PERTURBATION THEORY FOR COLLECTIVE MODES IN THE DYNAMICS OF SIMPLE AND COMPLEX LIQUIDS

I.M. MRYGLOD, V.M. KUPOROV

Institute for Condensed Matter Physics, Nat. Acad. of Sci. of Ukraine
(1, Swentsytskyi Str., Lviv 79011, Ukraine; e-mail: slw@ph.icmp.lviv.ua)

PACS 61.20.Gy, 61.20.Lc, 61.20.Ne, 47.10.-g

The perturbation theory formalism is used to calculate the corrections to the generalized collective modes caused by weak cross correlations between thermal and viscoelastic dynamic processes in simple and complex liquids. The general formulation of this perturbation theory up to the second order of magnitude inclusive is performed. The theory is tested by the example of the simplest dynamic models of liquids, and the obtained results are analyzed as compared with the accurate solutions for these models. It is shown that, under appropriate conditions, the results of perturbation theory allow one to reproduce the known solutions for these models.

When studying the nature of dynamic processes in simple and complex liquids, an important problem is to investigate their collective dynamics and to find the collective mode spectra that characterize the basic types of collective processes in a system. The hydrodynamic description is based upon the basis of dynamic variables including the densities of all conservative quantities, i.e. the particle density $\hat{n}_{\mathbf{k}}$ (partial densities of the components $\hat{n}_{\mathbf{k},\alpha}$ in the case of a multicomponent liquid), the flux density $\hat{\mathbf{J}}_{\mathbf{k}}$, and the total energy density $\hat{e}_{\mathbf{k}}$ determined in the Fourier transform space as follows:

$$\hat{n}_{\mathbf{k}} = \frac{1}{V}\sum_{j=1}^{N} \exp(i\mathbf{k}\mathbf{r}_j),$$

$$\hat{e}_{\mathbf{k}} = \frac{1}{V}\sum_{j=1}^{N} e_j \exp(i\mathbf{k}\mathbf{r}_j) = \frac{1}{V}\sum_{j=1}^{N}\left(\frac{p_j^2}{2m} + \sum_{l\neq j} V_{jl}\right) e^{i\mathbf{k}\mathbf{r}_j},$$

$$\hat{\mathbf{J}}_{\mathbf{k}} = \frac{1}{V}\sum_{j=1}^{N} m\dot{\mathbf{r}}_j \exp(i\mathbf{k}\mathbf{r}_j).$$

The time derivative of the particle density is related to the flux density, namely its projection $\hat{J}_{\mathbf{k}}$ onto the wave vector $\mathbf{k}$ (the longitudinal component):

$$\dot{\hat{n}}_{\mathbf{k}} = iL_N \hat{n}_{\mathbf{k}} = \frac{i\mathbf{k}}{m}\hat{\mathbf{J}}_{\mathbf{k}} = \frac{ik}{m}\hat{J}_{\mathbf{k}}.$$

Here, $\mathbf{r}_j$ denote the space coordinates of the j-th particle, $\mathbf{p}_j = m\dot{\mathbf{r}}_j$ is the momentum, m and e_j are the particle mass and energy, respectively, V_{jl} is the pair interaction potential, and iL_N is the Liouville operator for the N-particle system.

As is known, the hydrodynamic model often appears insufficient for the correct description of liquids [1–3] and cannot explain some results of experiments and numerical calculations. This concerns particularly a change of the sound velocity in liquids and dense gases (namely the registration of the fast sound phenomenon in [4–6] and the positive (or negative) sound dispersion [7, 8]), the appearance of propagator oscillation modes in ion systems [1], *etc.*

In this case, a more adequate description is realized with the use of extended models that consider both hydrodynamic variables and their time derivatives of higher fluxes, in particular, the derivative of the density flux (partial fluxes), energy density, and so on. For example, the thermoviscoelastic model of simple liquids includes the variables

$$\{n_{\mathbf{k}}, \dot{n}_{\mathbf{k}}, e_{\mathbf{k}}\} + \{\ddot{n}_{\mathbf{k}}, \dot{e}_{\mathbf{k}}\}.$$

For multicomponent liquids, we obtain, respectively,

$$\{n_{\mathbf{k},\alpha}, J_{\mathbf{k},\alpha}, e_{\mathbf{k}}\} + \{\dot{J}_{\mathbf{k},\alpha}, \dot{e}_{\mathbf{k}}\}.$$

At the same time, due to the inclusion of a large number of additional dynamic variables, the system of macrodynamic equations becomes rather complicated for the analytical consideration. Though, in some cases, it is possible to neglect cross correlations supposing them weak. Then a complex dynamic model is decomposed into several simpler ones, for which it is easy to analytically calculate the collective eigenmode spectrum. Models of this kind often can rather accurately describe the main processes in the system in certain ranges of the wave number. It is illustrated by the regions of partial and coherent dynamics [9] discovered in binary mixtures.

If the cross correlations in a certain region of wave vectors and frequencies become small, then the simplified models neglecting them allow one to understand the physical mechanism of the mode formation and provide some useful information for the interpretation of experiments. However, it is sometimes necessary to consider also the corrections to collective modes that will appear when allowing for weak cross correlations, in particular, between the energy and the partial densities, the energy and the density fluxes, or the density and momentum fluxes. In the case of the simplified models with no regard for cross correlations, it is easy to find the collective mode spectrum of the system analytically. Therefore, there arises an idea to develop and to use the corresponding perturbation theory. This problem is ideologically close to the stationary perturbation theory in quantum mechanics.

In this case, the unperturbed problem can be the collective mode problem for a system without cross correlations, whose kinetic matrix consists of several smaller blocks. The eigenvalues and eigenvectors for each of them can be searched for independently. It is evident that the eigenvectors of different blocks of the unperturbed problem will be orthogonal to one another. The perturbation of the kinetic matrix is chosen as a matrix of the same order as the initial one with the corresponding cross correlators of the static or dynamic nature (neglected in the unperturbed matrix) at the cross positions, and all other elements equal to zero.

The aim of this work is to construct the general formulation of such a perturbation theory and to demonstrate its application to some simple dynamic models of liquids.

