function of the longitudinal quasimomentum only, i.e. $\tau = \tau(x)$. Then, if Shubnikov–de Haas oscillations are fierce, the longitudinal crystal electroconductivity can be determined as follows:

$$\sigma_{zz}(B) = \sigma_0 + \sigma_{os}(B). \tag{3}$$

The individual components in Eq. ((3) are given by the formulas [11]

$$\sigma_0 = \frac{16\pi^2 e^2 m^* a}{h^4} \int\limits_{W(x)\le\zeta} \tau(x) |W'(x)|^2 dx, \tag{4}$$

$$\sigma_{os}(B) = \frac{32\pi^2 e^2 m^* a}{h^4} \sum_{l=1}^{\infty} (-1)^l f_l^\sigma \times$$

$$\times \int\limits_{W(x)\le\zeta} \tau(x) |W'(x)|^2 \cos\left(\pi l \frac{\zeta - W(x)}{\mu^* B}\right) dx, \tag{5}$$

$$f_l^\sigma = \frac{\pi^2 lkT/\mu^* B}{sh\left(\pi^2 lkT/\mu^* B\right)}. \tag{6}$$

The integration in formulas (4) and (5) is carried out over the positive x-values only. The quantity ζ is the chemical potential of an electron gas, the other notations either are standard or were explained above. The equation that determines the chemical potential of an electron gas in a quantizing magnetic field, provided that the Shubnikov–de Haas effect is pronounced, looks like

$$n_0 = \frac{4m^*}{ah^2} \int\limits_{W(x)\le\zeta} [\zeta - W(x)]\, dx + \frac{8\pi m^* kT}{ah^2} \times$$

$$\times \sum_{l=1}^{\infty} \frac{(-1)^l}{\mathrm{sh}\left(\pi^2 lkT/\mu^* B\right)} \int\limits_{W(x)\le\zeta} \sin\left(\pi l \frac{\zeta - W(x)}{\mu^* B}\right), \tag{7}$$

where n_0 is the concentration of charge carriers in the crystal bulk. Under conditions that the relaxation time is constant, $\tau(x) = \tau_0$, dispersion law (2) is fulfilled, and the FS is closed (for such an FS, $\zeta < 2\Delta$), formulas (4) and (5) read, respectively,

$$\sigma_0 = \frac{8\pi^2 e^2 m^* a \tau_0 \Delta^2}{h^4} (C_0 - C_2), \tag{8}$$

$$\sigma_{os}(B) = \frac{16\pi^2 e^2 m^* a \tau_0 \Delta^2}{h^4} \times$$

$$\times \sum_{l=1}^{\infty} (-1)^l f_l^\sigma \left\{ \cos\left(\pi l \frac{\zeta - \Delta}{\mu^* B}\right) \left[(C_0 - C_2) J_0\left(\frac{\pi l \Delta}{\mu^* B}\right) + \right.\right.$$

$$+ \sum_{r=1}^{\infty} (-1)^r (2C_{2r} - C_{2r+2} - C_{2r-2}) \times$$

$$\left.\left. \times J_{2r}\left(\frac{\pi l \Delta}{\mu^* B}\right)\right] - \sin\left(\pi l \frac{\zeta - \Delta}{\mu^* B}\right) \sum_{r=0}^{\infty} (-1)^r (2C_{2r+1} - \right.$$

$$\left. - C_{2r+3} - C_{|2r-1|}) J_{2r+1}\left(\frac{\pi l \Delta}{\mu^* B}\right)\right\}. \tag{9}$$

In these formulas,

$$C_0 = \arccos(1 - \gamma), \tag{10}$$

$$C_m = \frac{\sin m C_0}{m} \quad \text{for} \quad m \neq 0, \tag{11}$$

$$\gamma = \zeta/\Delta, \tag{12}$$

and $J_n(z)$ are the Bessel functions of the first kind of the real argument z.

