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# ELECTRON CROSS SCATTERING IN DISORDERED SYSTEMS

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Using the variation principle, the expression for the electrical resistivity of simple disordered metals and a fully ionized plasma in any order of perturbation theory in the electron-ion interaction is obtained. We consider the ion subsystem as static and temperature corrections for metals as insignificant. Decoupling parameters of high-order Green's functions, which are solutions of a quantum kinetic equation, are chosen from the condition of the full coincidence of the Boltzmann's equation and the quantum kinetic one in a low order of perturbation theory. In the second and third orders of perturbation theory, the earlier known results are obtained. For the first time, we have calculated the fourth-order effect which is related to the simultaneous scattering of electrons in a fully ionized plasma by an external electric field and the field of ions. But this effect is absent in the disordered metals in the limit of low temperatures.

which influences numerical values of the kinetic coefficients is high-order terms of perturbation theory. Many attempts to construct the series of perturbation theory in the electron-ion interaction were realized in the relaxation time approximation [7–15]. In other words, the perturbation theory was built not for the corresponding kinetic coefficient, but for the inverse time of relaxation which characterizes the given process of transfer. At the same time, the relaxation time approximation is true, as will be shown in the present work, only in the frame of perturbation theory which takes the terms of at most the third order into account. The correctness of this approximation in the fourth order remained unproved. But even in the third order of perturbation theory, the above-cited works were not successful, and the interest in this problem decreased significantly. The essential progress in the construction of perturbation theory was attained in the subsequent works [16–21]. There in the relaxation time approximation, the third-order term of perturbation theory was obtained and numerically calculated practically for all simple metals. Then the question arises: Which is the next step? The output beyond the relaxation time approximation requires the accurate consideration of fourth-order effects. We note that the fourth-order effects can be significant even for simple metals. They are of the order of  $\hbar/\varepsilon_F\tau$ , where  $\varepsilon_F$  is the Fermi energy, and  $\tau$  is the relaxation time. This dimensionless parameter is equal to several tenths for many simple metals. That is, the corresponding contribution to a kinetic coefficient can be of the order of tens of per cent. If these fourth-order effects are significant, then the Wiedemann–Franz law and the Hall effect would be satisfied only approximately for simple disordered metals. The corrections to the Lorentz and Hall constants would be of the order of the mentioned dimensionless parameter, i.e. of the second order in the pseudopotential. We note that no such corrections are present in the relaxation time approximation. Especially actual is this question about the necessity to sum the whole series of perturbation theory. Such a necessity appears, for exam-

## 1. Introduction

Simple metals and the majority of transition metals belong to systems possessing a small parameter. In many cases, a fully ionized plasma can be considered as a system with small parameter. This parameter is the ratio of the form-factor of the electron-ion interaction in a neighborhood of the Fermi level and the kinetic energy of a conduction electron at this level  $w(q)/\varepsilon_F$ . We note that the electron-electron interaction is not weak, as a rule. As a result, all characteristics of such systems can be represented by series of perturbation theory in this parameter. Each separate case is characterized by the particular series of perturbation theory, whose construction is a complicated task not solved generally up to now. For the resistance of simple liquid metals, the first term squared in the electron-ion interaction was obtained in [1]. It is the so-called Ziman formula. In the next decades, this formula induced a number of works related to its refinement and the extending of conditions of its validity (see references in [2, 3]). For example, the dynamics of the ion subsystem was considered in [4] in the second order of perturbation theory, the electron-electron interaction was explicitly taken into account in [5], and both factors were considered in [6]. However, the most crucial factor

ple, in calculations of the resistance of metallic hydrogen near the metal–dielectric transition [22–24].

One of the absolutely noninvestigated fourth-order effects is that caused by the cross scattering of conduction electrons in an external electric field and the field of ions. This work is devoted to its study. Moreover, we will consider not only the quantum case characteristic of disordered metals, but also the classical one characteristic of a fully ionized plasma. We will show also that the structure of the coefficient of resistance becomes more complicated if the terms of the fourth and higher orders are taken into account. The relaxation time stops to be a single structure-dependent quantity which determines this coefficient. The second structure-dependent quantity becomes the density of states of conduction electrons. The third one is the contribution due to the cross scattering. We will also show that, starting from the fourth-order terms, the introduction of the relaxation time does become an approximation. We note that the algorithm proposed by us to calculate the coefficient of resistance does not require one to introduce this approximation. We will use this term only in the traditional interpretation of the obtained results.

## 2. Coefficient of Resistance

We write the Hamiltonian of the electron subsystem of a simple metal or a fully ionized plasma within the diffraction model of a metal, where the electron–electron interaction is taken into account by the screening of the electron–ion interaction as [3]

$$H(t) = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} a_{\mathbf{k}}^{\dagger}(t) a_{\mathbf{k}}(t) + V^{-1} \sum_{\mathbf{k}, \mathbf{q}} w(\mathbf{q}) \rho(\mathbf{q}) a_{\mathbf{k}}^{\dagger}(t) a_{\mathbf{k}+\mathbf{q}}(t). \quad (1)$$

The diffraction model of a metal is quite satisfactory for simple metals and requires a significant correction only in the case of transition or rare-earth metals where several electron subsystems are present. But even in the last case, the electron–electron interaction is significant only in the crystalline state and only at low temperatures where its characteristic contribution to the electric resistance which is proportional to the square of the temperature can be separated. In formula (1),  $a_{\mathbf{k}}^{\dagger}$ ,  $a_{\mathbf{k}}$  are the operators of creation and annihilation of electrons in the state with a wave vector  $\mathbf{k}$ ,  $\varepsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m$  is the energy of a free electron gas,  $w(\mathbf{q})$  is the form-factor of

a local model pseudopotential of the electron–ion interaction,  $\rho(\mathbf{q})$  is the Fourier-transform of the density of ions, and  $V$  is the system volume. The ion subsystem is considered static, which is completely suitable for disordered metals and a plasma [3].