1. Generalized Hydrodynamic Matrix and Collective Excitation Spectrum

The problem of the calculation of the collective excitation spectrum of a system $\{z_\alpha(k)\}$ in the method of generalized collective modes is reduced to the problem of finding the eigenvalues and the eigenvectors of the generalized hydrodynamic (kinetic) matrix $\mathbf{T}(k)$ [1, 2]:

$$\mathbf{T}(k)\hat{\mathbf{X}}(k) = \hat{\mathbf{X}}(k)\hat{z}(k), \tag{1}$$

that includes two contributions $\mathbf{T}(k) = -i\Omega(k)+\tilde{\Phi}(k,0)$, where $i\Omega(k)$ and $\tilde{\Phi}(k,z)$ are the frequency matrix and the memory function matrix of the system, respectively, and $\hat{z}(k)$ is the diagonal matrix of eigenvalues: $z_{\alpha\beta}(k) = \delta_{\alpha\beta}z_\alpha(k)$. The frequency matrix is calculated in terms of the static equilibrium correlation functions as

$$i\Omega(k) = \langle iL\mathbf{P_k}\mathbf{P_{-k}}\rangle\langle\mathbf{P_k}\mathbf{P_{-k}}\rangle^{-1}.$$

In the symmetrized form, it is anti-Hermitian (which will be discussed in detail in what follows). Therefore, it makes only imaginary contributions to the eigenvalues $z_\alpha(k)$ that describe the characteristic frequencies of the propagator oscillation modes. The memory function matrix

$$\tilde{\Phi}(k,z) = \langle(1-\mathcal{P})iL_N\mathbf{P_k}(z+(1-\mathcal{P})iL_N)^{-1}\times$$

$$\times(1-\mathcal{P})iL_N\mathbf{P_{-k}}\rangle\langle\mathbf{P_k}\mathbf{P_{-k}}\rangle^{-1}$$

in the symmetrized form is Hermitian and has only real (and single-sign [10]) eigenvalues, which provides the decay of the time correlation functions with time. Here, $\mathbf{P_k}$ is the collection of basis dynamic variables used for the description of the system, $\langle...\rangle$ means the equilibrium statistical averaging, and $\mathcal{P} = \langle...\mathbf{P_{-k}}\rangle\langle\mathbf{P_k}\mathbf{P_{-k}}\rangle^{-1}\mathbf{P_k}$ is the Mori projection operator. The obtained eigenvalues $z_\alpha(k)$ yield the collective mode spectrum of the system calculated in the framework of a certain dynamic model determined by the collection $\{\mathbf{P_k}\}$, while the corresponding eigenvectors $\mathbf{x}_\alpha = \{\hat{X}_{i,\alpha}\}$ allow one to calculate the amplitudes of these modes that describe the contribution of a definite mode into the time correlation functions (TCF):

$$F_{ij}(k,t) = \sum_{\alpha=1}^{s} G_\alpha^{ij}(k)\exp(z_\alpha(k)t), \tag{2}$$

where the amplitudes $G_\alpha^{ij}(k)$ are searched for as

$$G_\alpha^{ij}(k) = \sum_{l=0}^{s-1}\hat{X}_{i,\alpha}\hat{X}_{\alpha,l}^{-1}F_{lj}(k,0). \tag{3}$$

If the basis of dynamic variables $\mathbf{P_k}$ can be divided into subclasses, the cross correlations between which are considered small due to some physical reasons, then the collective mode problem is simplified and can be rather easily solved analytically in the zero-order approximation with respect to these correlations. The elements of the cross correlations, that are considered small and not taken into account in the zero-order approximation, can be estimated with the help of perturbation theory in the first, second, and other orders in these quantities.

This problem looks rather similar to the problem of the stationary perturbation theory in quantum mechanics, but a more detailed investigation reveals the considerable and fundamental differences. First of all, it is the non-Hermitian character of the generalized hydrodynamic matrix, which, in the general case, results in the nonorthogonality of the eigenvectors corresponding to different eigenvalues. It is worth noting that the orthogonality plays an extremely important role in quantum mechanics for the construction of the perturbation theory formalism. That is why the further study requires a more detailed consideration of the properties of the generalized hydrodynamic matrix. First of all, in order to simplify the calculations of the collective modes, one can perform the orthogonalization of the matrix of static correlation functions $\langle \mathbf{P}_{\mathbf{k}}\mathbf{P}_{-\mathbf{k}}\rangle$ bringing it to the diagonal form. Such an orthogonalization can be carried out with the help of the Gram–Schmidt procedure using the technique of projection operators [11].

Second, it is necessary to ensure the orthogonality of the eigenvectors corresponding to different eigenvalues of the generalized hydrodynamic matrix and to solve the problem of normalization of the eigenvectors that, generally speaking, represent the functions of k. This aim can be achieved by additionally symmetrizing the generalized hydrodynamic matrix, which is reached due to the transformation $\mathbf{P}^{\alpha}_{\mathbf{k}} \to \mathbf{P}^{\alpha}_{\mathbf{k}}/\langle \mathbf{P}^{\alpha}_{\mathbf{k}}\mathbf{P}^{\alpha}_{\mathbf{k}}\rangle^{1/2}$.

In order to better formulate the general idea and the construction scheme of perturbation theory for the calculation of the collective modes, let us consider a simple example of the dynamic model of a liquid described by the collection of dynamic variables that includes such densities of the local motion integrals as the particle density $n_{\mathbf{k}}$, its first time derivative (or the flux density), and the time derivative of the momentum flux density, i.e. $\{n_{\mathbf{k}}, \dot{n}_{\mathbf{k}}, \ddot{n}_{\mathbf{k}}\}$. Neglecting the cross viscoelastic correlations, such a basis can be divided into two subclasses of variables $\{n_{\mathbf{k}}, \dot{n}_{\mathbf{k}}\} + \{\ddot{n}_{\mathbf{k}}\}$, each of them having its independent eigenvalues and corresponding eigenvectors. For such a model, one can proceed to the symmetrized and orthogonalized basis [12]. Then the generalized hydrodynamic matrix takes the form

$$\mathbf{T}_0(k) = \begin{pmatrix} 0 & ic_Tk & 0 \\ ic_Tk & 0 & 0 \\ 0 & 0 & (\frac{c_\infty^2}{c_T^2}-1)\frac{1}{\tau} \end{pmatrix}. \quad (4)$$