In the case of a closed FS and the dispersion law (2), Eq. (7) reads

$$n_0 = \frac{4m^*\Delta}{ah^2}\left[(\gamma-1)\,C_0 + \sqrt{2\gamma-\gamma^2}\right] +$$

$$+\frac{8m^*\pi kT}{ah^2}\sum_{l=1}^{\infty}\frac{(-1)^l}{sh\left(\pi^2 lkT/\mu^* B\right)}\left\{\sin\left(\pi l\frac{\zeta-\Delta}{\mu^* B}\right)\times\right.$$

$$\times\left[C_0 J_0\left(\frac{\pi l\Delta}{\mu^* B}\right) + 2\sum_{r=1}^{\infty}(-1)^r C_{2r} J_{2r}\left(\frac{\pi l\Delta}{\mu^* B}\right)\right] +$$

$$\left. + 2\cos\left(\pi l\frac{\zeta-\Delta}{\mu^* B}\right)\sum_{r=0}^{\infty}(-1)^r C_{2r+1} J_{2r+1}\left(\frac{\pi l\Delta}{\mu^* B}\right)\right\}. \tag{13}$$

In the case of an open FS, i.e. if $\gamma \geq 2$, one has to put $C_0 = \pi$ in formulas (8)–(13) and, additionally, put the radical in formula (13) to be equal zero.

By passing to the effective mass approximation in formulas (8), (9), and Eq. (13), we obtain, respectively,

$$\sigma_0 = \frac{16\pi^2 e^2 m^* a\tau_0\Delta^2}{3h^4}\left(\frac{2\zeta}{\Delta}\right)^{3/2}, \tag{14}$$

$$\sigma_{os}(B) = \frac{32\pi^{1/2}e^2 m^* a\tau_0\Delta^{1/2}(\mu^* B)^{3/2}}{h^4}\times$$

$$\times\sum_{l=1}^{\infty}\frac{(-1)^l f_l^{\sigma}}{l^{3/2}}\left[\sin\left(\frac{\pi l\zeta}{\mu^* B}\right)C\left(\sqrt{\frac{2l\zeta}{\mu^* B}}\right) -\right.$$

$$\left. - \cos\left(\pi l\frac{\zeta-\Delta}{\mu^* B}\right)S\left(\sqrt{\frac{2l\zeta}{\mu^* B}}\right)\right], \tag{15}$$

$$n_0 = \frac{8m^*\zeta}{3ah^2}\sqrt{\frac{2\zeta}{\Delta}} + \frac{8\pi m^* kT}{ah^2}\sqrt{\frac{\mu^* B}{\Delta}}\times$$

$$\times\sum_{l=1}^{\infty}\frac{(-1)^l}{l^{1/2}\mathrm{sh}\left(\pi^2 lkT/\mu^* B\right)}\left[\sin\left(\frac{\pi l\zeta}{\mu^* B}\right)C\left(\sqrt{\frac{2l\zeta}{\mu^* B}}\right) +\right.$$

$$\left. + \cos\left(\frac{\pi l\zeta}{\mu^* B}\right)S\left(\sqrt{\frac{2l\zeta}{\mu^* B}}\right)\right]. \tag{16}$$

In formulas (15) and (16), the functions $C(z)$ and $S(z)$ are the Fresnel cosine and sine integrals, respectively. The charge carrier concentration was assumed constant in both examined cases. It was determined by the formula

$$n_0 = \frac{4m^*\Delta}{ah^2}\left[(\gamma_0-1)\arccos(1-\gamma_0) + \sqrt{2\gamma_0-\gamma_0^2}\right], \tag{17}$$

where $\gamma_0 = \zeta_0/\Delta$, and ζ_0 is the Fermi energy of the electron gas in the crystal, provided the dispersion law (2), the zero absolute temperature, and the absence of a magnetic field.

Before passing to a more detailed analysis of the influence of layered-structure effects on the longitudinal electroconductivity of the crystal, we note that, in the quasiclassical approximation, when the conditions $\Delta/(\mu^* B) \gg 1$ and $\zeta/(\mu^* B) \gg 1$ are satisfied, formulas (9) and (15) for the oscillating part of the longitudinal electroconductivity give the same result,

$$\sigma_{os}(B) = \frac{16\sqrt{2}\pi^{1/2}e^2 m^* a\tau_0\Delta^{1/2}(\mu^* B)^{3/2}}{h^4}\times$$

$$\times\sum_{l=1}^{\infty}(-1)^l l^{-3/2}\sin\left(\frac{\pi l\zeta}{\mu^* B} - \frac{\pi}{4}\right). \tag{18}$$

It is not of surprise, because both the specific dispersion law for charge carriers and the FS finiteness – both in the magnetic field direction – are insignificant in the quasiclassical approximation. However, in formulas (9) and (15), those factors were taken into account explicitly. Therefore, in the framework of this approximation, it is impossible to distinguish the influence of layered-structure effects on the longitudinal electroconductivity in crystals with a closed FS, provided that the Fermi energies are identical.