According to the Kubo theory of a linear response, the coefficient of resistance in a steady homogeneous electric field for an isotropic metal or plasma has the form [3]

$$\sigma = -\frac{\hbar}{3V k_B T} \text{Im} \langle \langle \mathbf{I}(t) \mathbf{I}(t') \rangle \rangle_0. \quad (2)$$

Here,

$$\mathbf{I}(t) = \frac{e\hbar}{m} \sum_{\mathbf{k}} \mathbf{k} a_{\mathbf{k}}^{\dagger}(t) a_{\mathbf{k}}(t), \quad (3)$$

is the electron current operator, the double broken brackets mean the Green's two-time retarded function, and the lower index stands for the Fourier zeroth component.

The Fourier-transform of the Green's function which determines the coefficient of resistance will be searched by the motion equation method [25]. The relevant equation has the form

$$\begin{aligned} \hbar\omega \langle \langle a_{\mathbf{k}}^{\dagger}(t) a_{\mathbf{k}}(t) \mathbf{I}(0) \rangle \rangle_{\omega} &= \langle a_{\mathbf{k}}^{\dagger}(0) a_{\mathbf{k}}(0) \mathbf{I}(0) \rangle + \\ &+ V^{-1} \sum_{\mathbf{q}} \{ w(\mathbf{q}) \langle \langle a_{\mathbf{k}}^{\dagger}(t) \rho(-\mathbf{q}, t) a_{\mathbf{k}+\mathbf{q}}(t) \mathbf{I}(0) \rangle \rangle_{\omega} - \\ &- \langle \langle a_{\mathbf{k}+\mathbf{q}}^{\dagger}(t) \rho(\mathbf{q}, t) a_{\mathbf{k}}(t) \mathbf{I}(0) \rangle \rangle_{\omega} \}. \end{aligned} \quad (4)$$

For the Green's functions which are present on the right-hand side of this equation, the equation of motion is as follows:

$$\begin{aligned} (\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}} + \hbar\omega) \langle \langle a_{\mathbf{k}}^{\dagger}(t) \rho^i(-\mathbf{q}) a_{\mathbf{k}+\mathbf{q}}(t) \mathbf{I}(0) \rangle \rangle_{\omega} &= \\ = \langle a_{\mathbf{k}}^{\dagger}(0) \rho^i(-\mathbf{q}) a_{\mathbf{k}+\mathbf{q}}(0) \mathbf{I}(0) \rangle + \\ &+ V^{-1} w(\mathbf{q}) \{ \langle \langle a_{\mathbf{k}}^{\dagger}(t) \rho(-\mathbf{q}) \rho(\mathbf{q}) a_{\mathbf{k}}(t) \mathbf{I}(0) \rangle \rangle_{\omega} - \\ &- \langle \langle a_{\mathbf{k}+\mathbf{q}}^{\dagger}(t) \rho(\mathbf{q}) \rho(-\mathbf{q}) a_{\mathbf{k}+\mathbf{q}}(t) \mathbf{I}(0) \rangle \rangle_{\omega} \} + V^{-1} \times \\ &\times \sum_{\mathbf{q}'} \{ w(\mathbf{q}') \langle \langle a_{\mathbf{k}}^{\dagger}(t) \rho(-\mathbf{q}) \rho(-\mathbf{q}') a_{\mathbf{k}+\mathbf{q}+\mathbf{q}'}(t) \mathbf{I}(0) \rangle \rangle_{\omega} - \end{aligned}$$

$$-\langle\langle a_{\mathbf{k}+\mathbf{q}}^+(t)\rho(\mathbf{q})\rho(-\mathbf{q}')a_{\mathbf{k}+\mathbf{q}}(t)\mathbf{I}(0)\rangle\rangle_\omega\}. \quad (5)$$

The right-hand sides of such equations include Green's functions of two types, whose structures are determined, respectively, by weakly and strongly coupled means. For Green's functions of the second type, it is necessary again to construct the equation of motion, by continuing the chain of the equations of motion up to infinity. Such infinite chain of the equations of motion contains only Green's functions of the first type. To truncate this chain of equations for the required Green's function, we should carry out a decoupling of the type

$$\begin{aligned} &\langle\langle a_{\mathbf{k}}^+(t)\rho(-\mathbf{q},t)\rho(\mathbf{q})a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_\omega = \\ &= f(\mathbf{k}, \mathbf{k} + \mathbf{q}) \langle\rho(-\mathbf{q})\rho(\mathbf{q})\rangle \langle\langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_\omega. \end{aligned} \quad (6)$$

The function  $f(\mathbf{k}, \mathbf{k} + \mathbf{q})$  should be chosen in view of the condition of certain significant properties of the symmetry of the whole equation. We will specify this choice in what follows. Gathering the equations of a chain in a single one, we obtain the quantum kinetic equation

$$\begin{aligned} R(\mathbf{k}) + \sum_{\mathbf{k}'} T \{ \langle\langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_0 - \\ - \langle\langle a_{\mathbf{k}'}^+(t)a_{\mathbf{k}'}(t)\mathbf{I}(0)\rangle\rangle_0 \} = 0. \end{aligned} \quad (7)$$