In this case, it is easy to find the eigenvalues corresponding to two sound complex-conjugate modes $z_{\pm}^{(0)} = \pm ic_Tk$ and one relaxation mode $z_3^{(0)} = -(\frac{c_\infty^2}{c_T^2}-1)\frac{1}{\tau}$. Here, c_T and c_∞ stand for the isothermal and high-frequency velocities of sound, respectively, while τ is the correlation time for the time correlation density function. The isothermal velocity of sound c_T is determined from the equality $c_T^2k^2 = \langle \dot{n}_{\mathbf{k}}\dot{n}_{-\mathbf{k}}\rangle\langle n_{\mathbf{k}}n_{-\mathbf{k}}\rangle^{-1}|_{k\to0} = (k_BT/mS_{nn}(0))k^2$, where $S_{nn}(k) = \langle n_{\mathbf{k}}n_{-\mathbf{k}}\rangle$ is the static structural factor related to the compressibility of the system in the limit $k \to 0$; c_∞^2 is the so-called zero or high-frequency velocity of sound that allows only for elastic effects (similarly to a solid body) and is determined as $c_\infty^2k^2 = \langle \ddot{n}_{\mathbf{k}}\ddot{n}_{-\mathbf{k}}\rangle\langle \dot{n}_{\mathbf{k}}\dot{n}_{-\mathbf{k}}\rangle^{-1}|_{k\to0}$. The eigenvectors corresponding to these modes are also easy to find, and their normalization to 1 yields $\mathbf{x}_3^{(0)} = (0,0,1)$, $\mathbf{x}_{\pm}^{(0)} = \frac{1}{\sqrt{2}}(\mp1,1,0)$. It is evident that all of them are mutually orthogonal, but of most importance is the orthogonality of the vector $\mathbf{x}_3^{(0)}$ to the other eigenvectors. As will be shown below, it is important for the generalization of this simple example and the successive construction of the perturbation theory.

After including the cross correlations, the generalized hydrodynamic matrix can be presented in the form $\mathbf{T}(k) = \mathbf{T}_0(k) + \delta\mathbf{T}(k)$, where the symmetrized perturbation matrix

$$\delta\mathbf{T}(k) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & ik\sqrt{c_\infty^2 - c_T^2} \\ 0 & -ik\sqrt{c_\infty^2 - c_T^2} & 0 \end{pmatrix} \quad (5)$$

contains the expression $\sqrt{c_\infty^2 - c_T^2}$ that can be considered small at close values of c_∞ and c_T. In this case, there arises the typical problem of perturbation theory in the matrix form.

2. General Formulation of Perturbation Theory

Thus, the general statement of the problem is as follows: we are interested in the spectrum of eigenvalues and eigenvectors of the generalized hydrodynamic matrix that can be presented as the sum of two terms $\mathbf{T} = \mathbf{T}_0 + \lambda\delta\mathbf{T}$, where $\mathbf{T}_0$ is the matrix without regard for cross correlations and λ is some formal parameter which is considered small. Then the problem of finding the spectrum of eigenvalues and eigenvectors for the matrix $\mathbf{T}_0$ can be solved rather easily, while $\delta\mathbf{T}$ will represent the matrix of cross correlations that are small and can be taken into account when calculating the spectrum of eigenvalues and eigenvectors of the matrix $\mathbf{T}$ with the help of perturbation theory.

Thus, supposing that the cross correlations between certain variables are small, the corrections to the eigenmodes and the corresponding eigenvectors can be searched for as expansions in the frame of perturbation theory:

$$z_\alpha = z_\alpha^{(0)} + \lambda\delta z_\alpha^{(1)} + \lambda^2\delta z_\alpha^{(2)} + \ldots,$$

$$\mathbf{x}_\alpha = \mathbf{x}_\alpha^{(0)} + \lambda\delta\mathbf{x}_\alpha^{(1)} + \lambda^2\delta\mathbf{x}_\alpha^{(2)} + \ldots.$$

All these expansions can be substituted into the matrix equation (1). Then, with regard for the relation

$$\mathbf{T}_0(k)\hat{\mathbf{X}}^{(0)}(k) = \hat{\mathbf{X}}^{(0)}(k)\hat{z}^{(0)}(k),$$

one can equate the expressions of the same order of smallness on the left- and right-hand sides of this equation. This procedure yields a chain of linear equations (similarly to the case of the stationary perturbation theory in quantum mechanics), which allows one to find the corrections to the eigenvalues and the eigenvectors induced by a perturbation. We recall that, when finding a solution in quantum mechanics, it is important that the unperturbed eigenvectors corresponding to different eigenvalues of the Hamiltonian are orthogonal, as the Hamiltonian is Hermitian. In the case of the generalized hydrodynamic matrix, this statement is not true. However, it is important that the orthogonality condition will be satisfied for sure for some eigenvectors, because the basis of dynamic variables in this class of problems can be divided into independent subclasses: $\{\mathbf{P_k}\} = \{\mathbf{P_k^\mu}, \mathbf{P_k^\nu}\}$, the cross correlations between which are not taken into account in the zero-order approximation. For instance, such independent subclasses in the above-considered example are $\mathbf{P_k^\mu} = \{n_\mathbf{k}, \dot{n}_\mathbf{k}\}$, $\mathbf{P_k^\nu} = \ddot{n}_\mathbf{k}$. Each of these subclasses provides an independent matrix block, its independent eigenvalues, and the corresponding eigenvectors. Moreover, it is absolutely evident that the eigenvectors $\mathbf{x}_\alpha^{(0)}$ for different subclasses will be orthogonal, because they will have the structure $\mathbf{x}_\alpha^{(0)} = \{x_{1,\alpha}, \ldots, x_{\mu,\alpha}; 0, \ldots 0\}$ at $\alpha = 1, \ldots, \mu$ and $\mathbf{x}_\alpha^{(0)} = \{0, \ldots, 0; x_{\mu+1,\alpha}, \ldots, x_{\mu+\nu,\alpha}\}$ at $\alpha = \mu+1, ..., \mu+\nu$. The scalar product of such vectors can be determined as $(\mathbf{x}_\alpha\mathbf{x}_\beta) = \sum_i x_{i,\alpha}^* x_{i,\beta}$, and an arbitrary matrix $\mathbf{A}$ in the representation of the eigenvectors of the matrix $\mathbf{T}^{(0)}$ is determined as $\bar{A}_{\alpha\beta} = (\mathbf{x}_\alpha\mathbf{A}\mathbf{x}_\beta) = \sum_{i,j} x_{i,\alpha}^* A_{ij} x_{j,\beta}$. It is obvious that the cross correlation matrix $\delta\mathbf{T}$ in this representation will always have a zero diagonal $\delta\bar{T}_{\alpha\alpha} = 0$ and even more: the only non-zero components will be those $\delta\bar{T}_{\alpha\beta}$, for which α and β correspond to different subclasses. It is also worth finding the conjugate matrix $\bar{T}_{\alpha\beta}^* = \sum_{i,j} x_{i,\beta}^* T_{ij} x_{j,\alpha}$. Recurring to the above example, it should be noted that the only non-zero components in it are $\delta\bar{T}_{3\alpha}$ and $\delta\bar{T}_{\alpha 3}$, $\alpha = 1, 2(+,-)$.