However, if the concentration of charge carriers is constant, the layered-structure effects will affect electroconductivity oscillations even in the quasiclassical approximation, because the Fermi energy in a weak magnetic field is a little higher in the effective mass approximation than that in the case of the dispersion law (2). It is so because, if a finite width of the miniband is taken into account explicitly, the density of states turns out higher than that in the effective mass approximation. The field

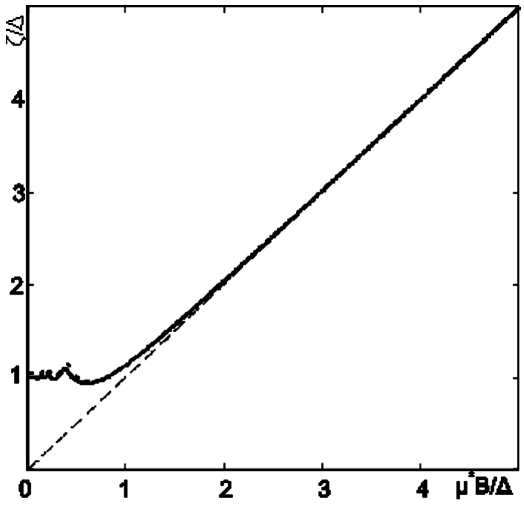

Fig. 1. Field dependences of the chemical potential at $\gamma_0 = 1$ and $kT/\Delta = 0.03$ for a layered crystal (solid curve) and in the effective mass approximation (dashed curve). The thin dashed line corresponds to the limiting case $\zeta = \mu^* B$

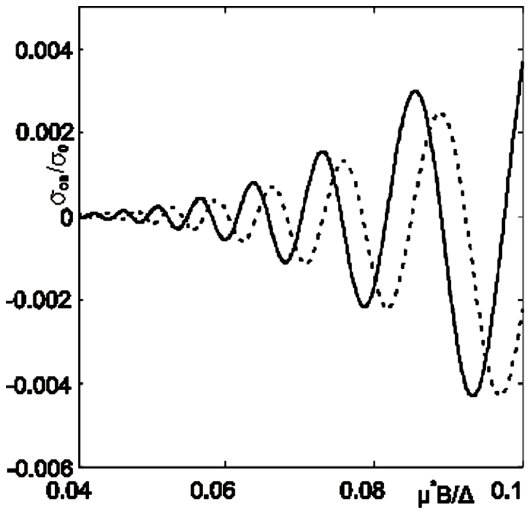

Fig. 2. Field dependences of the oscillating part of the longitudinal conductivity at $\gamma_0 = 1$ and $kT/\Delta = 0.03$ for the layered crystal (solid curve) and in the effective mass approximation (dashed curve)

dependences of the chemical potential in the electron gas are depicted in Fig. 1 in the range of magnetic fields $0 \leq \mu^* B/\Delta \leq 5$, at $kT/\Delta = 0.03$ and $\zeta_0/\Delta = 1$, and for the cases of a real layered crystal with the dispersion law (2) (solid curve) and in the effective mass approximation (dashed curve). The dependences were obtained without taking the Dingle factor into consideration. The difference of the latter from 1 can be neglected, when the condition that the scattering-induced broadening of energy levels is small in comparison with the distance between Landau levels is well satisfied within the whole examined range of magnetic fields. This condition will be discussed below in more details, when estimating the longitudinal conductivity numerically.

As the magnetic field grows, both curves approach each other, because, in the case of closed FS, Eq. (13) has the following asymptotic solution in strong quantizing magnetic fields [11]:

$$\zeta(B) = \mu^* B + \Delta \left[1 - \cos\left(\frac{f(\gamma_0)\,\Delta}{2\mu^* B}\right)\right], \tag{19}$$

where

$$f(\gamma_0) = (\gamma_0 - 1)\arccos(1 - \gamma_0) + \sqrt{2\gamma_0 - \gamma_0^2}. \tag{20}$$

Whence, it is evident that the single filled Landau subband becomes narrower in the ultraquantum limit, and this narrowing has to be taken into account in calculations of the longitudinal conductivity.