The free term

$$\begin{aligned} R(\mathbf{k}) = \langle a_{\mathbf{k}}^+(0)a_{\mathbf{k}}(0)\mathbf{I}(0)\rangle + V^{-1} \sum_{\mathbf{k}'} w(|\mathbf{k}' - \mathbf{k}|) \times \\ \times \left[ \frac{\langle a_{\mathbf{k}}^+(0)\rho(\mathbf{k}' - \mathbf{k})a_{\mathbf{k}'}(0)\mathbf{I}(0)\rangle}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} + i\delta} - \right. \\ \left. - \frac{\langle a_{\mathbf{k}'}^+(0)\rho(\mathbf{k} - \mathbf{k}')a_{\mathbf{k}}(0)\mathbf{I}(0)\rangle}{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + i\delta} \right] + \dots \end{aligned} \quad (8)$$

describes both the scattering of electrons by an external electric field in the zero order in the pseudopotential and the cross scattering by an external field and the field of ions with consideration of terms of higher orders in the pseudopotential. In terms of the Boltzmann's equation, these terms of the quantum kinetic equation should be named field and cross terms, respectively [26]. The integral terms of the equation

$$T \{ \langle\langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_0 - \langle\langle a_{\mathbf{k}'}^+(t)a_{\mathbf{k}'}(t)\mathbf{I}(0)\rangle\rangle_0 \} =$$

$$\begin{aligned} &= \frac{N}{V} w^2(|\mathbf{k}' - \mathbf{k}|) S(|\mathbf{k}' - \mathbf{k}|) \times \\ &\times \left\{ \frac{f(\mathbf{k}, \mathbf{k}') \langle\langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_0}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} + i\delta} - \right. \\ &\left. - \frac{f(\mathbf{k}', \mathbf{k}) \langle\langle a_{\mathbf{k}'}^+(t)a_{\mathbf{k}'}(t)\mathbf{I}(0)\rangle\rangle_0}{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + i\delta} \right\} + \dots \end{aligned} \quad (9)$$

describe the scattering of conduction electrons by only the field of ions, i.e. the collision integral in terms of the Boltzmann's equation. The dots stand for the terms of the same structure, but of higher orders in the pseudopotential. In addition, we introduced the two-particle structural factor of the ion subsystem

$$S(q) = N^{-1} \langle\rho(-\mathbf{q})\rho(\mathbf{q})\rangle, \quad \mathbf{q} \neq 0. \quad (10)$$

For the further analysis, we require a quantum kinetic equation in the least order of perturbation theory. Then the free term of the equation should be taken in the zero order in the pseudopotential. We have

$$\langle a_{\mathbf{k}}^+(0)a_{\mathbf{k}}(0)\mathbf{I}\rangle_0 = \frac{e\hbar}{m} \sum_{\mathbf{k}'} \mathbf{k}' \langle a_{\mathbf{k}}^+(0)a_{\mathbf{k}}(0)a_{\mathbf{k}'}^+(0)a_{\mathbf{k}'}(0)\rangle_0. \quad (11)$$

By the Wick–Bloch–de Dominicis theorem for noninteracting electrons [27],

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle_0 = \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle_0 \langle a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle_0 + \langle a_{\mathbf{k}}^+ a_{\mathbf{k}'} \rangle_0 \langle a_{\mathbf{k}} a_{\mathbf{k}'}^+ \rangle_0, \quad (12)$$

where

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}'} \rangle_0 = n_0(k) \Delta(\mathbf{k} - \mathbf{k}'), \quad (13)$$

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}'}^+ \rangle_0 = [1 - n_0(k)] \Delta(\mathbf{k} - \mathbf{k}'). \quad (14)$$

Here,  $n_0(k)$  is the Fermi–Dirac distribution function. Thus, the quantum kinetic equation in the least order of perturbation theory has the form

$$\begin{aligned} &\frac{e\hbar}{m} \mathbf{k} n_0(k) [1 - n_0(k)] = \\ &= i \frac{2\pi N}{V^2} \sum_{\mathbf{k}'} \frac{w^2(|\mathbf{k} - \mathbf{k}'|)}{\varepsilon^2(|\mathbf{k} - \mathbf{k}'|)} S(|\mathbf{k} - \mathbf{k}'|) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}) \times \\ &\times \{ f(k, k') \langle\langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_0 - \\ &- f(k, k') \langle\langle a_{\mathbf{k}'}^+(t)a_{\mathbf{k}'}(t)\mathbf{I}(0)\rangle\rangle_0 \}, \end{aligned} \quad (15)$$

where we used the Sokhotskii formula

$$\lim_{\delta \rightarrow 0} \frac{1}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} + i\delta} = P \frac{1}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}} - i\pi \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}). \quad (16)$$

### 3. Boltzmann's Equation

The Boltzmann's kinetic equation has revealed its high efficiency in the description of electron-transfer phenomena in metals and a plasma in the first nonzero order of perturbation theory in the pseudopotential. It was obtained on the basis of a clear physical reasoning and does not involve noncontrolled approximations. The quantum kinetic equation obtained by us contains just such an approximation which is the decoupling of Green's functions of higher orders. To minimize the uncertainty which arises at the decoupling, we can compare the quantum kinetic equation in the lower order of perturbation theory and the Boltzmann's kinetic equation. In the case of a steady electric field and without both a temperature gradient and a magnetic field, the Boltzmann's equation can be written in the form [28]

$$-\frac{e}{\hbar} \frac{\partial n(\mathbf{k})}{\partial \mathbf{k}} \mathbf{E} = \sum_{\mathbf{k}'} [P(\mathbf{k}, \mathbf{k}') - P(\mathbf{k}', \mathbf{k})]. \quad (17)$$

Here,  $n(\mathbf{k})$  is the distribution function for conduction electrons in the presence of an external electric field and the field of ions, and the probability of a transition of an electron as a result of the scattering by ions from the state with wave vector  $\mathbf{k}$  in a state with wave vector  $\mathbf{k}'$ ,

$$P(\mathbf{k}, \mathbf{k}') = n(\mathbf{k})[1 - n(\mathbf{k}')]L(\mathbf{k}, \mathbf{k}') \quad (18)$$

is the product of the probability of that the state with  $\mathbf{k}$  is occupied,  $n(\mathbf{k})$ , the probability of that the state with  $\mathbf{k}'$  is free,  $1 - n(\mathbf{k}')$ , and the transition probability  $L(\mathbf{k}, \mathbf{k}')$ . Without external influences on electrons, the probabilities of direct and reverse transitions must coincide (the detailed balancing principle), i.e.,

$$P_0(\mathbf{k}, \mathbf{k}') = P_0(\mathbf{k}', \mathbf{k}) \quad (19)$$

or

$$n_0(\mathbf{k})[1 - n_0(\mathbf{k}')]L(\mathbf{k}, \mathbf{k}') = n_0(\mathbf{k}')[1 - n_0(\mathbf{k})]L(\mathbf{k}', \mathbf{k}). \quad (20)$$