Thus, substituting the expansions of perturbation theory into Eq. (1) and keeping the terms up to the second order of magnitude inclusive, we obtain

$$\mathbf{T}_0(k)\mathbf{x}_\alpha^{(0)}(k) = \mathbf{x}_\alpha^{(0)}(k)\hat{z}_\alpha^{(0)}(k), \tag{6}$$

$$\delta\mathbf{T}(k)\mathbf{x}_\alpha^{(0)}(k) + \mathbf{T}_0(k)\delta\mathbf{x}_\alpha^{(1)} =$$

$$= \mathbf{x}_\alpha^{(0)}(k)\delta\hat{z}_\alpha^{(1)}(k) + \delta\mathbf{x}_\alpha^{(1)}(k)\hat{z}_\alpha^{(0)}(k), \tag{7}$$

$$\delta\mathbf{T}(k)\delta\mathbf{x}_\alpha^{(1)}(k) + \mathbf{T}_0(k)\delta\mathbf{x}_\alpha^{(2)} =$$

$$= \delta\mathbf{x}_\alpha^{(1)}(k)\delta\hat{z}_\alpha^{(1)}(k) + \delta\mathbf{x}_\alpha^{(2)}(k)\hat{z}_\alpha^{(0)}(k) + \delta\mathbf{x}_\alpha^{(0)}(k)\hat{z}_\alpha^{(2)}(k). \tag{8}$$

Without the additional conditions from (7), $\delta\mathbf{x}_\alpha^{(1)}$ is determined ambiguously, so we can impose the additional normalization condition $(\mathbf{x}_\alpha\mathbf{x}_\alpha^{(0)}) = 1$. When normalizing the unperturbed eigenvectors to 1, it results in the relations $(\mathbf{x}_\alpha^{(0)}\delta\mathbf{x}_\alpha^{(1)}) = 0$, $(\mathbf{x}_\alpha^{(0)}\delta\mathbf{x}_\alpha^{(2)}) = 0$. Multiplying Eq. (7) by $\mathbf{x}_\alpha^{(0)}$ and considering what was written above, one can see that all the first corrections to the modes $\delta z_\alpha^{(1)} = 0$. The first-order correction to the eigenvectors will be searched for in the form $\delta\mathbf{x}_\alpha^{(1)} = \sum_\gamma a_{\alpha\gamma}\mathbf{x}_\gamma^{(0)}$, where $a_{\alpha\gamma}$ are unknown coefficients of the first order of smallness. Substituting this expansion into (7) and multiplying it scalarly from left by the fixed $\mathbf{x}_\beta^{(0)}$ from another subclass, it is easy to show that $\delta\bar{T}_{\alpha\beta} = a_{\beta\alpha}(z_\beta^{(0)} - z_\alpha^{(0)})$. Hence, we obtain the following expression for the first correction to the eigenvectors:

$$\delta\mathbf{x}_\alpha^{(1)} = \sum_{\beta/} \frac{\delta\bar{T}_{\beta\alpha}}{z_\alpha^{(0)} - z_\beta^{(0)}}\mathbf{x}_\beta^{(0)}. \tag{9}$$

Performing similar operations with Eq. (8), one can find the second correction to the eigenvalues:

$$\delta z_{\alpha}^{(2)} = \sum_{\beta/} \frac{\delta \bar{T}^{*}_{\alpha\beta} \delta \bar{T}_{\beta\alpha}}{z_{\alpha}^{(0)} - z_{\beta}^{(0)}}. \quad (10)$$

Here, $\beta/$ denotes that the summation is performed over β from a subclass different from that of α. The condition for this perturbation theory to be valid is evidently the requirement $|\delta \bar{T}_{\alpha\beta}| \ll |z_{\alpha}^{(0)} - z_{\beta}^{(0)}|$.

3. Simplest Examples of Application

In order to illustrate the formulated formalism, we use it for the consideration of several simple examples that correspond to the known dynamic models for a simple liquid with available analytical solutions.

3.1. Hydrodynamic model

The hydrodynamic model is specified by the basis of variables $\{n_{\mathbf{k}}, \dot{n}_{\mathbf{k}}, h_{\mathbf{k}}\}$, where the enthalpy density $h_{\mathbf{k}} = e_{\mathbf{k}} - \langle e_{\mathbf{k}} n_{-\mathbf{k}} \rangle \langle n_{\mathbf{k}} n_{-\mathbf{k}} \rangle^{-1} n_{\mathbf{k}}$ appears instead of the energy density $e_{\mathbf{k}}$ due to the orthogonalization of the collection of densities of conservative variables. The symmetrized hydrodynamic matrix in the limit of small k has the form

$$\mathbf{T}(k) = \begin{pmatrix} 0 & ic_T k & 0 \\ ic_T k & D_l k^2 & \phi_{\dot{n}h} - i\omega_{\dot{n}h} \\ 0 & \phi_{h\dot{n}} - i\omega_{h\dot{n}} & D_h k^2 \end{pmatrix}, \quad (11)$$