In the effective mass approximation, formula (19) reads

$$\zeta(B) = \mu^* B + \frac{\Delta^3 f^2(\gamma_0)}{8(\mu^* B)^2}. \tag{21}$$

Note that all the formulas given above for the longitudinal conductivity were also obtained in the case where the interaction-induced broadening of energy levels is small in comparison with the distance between Landau levels. Only in this case, the broadening of energy levels can be associated directly with the relaxation time, and the scattering-induced shift of energy levels can be neglected. Then the approach based on the Boltzmann equation turns out completely equivalent to that based on the Kubo formalism. However, in the first part of work [5], the opposite case was considered, namely, when the scattering-induced broadening of energy levels is large in comparison with the distance between Landau levels. However, even in this case, the formula obtained in work [4] for the invariable component of the longitudinal conductivity coincides–to within an exponentially small (at low temperatures) correction–with formula (8) derived for the case of the open FS. The field dependences of the oscillating part in the longitudinal electroconductivity are shown in Fig. 2 in the magnetic field range $0.04 \leq \mu^* B/\Delta \leq 0.1$, for $kT/\Delta = 0.03$ and $\zeta_0/\Delta = 1$, and for the cases of a real layered crystal with the dispersion law (2) (solid curve) and in the effective mass approximation (dashed curve). The figure demonstrates that, owing to some difference between oscillation fre-

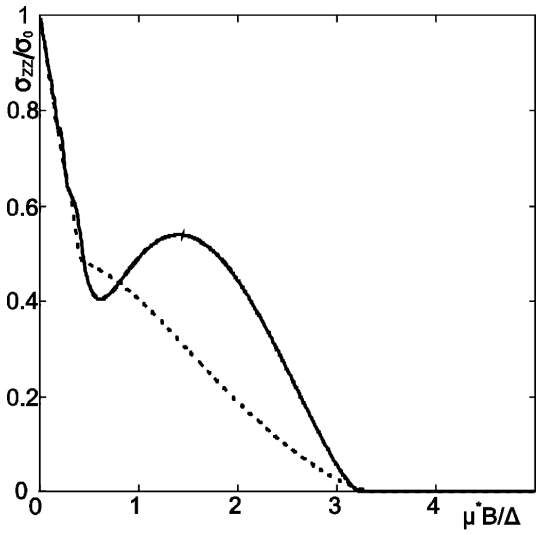

Fig. 3. Field dependences of the total longitudinal conductivity at $\gamma_0 = 1$ and $kT/\Delta = 0.03$ for a layered crystal (solid curve) and in the effective mass approximation (dashed curve)

quencies which was mentioned above, oscillations of the longitudinal electroconductivity in a layered crystal with the dispersion law (2) acquire, as the magnetic field induction grows, some phase retardation with respect to the same oscillations calculated in the effective mass approximation. Moreover, the relative contribution of oscillations in the case of the dispersion law (2) is larger than that in the effective mass approximation. This occurs because, if layered-structure effects are considered, the slope of the FS cross-section as a function of the longitudinal quasimomentum is less than that in the effective mass approximation, and, in addition, the longitudinal velocity of charge carriers is less in the former case. The analysis testifies that, if the Fermi energy is assumed constant, the considered oscillations coincide by their phase and frequency. However, the relative contribution of the oscillating part is somewhat larger, if the layered-structure effects are taken into account. In both cases, this contribution achieves 0.4% in the range of magnetic fields $0.04 \leq \mu^* B/\Delta \leq 0.1$.

Now let us calculate the total longitudinal conductivity in a magnetic field. As was already mentioned in work [11], one has to consider the narrowing of the single filled Landau subband. Therefore, the substitution $\gamma = (\zeta - \mu^* B)/\Delta$ has to be done in formulas (8) and (9) with regard for Eqs. (10)–(12). At the same time, formulas (14) and (15) derived in the effective mass approximation look like

$$\sigma_0 = \frac{32\sqrt{2}\pi^2 e^2 m^* a \tau_0 \Delta^{1/2}}{3h^4} \left(\zeta - \mu^* B\right)^{3/2}, \tag{22}$$

$$\sigma_{os}(B) = \frac{32\pi^{1/2} e^2 m^* a \tau_0 \Delta^{1/2} (\mu^* B)^{3/2}}{h^4} \times$$

$$\times \sum_{l=1}^{\infty} \frac{(-1)^l f_l^{\sigma}}{l^{3/2}} \left\{ \sin\left(\frac{\pi l \zeta}{\mu^* B}\right) C\left[\sqrt{\frac{2l(\zeta - \mu^* B)}{\mu^* B}}\right] - \right.$$

$$\left. - \cos\left(\frac{\pi l \zeta}{\mu^* B}\right) S\left[\sqrt{\frac{2l(\zeta - \mu^* B)}{\mu^* B}}\right] \right\}. \tag{23}$$