In order that the Boltzmann's equation satisfy automatically this requirement, it must have the following structure:

$$\begin{aligned} &-\frac{e}{\hbar} \frac{\partial n(\mathbf{k})}{\partial \mathbf{k}} \mathbf{E} = \\ &= \sum_{\mathbf{k}'} \left[ \frac{n(\mathbf{k}')[1 - n(\mathbf{k})]}{n_0(\mathbf{k}')[1 - n_0(\mathbf{k})]} - \frac{n(\mathbf{k})[1 - n(\mathbf{k}')] }{n_0(\mathbf{k})[1 - n_0(\mathbf{k}')] } \right] \times \end{aligned}$$

$$\times n_0(\mathbf{k}')[1 - n_0(\mathbf{k})]L(\mathbf{k}', \mathbf{k}). \quad (21)$$

For metals, the function  $n(\mathbf{r}, \mathbf{k})$  is slightly different from the equilibrium one which is the Fermi-Dirac function. In this case, the kinetic equation is linearized by deviations of the function  $n(\mathbf{r})$  from its equilibrium values. It is convenient to use the representation

$$n(\mathbf{k}) = n_0(k) - \Phi(\mathbf{k}) \frac{dn_0(k)}{d\varepsilon_k}. \quad (22)$$

Here,  $\Phi(\mathbf{k})$  is a new unknown function. Substituting relation (22) in the Boltzmann's equation and linearizing it, we obtain

$$\begin{aligned} &\frac{e\hbar}{mk_{\text{B}}T} \mathbf{k}n_0(k)[1 - n_0(k)]\mathbf{E} = \frac{1}{k_{\text{B}}T} \times \\ &\times \sum_{\mathbf{k}'} [\Phi(\mathbf{k}) - \Phi(\mathbf{k}')]n_0(k')[1 - n_0(k)]L(\mathbf{k}', \mathbf{k}), \end{aligned} \quad (23)$$

where we took into account that

$$\frac{dn_0(k)}{d\varepsilon_k} = \frac{n_0(k)[1 - n_0(k)]}{k_{\text{B}}T}. \quad (24)$$

Just this form of the Boltzmann's equation is convenient for the comparison with the quantum kinetic equation. To this end, it is necessary to write the Green's functions in the quantum kinetic equation in terms of the function  $\Phi(\mathbf{k})$  in the Boltzmann's equation. In the simplest way, it can be realized by comparing the formulas for the electric current involving both functions. By averaging the electric current operator over the Gibbs canonical with the initial Hamiltonian of the system, we obtain

$$I = \frac{e\hbar}{m} \sum_{\mathbf{k}} \mathbf{k}n(\mathbf{k}). \quad (25)$$

Substituting the distribution function for conduction electrons and taking their interaction with an external field and the field of ions, we get

$$I = \frac{e\hbar}{mk_{\text{B}}T} \sum_{\mathbf{k}} \mathbf{k}n_0(k)[1 - n_0(k)]\Phi(\mathbf{k}). \quad (26)$$

On the other hand, the electric current can be written in terms of the required Green's function with the use of the Kubo formula for the coefficient of resistance as

$$\begin{aligned} I &= \sigma E = \\ &= -\frac{\hbar}{3} \frac{e\hbar}{mk_{\text{B}}TV} \sum_{\mathbf{k}} \mathbf{k} \text{Im} \langle \langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0) \rangle \rangle_0 E. \end{aligned} \quad (27)$$

Hence, the sought connection is as follows:

$$-i\frac{\hbar}{3V}\langle\langle a_{\mathbf{k}}^+(t)a_{\mathbf{k}}(t)\mathbf{I}(0)\rangle\rangle_0 E = n_0(k)[1 - n_0(k)]\Phi(\mathbf{k}). \quad (28)$$

We now can write the quantum kinetic equation in the same form as the Boltzmann's equation:

$$\begin{aligned} \frac{e\hbar}{mk_{\text{B}}T}\mathbf{k}n_0(k)[1 - n_0(k)]E &= \frac{1}{k_{\text{B}}T} \times \\ &\times \sum_{\mathbf{k}'} \{f(k, k')n_0(k)[1 - n_0(k)]\Phi(\mathbf{k})L(\mathbf{k}, \mathbf{k}') - \\ &- f(k', k)n_0(k')[1 - n_0(k')]\Phi(\mathbf{k}')L(\mathbf{k}', \mathbf{k})\}. \end{aligned} \quad (29)$$

where the probability of the transition between the states

$$L(\mathbf{k}', \mathbf{k}) = 3\frac{2\pi N}{V\hbar} \frac{w^2(|\mathbf{k} - \mathbf{k}'|)}{\varepsilon^2(|\mathbf{k} - \mathbf{k}'|)} S(|\mathbf{k} - \mathbf{k}'|)\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}). \quad (30)$$

The fitting functions which appear at the decoupling of high-order Green's functions can be determined from the condition of the coincidence of the quantum kinetic equation and the Boltzmann's equation in the lowest order of perturbation theory. Obviously, we must take

$$f(k, k') = \frac{n_0(k')[1 - n_0(k)]}{n_0(k)[1 - n_0(k)]}. \quad (31)$$

These parameters characterize the asymmetry of states, in which the conduction is realized, and states, which are left by electrons which transfer the electric charge. In the case of elastic scattering where  $k = k'$ ,  $f(k, k') = f(k', k) = 1$ . Just the similarity of the structures of the Boltzmann's kinetic equation and the quantum kinetic equation allows us to interpret each term of the latter in terms of the former.