where the matrix elements corresponding to thermoviscous cross correlations include both static correlators (due to the elements of the frequency matrix $\omega_{\dot{n}h}$, $\omega_{h\dot{n}}$) and dynamic contributions taken into account by means of the elements of the memory functions $\phi_{h\dot{n}}$, $\phi_{\dot{n}h}$ with the known analytical expressions [2]. In the symmetrized form, we can write $\phi_{h\dot{n}} = -\phi_{\dot{n}h} = -ik^2 \xi(k,0)/\sqrt{TC_V m}$ and $i\omega_{\dot{n}h} = -i\omega_{h\dot{n}} = ik(\alpha_P/\kappa_T)\sqrt{TV/\rho C_V}$, where we operate with the limit values of the generalized isothermal compressibility κ_T, the linear coefficient of thermal expansion α_P, and the heat capacity C_V at $k \to 0$, the non-hydrodynamic transport coefficient (so-called thermal viscosity) $\xi(k,0)\,|_{k\to 0} \to 0$, the mass density ρ, and the thermodynamic temperature T.

In the zero-order approximation, one can often neglect the cross correlations of the thermal density with the particle density and its flux, by dividing matrix (11) into two terms

$$\mathbf{T}(k) = \mathbf{T}_0(k) + \delta \mathbf{T}(k),$$

where

$$\delta \mathbf{T}(k) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \phi_{\dot{n}h} - i\omega_{\dot{n}h} \\ 0 & \phi_{h\dot{n}} - i\omega_{h\dot{n}} & 0 \end{pmatrix}. \quad (12)$$

The generalized collective modes of the model with the matrix $\mathbf{T}_0(k)$ without cross correlations will have the form (to within k^2)

$$z_{\pm}^{(0)} = -D_l k^2/2 \pm ic_T k, \quad z_3^{(0)} = -D_h k^2. \quad (13)$$

Here, $D_h = (V/C_V)\lambda$, where C_V and λ are the limit values of the generalized heat capacity $C_V(k)$ and the generalized heat conduction $\lambda(k,z)$ at $k \to 0$, $z = 0$, respectively, whereas D_l is the longitudinal viscosity coefficient.

The eigenvectors of the unperturbed problem normalized to 1 can be found easily: $\mathbf{x}_h^{(0)} = (0,0,1)$, $\mathbf{x}_{\pm}^{(0)} = (1/\sqrt{2})(\pm 1, 1, 0)$.

On the following stage, one can include the cross correlations with the use of perturbation theory. It is obvious that, among the matrix elements $\delta \bar{T}_{\alpha\beta}$, the only non-zero ones will be $\delta \bar{T}_{3\alpha}$ and $\delta \bar{T}_{\alpha 3}$ at $\alpha = 1, 2(+,-)$.

According to (10), the second-order corrections to the modes are searched for as

$$\delta z_3^{(2)} = \sum_{\alpha=1}^{2} \frac{\delta \bar{T}^{*}_{3\alpha} \delta \bar{T}_{\alpha 3}}{z_3^{(0)} - z_{\alpha}^{(0)}},$$

$$\delta z_{\alpha}^{(2)} = \frac{\delta \bar{T}^{*}_{\alpha 3} \delta \bar{T}_{3\alpha}}{z_{\alpha}^{(0)} - z_3^{(0)}}, \qquad \alpha = 1, 2(+,-). \quad (14)$$

Calculating the product $\delta \bar{T}_{3\alpha} \delta \bar{T}_{\alpha 3}$ accurate to k^2, we obtain

$$\delta \bar{T}_{3\alpha} \delta \bar{T}_{\alpha 3} = -k^2 \left(\frac{\alpha_P}{\kappa_T} \right)^2 \frac{TV}{2\rho C_V}. \quad (15)$$

Substituting (15) into (14) and taking the values of $z_{\alpha}^{(0)}$ into account, we find the corrections to the eigenvalues in the second order of perturbation theory:

$$\delta z_3^{(2)} = -\frac{V^2 T \lambda}{2\rho C_V^2 c_T^2} \left(\frac{\alpha_P}{\kappa_T} \right)^2 k^2, \quad (16)$$

$$\delta z_{\pm}^{(2)} = -\frac{V^2 T \lambda}{2\rho C_V^2 c_T^2} \left(\frac{\alpha_P}{\kappa_T} \right)^2 k^2 \pm$$

$$\pm i\left(\frac{\alpha_P}{\kappa_T}\right)^2\frac{TV}{2\rho C_V c_T^2}c_T k. \tag{17}$$

Hence,

$$z_3 \simeq z_3^{(0)} + \delta z_3^{(2)} = -\frac{V\lambda}{C_V}\left(1+\frac{VT}{2\rho C_V c_T^2}\left(\frac{\alpha_P}{\kappa_T}\right)^2\right)k^2, \tag{18}$$

$$z_\pm \simeq z_\pm^{(0)} + \delta z_\pm^{(2)} = -\left(\frac{D_l}{2}+\frac{V^2T\lambda}{2\rho C_V^2 c_T^2}\left(\frac{\alpha_P}{\kappa_T}\right)^2\right)k^2\pm$$

$$\pm i c_T k\left(1+\left(\frac{\alpha_P}{\kappa_T}\right)^2\frac{TV}{2\rho C_V c_T^2}\right). \tag{19}$$

One can see that, in the case of sound perturbations and the thermal relaxation mode, the correction renormalizes the damping coefficients $\sim k^2$. For the sound modes, the velocity of sound is also renormalized. It is also easy to see that all the relations include the quantity (α_P/κ_T) that actually plays the role of a small parameter of perturbation theory. Recalling the expression for the generalized isothermal compressibility $1/\kappa_T(k) = nk_BT/S_{nn}(k)$ and comparing it to the above expression for the isothermal velocity of sound, one can notice a simple relation existing between them: $1/\kappa_T = c_T^2/\rho$, $\rho = nm$. This fact allows us to rewrite the second term in (17) and explicitly obtain the renormalization factor of the velocity of sound in (19) in the well-known form $\gamma = 1 + TV\alpha_P^2/\kappa_T C_V$. That is, taking the cross correlations between the energy and density fluctuations into account results in the renormalization of the isothermal velocity of sound into the adiabatic one $c_s = c_T\sqrt{\gamma}$. In this case, we used the relation $\sqrt{1+x} \approx 1 + x/2$ that is valid at small x. This means that, in the case of the hydrodynamic model, the perturbation theory yields the standard form of sound modes with the adiabatic velocity of sound:

$$z_\pm \simeq z_\pm^{(0)} + \delta z_\pm^{(2)} = -\Gamma k^2 \pm i c_s k, \tag{20}$$

where the sound damping coefficient $2\Gamma = D_l + (\gamma - 1)(V/C_V)\lambda$, and the formal smallness parameter of perturbation theory is presented by the quantity $\gamma - 1 = (c_s^2 - c_T^2)/c_T^2$.