The field dependences of the total longitudinal electroconductivity in the range of magnetic fields $0 \leq \mu^* B/\Delta \leq 5$ and at $kT/\Delta = 0.03$ and $\zeta_0/\Delta = 1$ are exhibited in Fig. 3 for the cases of a real layered crystal with the dispersion law (2) (solid curve) and in the effective mass approximation (dashed curve). The figure demonstrates that, in a weak magnetic field, the total conductivity is slightly different in both considered cases, although the contribution of the oscillating term is more pronounced if the layered-structure effects are taken into account. In a very strong magnetic field, they again weakly differ from each other, because formulas (19) and (21) give practically the same result here. However, there exists a certain "optimal" range of magnetic fields $0.5 \leq \mu^* B/\Delta \leq 3$, where the layered-structure effects are the most pronounced owing to both the slower dependence of the FS cross-section area on the longitudinal quasimomentum and the smaller longitudinal velocity of charge carriers. Hence, one can see that the layered-structure effects can be highly expressed in crystals with a closed FS as well.

Consider now, for example, the Kapitsa longitudinal effect [12] which consists in the linear dependence of the longitudinal magnetoresistance on the magnetic field induction. Expanding Eq. (8) in a power series in the small parameter $\mu^* B/\Delta$ and confining the expansion to linear terms, we obtain the following formula for the relative magnetoresistance:

$$\frac{\Delta\rho(B)}{\rho(0)} = \frac{2\sin C_0}{C_0 - C_2} \frac{\mu^* B}{\Delta}. \tag{24}$$

For instance, at $\gamma = 1$ and $\mu^* B/\Delta = 0.1$, taking into account that $C_0 = \pi/2$ and $C_2 = 0$, we obtain a value of 12.7% for the relative magnetoresistance. At the same time, in the effective mass approximation, by expanding expression (22) in a Taylor series, we obtain the formula

$$\frac{\Delta\rho(B)}{\rho(0)} = 1.5\sqrt{\gamma}\frac{\mu^* B}{\Delta}. \tag{25}$$

Under the conditions given above, we obtain a value of 15% for the relative magnetoresistance, i.e. a little larger than in the case where the layered-structure effects were taken into account. From the presented results, it follows that, in the range of magnetic fields $0 \le \mu^* B/\Delta \le 0.1$, the contribution of the Kapitsa effect to the longitudinal magnetoresistance of a crystal is much larger than that of the Shubnikov–de Haas one.

Note that it is impossible to explain the Kapitsa longitudinal effect within the conventional approaches. Those approaches can explain only the transverse Kapitsa effect, and its physical reason is understood as if the role of mean free paths of charge carriers in strong magnetic fields is gradually transferred to the radii of their cyclotron orbits [13, 14].

Now let us estimate the longitudinal electroconductivity of the crystal in the absence of a magnetic field, because it is a reference point for the characteristics depicted in Figs. 2 and 3. Consider the region of the charge carrier scattering at charged impurities. Since the relaxation time is assumed to be constant, it can be determined by the formula

$$\tau_0 = \frac{2\pi l m^*_{es}}{h k_0}. \tag{26}$$

In this expression, l is the mean free path of charge carriers; k_0 the equivalent radius of a Fermi sphere that simulates the real FS in the layered crystal to consider the charge carrier scattering as isotropic; and m^*_{es} the effective mass of charge carriers on that sphere. The formulas for those quantities were obtained using the conditions that the Fermi energy and the concentration of charge carriers are the same as in a real crystal. At $\gamma = 1$, these are

$$k_0 = \sqrt[3]{\frac{12\pi^2 m^* \Delta}{a h^2}}, \tag{27}$$

$$m^*_{es} = \frac{h^2 k_0^2}{8\pi^2 \Delta}. \tag{28}$$

Substituting Eqs. (27) and (28) into Eq. (8), taking into account that $\gamma = 1$, and introducing the notation $N = l/a$, we obtain the following final formula for the longitudinal conductivity of a layered crystal in the absence of a magnetic field:

$$\sigma_0 = \frac{2\pi e^2 m^* a^2 \Delta}{h^3} \sqrt[3]{\frac{12\pi^2 m^* \Delta}{a h^2}} N. \tag{29}$$