#### 4. Variation Principle

Together with the quantum kinetic equation, we may consider the functional, the variation of which gives the equation analogously to the Boltzmann's equation [28],

$$(\Phi, \hat{L}\Phi) = (\Phi, X), \quad (32)$$

where

$$(\Phi, X) = \sum_{\mathbf{k}} \Phi(\mathbf{k})R(\mathbf{k})E, \quad (33)$$

$$(\Phi, \hat{L}\Phi) = -\frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}'} [\Phi(\mathbf{k}) - \Phi(\mathbf{k}')] T [\Phi(\mathbf{k}) - \Phi(\mathbf{k}')]. \quad (34)$$

If we restrict ourselves by the lowest order of perturbation theory, this equation has the form

$$\begin{aligned} \frac{e\hbar}{m} \sum_{\mathbf{k}} \Phi(\mathbf{k})\mathbf{k}n_0(\mathbf{k})[1 - n_0(\mathbf{k})]E &= \\ &= \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}'} [\Phi(\mathbf{k}) - \Phi(\mathbf{k}')]^2 n_0(\mathbf{k}')[1 - n_0(\mathbf{k})]L(\mathbf{k}', \mathbf{k}). \end{aligned} \quad (35)$$

The calculation of the entropy generation rate in a circuit with resistivity  $\rho$ , where a current  $I$  flows, with the help of the Boltzmann's equation gives the coefficient of resistance in the form [28]

$$\rho = \frac{k_{\text{B}}T(\Phi, \hat{L}\Phi)E^2}{(\Phi, X)^2}. \quad (36)$$

Since the structures of the quantum kinetic equation and the Boltzmann's equation are identical, the same formula is true, while we use the quantum kinetic equation in any order of perturbation theory. The significance of such a formula consists in that the function  $\Phi(\mathbf{k})$  which ensures the minimum of the relevant functional allows one to get the most exact value of the resistance of the system.

We now consider the case of elastic scattering. Then the solution of the kinetic equation written in the second order of perturbation theory is known. To within a constant multiplier, it equals

$$\Phi(\mathbf{k}) = \mathbf{k}E. \quad (37)$$

In this case,

$$\begin{aligned} \frac{1}{k_{\text{B}}T}(\Phi, X) &= \frac{e\hbar}{mk_{\text{B}}T} \sum_{\mathbf{k}} k^2 n_0(\mathbf{k})[1 - n_0(\mathbf{k})]E = \\ &= \frac{eV}{\pi^2 \hbar k_{\text{F}}} \int_0^{\infty} \delta(k - k_{\text{F}})k^4 dk = \frac{eVk_{\text{F}}^3}{\pi^2 \hbar}. \end{aligned} \quad (38)$$

Respectively,

$$\begin{aligned} \frac{1}{k_{\text{B}}T}(\Phi, \hat{L}\Phi) &= \frac{1}{2k_{\text{B}}T} \sum_{\mathbf{k}, \mathbf{k}'} (k^2 + k'^2 - 2\mathbf{k}\mathbf{k}')n_0(\mathbf{k}') \times \\ &\times [1 - n_0(\mathbf{k})]L(\mathbf{k}', \mathbf{k}) = \frac{V^2 m k_{\text{F}}}{2(2\pi)^2 \pi^2 \hbar^2} \int_0^{\infty} dk' k'^2 \times \end{aligned}$$

$$\times \int_{-1}^1 [k^2 + k'^2 - 2kk' \cos(\theta)] L(\mathbf{k}', \mathbf{k}) d \cos(\theta). \quad (39)$$

Since

$$L(\mathbf{k}', \mathbf{k}) = 3 \frac{2\pi n}{\hbar} \frac{w^2(|\mathbf{k} - \mathbf{k}'|)}{\varepsilon^2(|\mathbf{k} - \mathbf{k}'|)} S(|\mathbf{k} - \mathbf{k}'|) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}), \quad (40)$$

we obtain

$$\begin{aligned} (\Phi, \hat{L}\Phi) &= \frac{3V^2 m^2 n}{4\pi^3 \hbar^5} \int_0^{2k_F} \frac{w^2(q)}{\varepsilon^2(q)} S(q) q^3 dq = \\ &= \frac{3V^2 m k_F^3}{\pi^2 \hbar^2} \frac{1}{\tau(k_F)}, \end{aligned} \quad (41)$$

where the reciprocal relaxation time

$$\frac{1}{\tau(k_F)} = \frac{mn}{4\pi \hbar^3 k_F^3} \int_0^{2k_F} \frac{w^2(q)}{\varepsilon^2(q)} S(q) q^3 dq. \quad (42)$$

Then the coefficient of resistivity has the form

$$\rho = \frac{m}{e^2 n} \frac{1}{\tau(k_F)}. \quad (43)$$

Thus, the exact solution of the quantum kinetic equation in the case of elastic scattering minimizes the corresponding functional. In this case, the uses of the variation method and the kinetic equation give the same result. The advantage of the variation method is manifested in the case where the exact solution cannot be found. It is more convenient to seek the approximate solution, by minimizing a functional. If we take the same trial function to determine the resistance in the third order of perturbation theory, we get the known third-order contribution to the reciprocal relaxation time [3, 16, 17, 22]

$$\begin{aligned} \frac{1}{\tau_3(k_F)} &= \frac{2\pi n \pi^2 \hbar^2}{V^3 m \hbar^3 k_F^3 k_B T} \times \\ &\times \sum_{\mathbf{k}, \mathbf{k}', \mathbf{k}''} (k^2 + k'^2 - 2\mathbf{k}\mathbf{k}') n_0(\mathbf{k}') [1 - n_0(\mathbf{k})] \times \\ &\times \frac{w(|\mathbf{k} - \mathbf{k}'|)}{\varepsilon(|\mathbf{k} - \mathbf{k}'|)} \frac{w(|\mathbf{k}' - \mathbf{k}''|)}{\varepsilon(|\mathbf{k}' - \mathbf{k}''|)} \frac{w(|\mathbf{k}'' - \mathbf{k}|)}{\varepsilon(|\mathbf{k}'' - \mathbf{k}|)} \times \\ &\times S(\mathbf{k} - \mathbf{k}', \mathbf{k}' - \mathbf{k}'', \mathbf{k}'' - \mathbf{k}) \frac{\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}''}}. \end{aligned} \quad (44)$$

Using the convolution approximation for the three-particle structural factor allows one to significantly decrease of the integration multiplicity [3, 16, 17, 22].