Keeping the terms of the order of k^3 in the expansion of the hydrodynamic modes in terms of the wave number and considering the correction to the velocity of sound $c(k) = c_T\sqrt{1-(D_l^2/4c_T^2)k^2}$ in (13), we obtain

$$z_\pm \simeq z_\pm^{(0)} + \delta z_\pm^{(2)} = -\Gamma k^2 \pm i c_s k\sqrt{1-\frac{D_l^2}{4c_T^2}k^2}. \tag{21}$$

As concerns the damping coefficients, the analysis of expressions (18)-(19) demonstrates the renormalization of the longitudinal viscosity and the coefficients of sound damping and heat conduction in the expressions for the corresponding modes, which is caused by the allowance for thermoviscous correlations.

3.2. Viscoelastic model

In the case of simple liquids, this dynamic model is based upon the basic variables that include the partial particle density and two of its time derivatives $\{n_{\mathbf{k}}, \dot{n}_{\mathbf{k}}, \ddot{n}_{\mathbf{k}}\}$. The variable $\ddot{n}_{\mathbf{k}}$ describes a change of the momentum flux and also allows for elastic effects. In practice, this model can be used for the description of the systems, in which the effects of thermal expansion are inessential, for instance, in the case of some liquid metals. The orthogonalization results in the physically equivalent collection $\{n_{\mathbf{k}}, \dot{n}_{\mathbf{k}}, n_{3,\mathbf{k}}\}$, where $n_{3,\mathbf{k}} = \ddot{n}_{\mathbf{k}} - \langle \ddot{n}_{\mathbf{k}} n_{-\mathbf{k}}\rangle\langle n_{\mathbf{k}} n_{-\mathbf{k}}\rangle^{-1} n_{\mathbf{k}}$. This model is formally similar to the previous one, as two of its first variables will be the same, while the energy density is replaced by the stress tensor density, whose correlations with the other two variables can be neglected in the zero-order approximation of perturbation theory. Thus, the elements of the kinetic matrix corresponding to the correlations with n_3 or $\ddot{n}$ will be considered small. The further consideration of the system is formally reduced to the thermoviscous one by means of replacing h by n_3.

Some differences will appear in the values of the principal and cross elements of the frequency matrix and the memory function matrix. The unperturbed model in the symmetrized basis will yield the generalized kinetic matrix (4).

The unperturbed eigenvalues will be $z_\pm^{(0)} = \pm i c_T k$, $z_3^{(0)} = (\frac{c_\infty^2}{c_T^2} - 1)/\tau$, i.e. two conjugate sound modes and a relaxation one corresponding to the viscoelastic Maxwell relaxation [13], whereas the corresponding eigenvectors are $\mathbf{x}_3^{(0)} = (0,0,1)$, $\mathbf{x}_\pm^{(0)} = \frac{1}{\sqrt{2}}(\mp 1, 1, 0)$. The symmetrized matrix of cross correlations is specified by expression (5).

Thus, we have

$$\delta\bar{T}_{3\alpha}\delta\bar{T}_{\alpha 3} = -\frac{1}{2}k^2(c_\infty^2 - c_T^2), \tag{22}$$

and the corrections to the eigenvalues at arbitrary τ have the form

$$\delta z_3^{(2)} = -c_T^2 k^2 \tau \frac{(c_\infty^2/c_T^2 - 1)^2}{(c_\infty^2/c_T^2 - 1)^2 + c_T^2 k^2 \tau^2}, \quad (23)$$

$$\delta z_\pm^{(2)} = \frac{-c_T^2 k^2 \tau (c_\infty^2/c_T^2 - 1)^2 \pm i c_T \tau^2 (c_\infty^2 - c_T^2) k^3}{2[(c_\infty^2/c_T^2 - 1)^2 + c_T^2 k^2 \tau^2]}. \quad (24)$$

The correlation time τ for the time correlation density function takes both thermal and viscous effects into account, and its expression follows from the Landau–Placzek relation for the dynamic structural factor of a simple liquid [13, 14] that includes both the thermal contribution $\sim k^2$ and the viscoelastic one.

That is why it is appropriate to consider separately the case where thermal effects are taken into account ($\gamma \neq 1$) and the case where they can be neglected as compared to viscous ones ($\gamma = 1$). In the first case, we obtain the hydrodynamic behavior of τ ($\tau \sim k^{-2}$). Therefore, the non-zero memory function $\phi_2 = (\frac{c_\infty^2}{c_T^2} - 1)\frac{1}{\tau} \sim k^2$ will behave itself in the same way as in the hydrodynamic model. In the second case, the hydrodynamic contribution into the correlation time contains the quantity $\gamma - 1 \approx 0$ that disappears when neglecting the thermal fluctuations. Thus, it is worth taking into account the next contribution related to the longitudinal viscosity D_l, namely $\tau|_{k\to 0} \to D_l/c_T^2$.