The corresponding analysis showed that, for the parameters $a = 1$ nm, $m^* = 0.01 m_0$, and $\Delta = 0.01$ eV, the Dingle factor does not substantially influence the chemical potential oscillations and the oscillating part of longitudinal conductivity, if $N \ge 1000$. Hence, in accordance with formula (29), the minimum value of longitudinal conductivity in the layered crystal under given conditions can amount to $1.06 \times 10^3\ \Omega^{-1}\mathrm{m}^{-1}$. In the effective mass approximation, for the same relaxation time and charge carrier concentration, this conductivity is about 20% higher and amounts to $1.272 \times 10^3\ \Omega^{-1}\mathrm{m}^{-1}$. This result has a simple physical interpretation: any restriction imposed on the free motion of charge carriers in the direction perpendicular to the layers reduces the conductivity of the crystal in this direction.

At last, let us obtain an explicit asymptotic expression for the electroconductivity in a layered crystal in strong ultraquantum magnetic fields. For this purpose, let us substitute the quantities τ_0 from Eq. (26) and $\zeta(B)$ from Eq. (21) into formula (5) immediately. Then, the multiplier $(-1)^l$ under the sum over l is compensated, the cosine can be substituted by 1, and the integration over x can be executed within the limits from 0 to $\frac{f(\gamma_0)\Delta}{2\mu^* B}$. When integrating, the square of the longitudinal velocity of charge carriers in the integrand has to be replaced by its principal term of the expansion in x, i.e. the quadratic one. In addition, the numerical analysis demonstrates that, at $\mu^* B/kT \gg 1$,

$$\sum_{l=1}^{\infty} f_l^{\sigma} = \frac{2.467\mu^* B}{\pi^2 kT}. \tag{30}$$

Executing all the quoted transformations and combining all numerical multipliers into a single one, we obtain the following final asymptotic expression for the longitudinal conductivity:

$$\sigma_{zz}(B) = 1.285 \frac{e^2 m^* a^2 \Delta^4 f^3(\gamma_0)}{h^3 (\mu^* B)^2 kT} \sqrt[3]{\frac{m^* \Delta}{a h^2}} N. \tag{31}$$

However, it is valid only for very strong magnetic fields. For instance, for the parameters given above and $kT/\Delta = 0.03$, the longitudinal conductivity of the crystal in a magnetic field of 60 T amounts to $0.675\ \Omega^{-1}\mathrm{m}^{-1}$, i.e. its resistance becomes about 1600 times as high as the resistance in the absence of a magnetic field. Hence, we obtained that, in ultraquantum magnetic fields, the magnetoresistivity obeys the law $\rho_{zz} \propto TB^2$. This law includes the dependence on both the field and the temperature and can be used for the experimental verification of whether the approximations made in this paper and the corresponding model calculations were correct.

However, this asymptotic law has a substantial shortcoming from the formal point of view. According to the

law, the total longitudinal resistance of the crystal tends to zero as $T \to 0$ in strong magnetic fields. It is difficult to explain such a behavior from physical reasons, if we are in the region of the charge carrier scattering by charged impurities (although this law has no physically unreasonable consequences at real temperatures). Therefore, the question arises: Does another asymptotic law free of this shortcoming exist?

Before answering this question, we note that the same problem on the longitudinal conductivity in layered crystals was considered in work [5]. However, it was done only for crystals with highly open Fermi surfaces, i.e. for which $\zeta_0/\Delta \gg 1$. Moreover, the consideration was carried out in the approximation $\zeta_0/(\mu^* B) \gg 1$. The result obtained in work [5] for the constant component of the conductivity at low temperatures, when $\zeta_0/(kT) \gg 1$, practically coincides with our one presented in work [11] for the case of a constant relaxation time. However, the dependence of the oscillating part of the longitudinal conductivity on the magnetic field is slightly different. If we abstract from the Dingle factor which was also taken into account in work [5], this difference is mainly associated in the case of weak magnetic fields with the fact that the oscillating part of the conductivity was considered under conditions slightly different from those analyzed in work [11] and this paper, namely, $\Delta/(\mu^* B) \gg 1$ and $\omega_c \tau_0 \ll 1$. In this case, the Landau subbands are smeared very much due to the interaction between charge carriers and a random potential created by charged impurities. Under those conditions, the oscillating part of the longitudinal conductivity cannot longer be expressed in terms of the squared longitudinal velocity of charge carriers, as was done in work [11] and is done in this paper.