## 5. Cross Term of the Quantum Kinetic Equation

While analyzing the terms of the fourth and higher orders of perturbation theory, it is not sufficient only to calculate the reciprocal relaxation time to within the fourth-order terms. It is necessary to consider a number of finer effects. One of such fourth-order unstudied effects is the cross scattering of conduction electrons by an external electric field and the field of ions. In this case, we will take the exact solution of the kinetic equation in the second order of perturbation theory as a trial function. We have

$$(\Phi, X) = (\Phi, X_1) + (\Phi, X_2), \quad (45)$$

$$(\Phi, X_1) = \sum_{\mathbf{k}} \mathbf{k} \langle a_{\mathbf{k}}^+(0) a_{\mathbf{k}}(0) \mathbf{I}(0) \rangle E, \quad (46)$$

$$\begin{aligned} (\Phi, X_2) &= \frac{1}{V} \sum'_{\mathbf{k}, \mathbf{k}'} \mathbf{k} w(|\mathbf{k} - \mathbf{k}'|) \times \\ &\times \left[ \frac{\langle a_{\mathbf{k}}^+(0) \rho^i(\mathbf{k}' - \mathbf{k}) a_{\mathbf{k}'}(0) \mathbf{I}(0) \rangle}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} + i\delta} - \right. \\ &\left. - \frac{\langle a_{\mathbf{k}'}^+(0) \rho^i(\mathbf{k} - \mathbf{k}') a_{\mathbf{k}}(0) \mathbf{I}(0) \rangle}{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + i\delta} \right]. \end{aligned} \quad (47)$$

Using the Sokhotskii formula and interchanging the summation indices in the second term under the summation symbol, we get

$$\begin{aligned} (\Phi, X_2)_2 &= \frac{1}{V} \sum'_{\mathbf{k}, \mathbf{k}'} (\mathbf{k} - \mathbf{k}') w(|\mathbf{k} - \mathbf{k}'|) \times \\ &\times \frac{\langle a_{\mathbf{k}}^+(0) \rho^i(\mathbf{k} - \mathbf{k}') a_{\mathbf{k}}(0) \mathbf{I}(0) \rangle}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}} E. \end{aligned} \quad (48)$$

In turn, the equation of motion for the thermodynamic means has the form

$$\begin{aligned} (\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}}) \langle a_{\mathbf{k}}^+ \rho^i(-\mathbf{q}) a_{\mathbf{k}+\mathbf{q}} \mathbf{I} \rangle &= \frac{e\hbar}{Vm} \sum_{\mathbf{k}', \mathbf{q}'} \mathbf{k}' w(q') \times \\ &\times \left\{ \langle a_{\mathbf{k}}^+ \rho^i(-\mathbf{q}) \rho^i(-\mathbf{q}') a_{\mathbf{k}+\mathbf{q}+\mathbf{q}'} a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle - \right. \\ &\left. - \langle a_{\mathbf{k}+\mathbf{q}'}^+ \rho^i(-\mathbf{q}) \rho^i(\mathbf{q}') a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle + \right. \end{aligned}$$

$$+ \langle a_{\mathbf{k}}^+ \rho^i(-\mathbf{q}) \rho^i(\mathbf{q}') a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'+\mathbf{q}'} \rangle - \langle a_{\mathbf{k}}^+ \rho^i(-\mathbf{q}) \rho^i(\mathbf{q}') a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'+\mathbf{q}'}^+ a_{\mathbf{k}'} \rangle \}. \quad (49)$$

It is sufficient to calculate the thermodynamic means on the right-hand side of the last equation in the electron-ion interaction. In this case,

$$(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}}) \langle a_{\mathbf{k}}^+ \rho^i(-\mathbf{q}) a_{\mathbf{k}+\mathbf{q}} \mathbf{I} \rangle = \frac{N e \hbar}{V m} w(q) S(q) \times \sum_{\mathbf{k}'} \mathbf{k}' \left\{ \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle - \langle a_{\mathbf{k}+\mathbf{q}}^+ a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle + \langle a_{\mathbf{k}}^+ a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'+\mathbf{q}} \rangle - \langle a_{\mathbf{k}}^+ a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'+\mathbf{q}}^+ a_{\mathbf{k}'} \rangle \right\}. \quad (50)$$

In the zero order in the electron-ion interaction,

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle = \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle [1 - \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle] \Delta(\mathbf{k} - \mathbf{k}') + \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle \langle a_{\mathbf{k}'}^+ a_{\mathbf{k}'} \rangle. \quad (51)$$

Here, the second term on the right-hand side gives no contribution to the sum over  $\mathbf{k}'$  due to the spherical symmetry of the problem. Analogously,

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}'}^+ a_{\mathbf{k}'+\mathbf{q}} \rangle = \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle [1 - \langle a_{\mathbf{k}+\mathbf{q}}^+ a_{\mathbf{k}+\mathbf{q}} \rangle] \times \Delta(\mathbf{k} - \mathbf{k}' + \mathbf{q}) + \langle a_{\mathbf{k}+\mathbf{q}}^+ a_{\mathbf{k}+\mathbf{q}} \rangle \langle a_{\mathbf{k}'}^+ a_{\mathbf{k}'+\mathbf{q}} \rangle. \quad (52)$$