First, we consider the case $\gamma \neq 1$ in more details, where thermal processes are taken into account implicitly, and the correlation time is determined from the Landau–Placzek relation

$$\tau = \frac{\gamma - 1}{\gamma} \left(\frac{V\lambda}{C_V} k^2 \right)^{-1},$$

i.e., $\tau \sim k^{-2}$. Using the notation $(c_\infty^2/c_T^2 - 1)(1/\tau) \equiv dk^2$ and taking the relation $z_3^{(0)} = -dk^2$ into account, we obtain

$$z_3 \simeq z_3^{(0)} + \delta z_3^{(2)} = -dk^2 \left(1 - \frac{c_\infty^2 - c_T^2}{c_T^2} \right), \quad (25)$$

$$z_\pm \simeq z_\pm^{(0)} + \delta z_\pm^{(2)} = -\frac{d(c_\infty^2 - c_T^2)}{2c_T^2} k^2 \pm$$

$$\pm i c_T k \left(1 + \frac{c_\infty^2 - c_T^2}{2c_T^2} \right). \quad (26)$$

As one can see, the formal parameter of perturbation theory is presented here by the difference between the isothermal and zero velocities of sound $(c_\infty^2 - c_T^2)/c_T^2$. The use of the approximate formula $1 + x/2 \approx \sqrt{1 + x}$ yields $1 + (c_\infty^2 - c_T^2)/2c_T^2 \approx c_\infty/c_T$ in (26), that is, there takes place the renormalization of the velocity of sound from the isothermal to the high-frequency one: $c_T \to c_\infty$. As is known, $c_\infty > c_s > c_T$. It is worth noting that the accurate solution for the case $\tau \sim k^{-2}$ yields the sound modes exactly with the high-frequency velocity of sound $z_\pm = \pm ikc_\infty + Dk^2$. Such a situation takes place in liquids, in which a sound of high frequency (from here we borrow the term "high-frequency") behaves in a liquid as in an elastic medium similar to a solid body (the case of low-compressible liquids, for example, liquid metals). This fact explains the effective increase of the velocity of sound in liquids as a result of the elastic mechanism of sound propagation.

However, from the physical point of view, the case where thermal effects are completely excluded, i.e. $\gamma = 1$ and viscous processes are considered dominant will be more proper for this model [12]. In this case, the correlation time $\tau = const$ is finite even in the hydrodynamic limit, and there arises an unperturbed relaxation mode with a finite lifetime $z_3^{(0)} = (c_\infty^2/c_T^2 - 1)(1/\tau) \equiv d_0 = const$, whereas corrections (23)–(24) at small k will depend on the relation between the quantities appearing in the denominator, namely k and the characteristic parameter $k_1 = (c_\infty^2/c_T^2 - 1)/c_T^3\tau = (c_\infty^2 - c_T^2)/c_T D_l$, where it is taken into account that $\tau|_{k\to 0} \to D_l/c_T^2$.

If $k \ll k_1$, then the corrections to the damping coefficients will be of the order of k^2 [12]: $\delta z_3^{(2)} = -c_T^2 \tau k^2 = -D_l k^2$, Re $(\delta z_\pm^{(2)}) = -c_T^2 \tau k^2/2 = -D_l k^2/2$. Of higher interest are the corrections to the imaginary parts of the collective modes that make changes to the dispersion law. For instance, formally passing to the hydrodynamic limit in (24) and keeping the terms $\sim k^3$, we obtain the quadratic correction to the velocity of sound [12]

$$\delta c^{(2)} = c_T (D_l/d_0) k^2 = c_T \frac{D_l^2}{2(c_\infty^2 - c_T^2)} k^2 \equiv c_T \beta k^2. \quad (27)$$

Thus, we have a positive correction to the linear dispersion caused by viscous processes that is really observed experimentally, which is reported in the literature [8]. This result explains qualitatively the real behavior of the dispersion dependence.

At $k \sim k_1$, the situation will be different. According to (23)–(24), the corrections to the damping coefficients will behave in a more complicated way, namely as $adk^2/(d + ak^2)$, whereas the positive correction to the dispersion has the form $bk^3/(d + ak^2)$.

For the viscoelastic model, one can find the accurate solution of the eigenvalue problem in the hydrodynamic limit searching for it in the form of the expansion $z(k) = d_0 + ick + dk^2 + bk^3 + \ldots$. The first order of k enters the matrix $\mathbf{T}(k)$ only with the imaginary unit, hence all the contributions of odd orders, except for the first and third ones, will be imaginary. In particular, calculating the mode spectrum of the system for the case $\tau = \mathrm{const} = D_l/c_T^2$ to within k^3, we find

$$z_\pm = -D_l k^2/2 \pm ic_T k(1+\beta k^2),$$

$$z_3 = -\frac{c_\infty^2 - c_T^2}{D_l} + D_l k^2, \tag{28}$$

which does not conflict with the solution with corrections (23)–(24) of perturbation theory. The correction to the velocity of sound $\sim k^2$ can be obtained keeping the k^3 terms in the expansion of $z(k)$ and extracting the imaginary part of the corresponding contribution. The accurate result gives

$$\beta = \frac{D_l^2}{8}\frac{5 - c_\infty^2/c_T^2}{c_\infty^2 - c_T^2} = \frac{D_l^2}{2(c_\infty^2 - c_T^2)} + \frac{D_l^2(1 - c_\infty^2/c_T^2)}{8(c_\infty^2 - c_T^2)}, \tag{29}$$

where the first term coincides with correction (27) by perturbation theory derived from the general expression (17), while the second one $\sim c_\infty^2/c_T^2 - 1$ follows from the continuation of the expansion of $z(k)$ up to k^3 [see relation (21 and the text before it] and the known expansion $\sqrt{1+x} \simeq 1 + x/2$.

In addition to the positive dispersion of the velocity of sound, we note the appearance of the terms $\sim D_l k^2$ in the relaxation modes in the same way as in the hydrodynamic model.

3.3. Thermoviscoelastic model

This model can be considered as a generalization of the two previous ones and is based upon the collection of dynamic variables $\{n_\mathbf{k}, \dot{n}_\mathbf{k}, \ddot{n}_\mathbf{k}\} + \{h_\mathbf{k}\}$ or $\{n_\mathbf{k}, \dot{n}_\mathbf{k}, n_{3,\mathbf{k}}\} + \{h_\mathbf{k}\}$ taking thermal effects into account (in addition to viscoelastic ones), whose correlations can be supposed small and considered as a perturbation. Thus, the unperturbed model is chosen in the form $\{n_\mathbf{k}, \dot{n}_\mathbf{k}, n_{3,\mathbf{k}}\}$, while the cross correlations of the energy $h_\mathbf{k}$ with other variables are introduced as perturbations.