Moreover, the authors of work [5] considered only such magnetic fields, in which the FS remains open even in the case $\Delta/(\mu^* B) \ll 1$ and $\omega_c \tau_0 \gg 1$. Hence, the technique developed in that work demands modifications for the case of closed surfaces with regard for a "squeezing" of the FS under the action of a strong ultraquantum magnetic field.

In this work, we do not make such a modification in the case of a quasiclassical magnetic field, because the majority of experiments dealing with galvano-magnetic phenomena in layered crystals are carried out under the conditions $\Delta/(\mu^* B) \gg 1$ and $\omega_c \tau_0 \gg 1$ [15–17], when the approach presented in work [11] and this one is applicable, although the Dingle factor is taken into consideration at the treatment of experimental data, because it allows the relaxation time of charge carriers to be determined directly, at least on the extreme FS cross-sections. However, let us make the specified modification in the case of strong ultraquantum magnetic fields. In work [5], it was shown that, if charge carriers are scattered at the random potential of charged impurities, the longitudinal electroconductivity of the crystal can be considered as a diffusion one in the case where $\Delta/(\mu^* B) \ll 1$, $\omega_c \tau_0 \gg 1$, $\zeta_0/\Delta \gg 1$, $\zeta_0/(\mu^* B) \gg 1$ as well. Here, ω_c is the cyclotron frequency. Longitudinal conductivity can be determined at $T = 0$ by the formula

$$\sigma_{zz} = \frac{8\pi^3 e^2 m^* a \tau_0 \Delta^2}{h^4} \left(1 - \cos \frac{\pi \zeta_0}{\mu^* B}\right). \tag{32}$$

This formula differs from that presented in work [11] for the constant component of the electroconductivity for the open FS in that the constant relaxation time τ_0 is substituted by the time depending on the magnetic field, and this dependence is completely determined by the expression in parentheses. However, the authors of work [5] recognized that formula (32) is inapplicable, when, due to a high FS openness, the Landau level intersects the Fermi one, because the longitudinal electroconductivity vanishes at such magnetic fields, the situation being unphysical. However, the condition of strong FS openness is not obligatory for a formula of the type (32) to be correct, because, as was indicated in paper [5], formula (32) was obtained by solving the kinetic Boltzmann equation for *every* Landau subband, totally or partially filled. Therefore, for this formula to be valid, it does not matter how many Landau subbands are filled. Hence, formula (32) will be generalized to the case of ultraquantum fields and a closed FS in two ways: considering the single Landau subband with the number $n = 0$ as 1) completely or 2) partially filled. In both those cases, the Fermi energy is not assumed constant, but it is dependent on the magnetic field according to formulas (19) and (21).

Following the first way, the quantity Δ in formula (32) is substituted by the quantity $\frac{\Delta^3 f^2(\gamma_0)}{16(\mu^* B)^2}$, and the quantity ζ_0 by quantity (21). Then, taking formulas (26)–(28) into account, we obtain the following final expression for the longitudinal conductivity:

$$\sigma_{zz} = \frac{\pi^2 e^2 m^* a^2 \Delta^5 f^4(\gamma_0)}{64 h^3 (\mu^* B)^4} \sqrt[3]{\frac{12\pi^2 m^* \Delta}{a h^2}} N. \tag{33}$$

For the parameters given above and a magnetic field induction of 60 T, the crystal conductivity amounts to $2.124 \times 10^{-5}\ \Omega^{-1}\mathrm{m}^{-1}$. Thus, if the single Landau subband is considered as completely filled, the asymptotic law $\rho_{zz} \propto B^4$ is obtained.