Here, the second term on the right-hand side equals zero, since only the diagonal elements of the one-particle density matrix are nonzero in the zero order in the electron-ion interaction. Thus,

$$(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}}) \langle a_{\mathbf{k}}^+(0) \rho^i(-\mathbf{q}) a_{\mathbf{k}+\mathbf{q}}(0) \mathbf{I}(0) \rangle = \frac{N e \hbar}{V m} w(q) S(q) \times \left\{ \mathbf{k} n_0(\mathbf{k}) [1 - n_0(\mathbf{k})] - (\mathbf{k} + \mathbf{q}) \times n_0(\mathbf{k} + \mathbf{q}) [1 - n_0(\mathbf{k} + \mathbf{q})] + \mathbf{q} n_0(\mathbf{k}) [1 - n_0(\mathbf{k} + \mathbf{q})] \right\}. \quad (53)$$

We have

$$(\Phi, X_2) = E \frac{N e \hbar}{V^3 m} \sum_{\mathbf{k}, \mathbf{k}'} w^2(|\mathbf{k} - \mathbf{k}'|) S(|\mathbf{k} - \mathbf{k}'|) \times$$

$$\times \left\{ 2(k^2 - \mathbf{k}\mathbf{k}') \frac{n_0(\mathbf{k}) [1 - n_0(\mathbf{k})]}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} - (\mathbf{k} - \mathbf{k}')^2 \frac{n_0(\mathbf{k}) [1 - n_0(\mathbf{k}')] }{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} \right\}. \quad (54)$$

In the second order in the electron-ion interaction, the next term of the quantum kinetic equation reads

$$(\Phi, X_1) = \sum_{\mathbf{k}} \mathbf{k} \langle a_{\mathbf{k}}^+(0) a_{\mathbf{k}}(0) \mathbf{I}(0) \rangle E = \frac{e \hbar}{m} \sum_{\mathbf{k}, \mathbf{k}'} \mathbf{k} \mathbf{k}' \langle a_{\mathbf{k}}^+(0) a_{\mathbf{k}}(0) a_{\mathbf{k}'}^+(0) a_{\mathbf{k}'}(0) \rangle E. \quad (55)$$

In this equation, the thermodynamic means on the right-hand side should be calculated in the second order of perturbation theory. To this end, we represent the mean of the product of four operators of creation and annihilation by the sum, whose terms are expressed through all possible binary products:

$$\langle a_{\mathbf{k}}^+(0) a_{\mathbf{k}}(0) a_{\mathbf{k}'}^+(0) a_{\mathbf{k}'}(0) \rangle = \langle \langle a_{\mathbf{k}}^+(0) a_{\mathbf{k}}(0) \rangle \rangle \times \langle \langle a_{\mathbf{k}'}^+(0) a_{\mathbf{k}'}(0) \rangle \rangle + \langle a_{\mathbf{k}}^+(0) a_{\mathbf{k}}(0) \rangle \langle a_{\mathbf{k}'}^+(0) a_{\mathbf{k}'}(0) \rangle. \quad (56)$$

Here, the internal averaging is performed only over coordinates of the electron subsystem, whereas the external one is realized over coordinates of the ion subsystem. The second term gives no contribution to the corresponding sum due to its symmetry under the summation over the wave vectors  $\mathbf{k}$  and  $\mathbf{k}'$ . In the first order of perturbation theory, each factor of the first term can be written as follows:

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}'} \rangle = - \langle a_{\mathbf{k}'} a_{\mathbf{k}}^+ \rangle = w(|\mathbf{k} - \mathbf{k}'|) \rho^i(\mathbf{k}' - \mathbf{k}) \frac{n_0(k) - n_0(k')}{\varepsilon_k - \varepsilon_{k'}}. \quad (57)$$

Now, in the second order of perturbation theory, we have

$$(\Phi, X_1)_2 = - \frac{N e \hbar}{V^2 m} \sum_{\mathbf{k}, \mathbf{k}'} \mathbf{k} \mathbf{k}' w^2(|\mathbf{k} - \mathbf{k}'|) \times S(|\mathbf{k} - \mathbf{k}'|) \frac{[n_0(k) - n_0(k')]^2}{(\varepsilon_k - \varepsilon_{k'})^2} E. \quad (58)$$

We take also into account that

$$[n_0(k) - n_0(k')]^2 = n_0(k) [1 - n_0(k')] + n_0(k') \times$$

$$\times [1 - n_0(k)] - n_0(k)[1 - n_0(k)] - n_0(k')[1 - n_0(k')]. \quad (59)$$

Combining  $(\Phi, X_1)_2$  and  $(\Phi, X_2)_2$ , we obtain

$$\begin{aligned} & (\Phi, X_1)_2 + (\Phi, X_2)_2 = \\ & = \frac{N}{V^3 k_B T} \frac{e\hbar}{m} \sum'_{\mathbf{k}, \mathbf{k}'} \frac{k^2}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} w^2(|\mathbf{k} - \mathbf{k}'|) S(|\mathbf{k} - \mathbf{k}'|) \times \\ & \times \left\{ 2k^2 \frac{n_0(\mathbf{k})[1 - n_0(\mathbf{k})]}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} - (k^2 + k'^2) \frac{n_0(\mathbf{k})[1 - n_0(\mathbf{k}')] }{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} \right\}. \end{aligned} \quad (60)$$

or, in a more symmetric form,

$$\begin{aligned} & (\Phi, X_1)_2 + (\Phi, X_2)_2 = \\ & = \frac{N}{V^3 k_B T} \frac{e\hbar}{m} \sum'_{\mathbf{k}, \mathbf{k}'} \frac{k^2}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} w^2(|\mathbf{k} - \mathbf{k}'|) S(|\mathbf{k} - \mathbf{k}'|) \times \\ & \times \left\{ 2n_0(\mathbf{k})[1 - n_0(\mathbf{k})] - n_0(\mathbf{k})[1 - n_0(\mathbf{k}')] - \right. \\ & \left. - n_0(\mathbf{k}')[1 - n_0(\mathbf{k})] \right\} E. \end{aligned} \quad (61)$$

This formula can be significantly simplified in the case of a strongly degenerate or classical electron gas. The first and second situations are realized, respectively, for metals and a fully ionized classical plasma of metals. For metals,  $k_B T \ll \varepsilon_F$ . In this case, formula (61) takes the form

$$\begin{aligned} & \frac{(\Phi, X_1)_2}{k_B T} + \frac{(\Phi, X_2)_2}{k_B T} = \frac{N}{k_B T V^3} \frac{e\hbar}{m} \times \\ & \times \sum'_{\mathbf{k}, \mathbf{k}'} \frac{k^2}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})^2} w^2(|\mathbf{k} - \mathbf{k}'|) S(|\mathbf{k} - \mathbf{k}'|) \times \\ & \times n_0(\mathbf{k})[1 - n_0(\mathbf{k})] \left\{ 2 - \frac{1 - n_0(\mathbf{k}')}{1 - n_0(\mathbf{k})} - \frac{n_0(\mathbf{k}')}{n_0(\mathbf{k})} \right\} E. \end{aligned} \quad (62)$$

For extremely low temperatures, the relation

$$\frac{n_0(\mathbf{k})[1 - n_0(\mathbf{k})]}{k_B T} = \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \quad (63)$$

yields

$$\delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \frac{1}{1 - n_0(\mathbf{k})} = \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \frac{1}{n_0(\mathbf{k})} = 2\delta(\varepsilon_{\mathbf{k}} - \varepsilon_F), \quad (64)$$

and we get

$$\frac{(\Phi, X_1)_2}{k_B T} + \frac{(\Phi, X_2)_2}{k_B T} = 0. \quad (65)$$

Thus, for a strongly degenerate electron gas, the contribution of the cross processes of scattering to the electric resistance is zero. It is natural that this contribution can be nonzero due to the temperature-dependent corrections.

We now consider the classical electron gas. In this case, we can neglect the products of the distribution functions of conduction electrons as compare with the distribution function to the first power. Then

$$\begin{aligned} & (\Phi, X_1)_2 + (\Phi, X_2)_2 = \frac{N}{V^3} \frac{4e\hbar m}{\hbar^3} \times \\ & \times \sum'_{\mathbf{k}, \mathbf{k}'} \frac{n_0(k)}{(k + k')(k - k')} w^2(|\mathbf{k} - \mathbf{k}'|) S(|\mathbf{k} - \mathbf{k}'|) E. \end{aligned} \quad (66)$$

We now maximally simplify the formula to facilitate the numerical calculations. After the transition from the summation to the integration in a spherical system of coordinates, we will integrate only with respect to two moduli of the wave vectors and the angle between them:

$$\begin{aligned} & (\Phi, X_1)_2 + (\Phi, X_2)_2 = E \frac{N}{V} \frac{2e\hbar m}{\pi^4 \hbar^3} \int_0^\infty dk k n_0(k) \times \\ & \times \int_0^\infty dk' \frac{k'}{(k + k')(k - k')} \int_{|k - k'|}^{k + k'} w^2(q) S(q) q dq. \end{aligned} \quad (67)$$

It is seen that the integrand has a singularity, namely the first-order pole. That is, the given integral exists in the sense of the principal value.

## 6. Discussion and Conclusions

We have obtained the formula for the coefficient of resistance of simple disordered metals which is true in any order of perturbation theory in the electron-ion interaction. This formula is based on the variation principle. In this case, the ion subsystem is considered static.



The decoupling parameters of higher-order Green's functions which arise at the derivation of the quantum kinetic equation are chosen from the condition of coincidence of the Boltzmann's equation and the quantum kinetic equation in the lowest order of perturbation theory. This allows us to give a clear physical sense to the decoupling procedure and to maximally bring together the structures of these two equations with regard for the higher-order terms of perturbation theory in the quantum kinetic equation.

For the first time, we consider the cross term of the kinetic equation and its contribution to the electric resistance. In the limit of low temperatures, it turns out zero for simple disordered metals in the fourth order of perturbation theory and nonzero for a fully ionized classical plasma produced of these metals. This fact does not exclude the possibility of that the cross processes in higher orders of perturbation theory give a nonzero contribution to kinetic coefficients for metals as well. Thus, the relaxation time approximation for metals is more exact than that for a fully ionized plasma.

Thus, the cross processes in metals give no contribution to the Lorentz constant for the Wiedemann–Franz law and to the Hall constant for the Hall effect. But, for a fully ionized plasma, such a contribution is nonzero. This completely agrees with the available experimental results which testify that, in disordered metals, the Wiedemann–Franz law is fulfilled with a high accuracy, and the Hall constant is practically the same as that for free electrons.

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#### ПЕРЕХРЕСНЕ РОЗСІЮВАННЯ ЕЛЕКТРОНІВ У НЕВПОРЯДКОВАНИХ СИСТЕМАХ

Т.В. Швець

Резюме

На основі варіаційного принципу отримано вираз для коефіцієнта електричного опору простих неупорядкованих металів і повністю іонізованої плазми, вірний у довільному порядку теорії збурень за електрон-іонною взаємодією. При цьому іонна підсистема вважається статичною. Параметри розчленення функцій Гріна старших порядків, що виникають при отриманні квантового кінетичного рівняння, вибрані з умови збігу рівняння Больцмана і квантового кінетичного рівняння у нижчому порядку теорії збурень. У другому і третьому порядках теорії збурень відтворені вже відомі раніше результати. Вперше враховано ефект четвертого порядку, пов'язаний з одночасним розсіюванням електронів провідності зовнішнім електричним полем і полем іонів. Цей ефект у четвертому порядку теорії збурень виявився відмінним від нуля для повністю іонізованої плазми і рівним нулю у межі низьких температур для неупорядкованих металів.