Following the second way, let us use the general formula (4), in which the relaxation time is determined identically to how it was done in the first way, and the integration over x, in accordance with Eq. (19), is carried out from 0 to $\frac{f(\gamma_0)\Delta}{2\mu^* B}$, with the integrand being restricted to the square-law approximation with respect to x. Then, the longitudinal conductivity of the crystal looks like

$$\sigma_{zz} = \frac{\pi e^2 m^* a^2 \Delta^4 f^3(\gamma_0)}{3h^3 (\mu^* B)^3} \sqrt[3]{\frac{12\pi^2 m^* \Delta}{ah^2}} N. \qquad (34)$$

For the above-given parameters of the problem and a magnetic field induction of 60 T, the crystal conductivity amounts to 5.018×10^{-3} $\Omega^{-1}\mathrm{m}^{-1}$. Thus, if the single Landau subband is considered as partially filled, the asymptotic law $\rho_{zz} \propto B^3$ is obtained. Hence, in a magnetic field of 60 T, the crystal electroconductivity becomes almost eight, in the first case, and almost six, in the second case, orders of magnitude as low as that in the absence of a magnetic field. It is clear that the results obtained are to be experimentally verified. However, the overwhelming majority of available experiments for galvano-magnetic phenomena in crystals with a superlattice were carried out for highly open FSes (see, e.g., work [17] and the relevant references therein).

3. Conclusions

Hence, we proved that the layered-structure effects are essential not only for open, but also for closed FSes. At a constant concentration of charge carriers, those effects are essential even in the range where the quasiclassical approximation is valid. In addition, there exists an optimal interval of the magnetic field induction, where those effects are most brightly pronounced. In the range where the quasiclassical approximation is valid, they manifest themselves as an increase of the relative contribution of Shubnikov–de Haas oscillations to the total conductivity and as some phase retardation of the oscillations. In this case, the field dependence of the chemical potential almost does not influence the oscillation manifestation character. In the optimal interval of magnetic fields, the field dependence of the chemical potential considerably affects the character of manifestations of the layered-structure effect. Concerning the Kapitsa effect, the inverse situation takes place: the layered-structure effects diminish the coefficient of proportionality between the magnetoresistance and the magnetic field induction. Moreover, the longitudinal Kapitsa effect, provided the charge carrier scattering at impurities, can be explained only if the FS squeezing under the action of a magnetic field, i.e. the field dependence of the chemical potential, is taken into account. Hence, crystals with closed FSes and high degrees of the filling of the miniband should also be regarded as layered ones. In addition, at rigorous calculations of the longitudinal electroconductivity, the FS extension in the magnetic field direction and the dependence of this extension on the magnetic field induction have to be taken into consideration. In strong magnetic fields, the following asymptotic laws can be obtained, depending on the intensity of the charge carrier scattering by charged impurities and the way of how the filling of the single Landau subband is simulated: $\rho_{zz} \propto TB^2$, if the conductivity is considered as a drift one; $\rho_{zz} \propto B^3$, if the conductivity is considered as a diffusion one and the single Landau subband is partially filled; and $\rho_{zz} \propto B^4$, if the conductivity is considered as a diffusion one and the single Landau subband is filled completely. The field dependence of the chemical potential is also essential for the explanation of those dependences.

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ЧИ ВИРАЖЕНІ ЕФЕКТИ ШАРУВАТОСТІ ПРИ ЗАМКНЕНИХ ПОВЕРХНЯХ ФЕРМІ?

П.В. Горський

Р е з ю м е

У статті на прикладі поздовжньої електропровідності у квантуючому магнітному полі, перпендикулярному до шарів, показано, що ефекти шаруватості можуть бути виражені не лише у кристалах із сильно відкритими поверхнями Фермі (ПФ), як це традиційно вважається, але й у кристалах із замкненими ПФ. Розрахунки проведено в наближенні сталого часу релаксації. У слабких магнітних полях ефекти шаруватості виражаються у відставанні осциляцій Шубнікова–де-Гааза (ШДГ) за фазою і у деякому збільшенні їх відносного внеску. В області сильних магнітних полів існує оптимальний діапазон, в якому ефекти шаруватості виявляються у різко немонотонній залежності поздовжньої електропровідності від магнітного поля. Крім того, показано, що ефекти шаруватості ведуть до зниження коефіцієнта пропорційності між магнітоопором і індукцією у поздовжньому ефекті Капіци. Розглянуто також поздовжню електропровідність шаруватих кристалів в ультраквантових магнітних полях і показано, що залежно від того, як моделюється заповнення єдиної підзони Ландау і від того, розглядається поздовжня провідність як дрейфова, чи як дифузійна, можна отримати такі закони зміни магнітоопору з магнітним полем: $\rho_{zz} \propto TB^2$, $\rho_{zz} \propto B^3$ та $\rho_{zz} \propto B^4$.