The generalized hydrodynamic matrix without cross correlations for such a model in the symmetrized basis has the form

$$\mathbf{T}_0(k) = \begin{pmatrix} 0 & ic_T k & 0 & 0 \\ ic_T k & 0 & -ik\sqrt{c_\infty^2 - c_T^2} & 0 \\ 0 & ik\sqrt{c_\infty^2 - c_T^2} & (\frac{c_\infty^2}{c_T^2} - 1)\frac{1}{\tau} & 0 \\ 0 & 0 & 0 & \frac{V}{C_V}\lambda k^2 \end{pmatrix}, \tag{30}$$

whereas the thermoviscous cross correlations are considered as perturbations

$$\delta\mathbf{T}(k) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\omega_{\dot{n}h} \\ 0 & 0 & 0 & \phi_{3h} \\ 0 & -i\omega_{h\dot{n}} & \phi_{h3} & 0 \end{pmatrix}. \tag{31}$$

Here, in contrast to the previous case, the correlation time τ remains finite in the limit $k \to 0$ even at $\gamma \neq 1$ and tends to D_l/c_T^2 in the hydrodynamic limit, while ϕ_{h3} and ϕ_{3h} stand for the memory functions constructed on the variables h and n_3.

As for the initial conditions, we assume that $c_\infty^2 - c_T^2$ is not small, and $k \ll k_1$ in the hydrodynamic limit. Using the accurate solution for the viscoelastic model (28) in the hydrodynamic limit, we obtain the following eigenvalues for the unperturbed model (30):

$$z_\pm^{(0)} = -D_l k^2/2 \pm ic_T k(1+\beta k^2),$$

$$z_3^{(0)} = -\frac{c_\infty^2 - c_T^2}{D_l} + D_l k^2,$$

$$z_4^{(0)} = -\frac{V}{C_V}\lambda k^2, \tag{32}$$

where β is determined from (29).

For the "unperturbed" eigenvectors normalized to 1, we find $\mathbf{x}_\pm^{(0)} = (1/\sqrt{2})\{\pm 1, 1, 0, 0\}$, $\mathbf{x}_3^{(0)} = \{0, 0, 1, 0\}$, and $\mathbf{x}_4^{(0)} = \{0, 0, 0, 1\}$.

The cross correlation matrix in the symmetrized basis will have non-zero elements at the corresponding cross-correlation positions.

The small parameter of perturbation theory is again presented by α_P/κ_T or the characteristic quantity $(\alpha_P^2 TV)/(\kappa_T C_V) = \gamma - 1$. The corrections to the eigenmodes of the system induced by the inclusion of the cross correlations (31) will be calculated with the help of the above expressions of perturbation theory (10). We have

$$\delta\bar{T}_{4\pm}\delta\bar{T}_{\pm 4} = -\frac{\alpha_P^2 TV}{2\kappa_T C_V} c_T^2 k^2, \quad \delta\bar{T}_{43}\delta\bar{T}_{34} = \phi_{h3}\phi_{3h} \sim k^4,$$

because the symmetrized functions ϕ_{h3} and ϕ_{3h} are proportional to k^2. After that, the use of expressions (10) and the accurate modes (32) allows one to obtain the corrections to the sound and thermal modes:

$$z_\pm = -D_l k^2/2 - \frac{\alpha_P^2 TV((V/C_V)\lambda + D_l/2)}{2\kappa_T C_V} k^2 \pm$$

$$\pm i c_T (1+\beta k^2)\left(1 + \frac{\alpha_P^2 TV}{2\kappa_T C_V}\right) k, \tag{33}$$

$$z_4 = -\frac{V}{C_V}\lambda k^2 \left(1 - \frac{\alpha_P^2 TV}{\kappa_T C_V}\right) - \frac{\alpha_P^2 TV}{\kappa_T C_V} D_l k^2. \tag{34}$$

In other words, we again observe the renormalization of the isothermal velocity into the adiabatic one $c_s = c_T\sqrt{\gamma}$, if we use the expansion $1 + x/2 \approx \sqrt{1+x}$, the renormalization of the damping coefficients, and replace $c_T^2 \tau k^2/2$ by $c_s^2 \tau k^2/2$. Moreover, we get the renormalization of the thermal capacity into $C_P = \gamma C_V$, by taking the relation $(1+x)^{-1} \approx 1 - x$ at small x into account. In this case, one can state that the correlation time is renormalized into $\tau_{k\to 0} \to D_l/c_s^2$. As for the correction to z_3, it will be proportional to k^4 in the hydrodynamic limit. It is worth noting that $z_3^{(0)} = \text{const} \neq 0$ in the limit $k \to 0$, which is typical of excitations of the kinetic type. For the thermal mode $z_4^{(0)}$, the obtained correction is proportional to k^2, which means the renormalization of the heat conduction caused by the mutual correlations of thermal and viscoelastic processes.

Thus, we finally find

$$z_\pm = -D_l k^2/2 - (\gamma - 1)\frac{V}{C_V}\lambda k^2 \pm i(c_s + \delta c_s^{(2)})k$$

$$z_3 = -\frac{c_\infty^2 - c_s^2}{D_l} + D_l^2 k^2$$

$$z_4 = -\frac{V}{C_P}\lambda k^2 - (\gamma - 1) D_l k^2. \tag{35}$$

As one can see, there appears a quadratic positive dispersion of the adiabatic velocity of sound induced by the viscoelastic correction (29). The consideration of the renormalization of the sound velocity into the adiabatic one yields the quadratic correction in the form

$$\delta c_s = c_s \frac{D_l^2}{8}\frac{5 - c_\infty^2/c_T^2}{c_\infty^2 - c_T^2} k^2, \tag{36}$$

which provides the positive dispersion of the adiabatic velocity of sound.

It is evident that a more accurate solution requires the study of the thermoviscoelastic model also including thermoelastic correlations $\ddot{n} - h$, as well as correlations with the enthalpy flux $\dot{h}$, i.e. we should start within the five-variable model [15]. This would expand and complicate the description due to the appearance of the effects of thermodiffusion and thermal elasticity.

4. Conclusions

Thus, the corrections to the generalized collective modes caused by the cross correlations between dynamic processes of different nature can be calculated with the use of perturbation theory. We propose a general formalism of such an approach in the dynamic theory of liquids. General expressions of perturbation theory up to the second order of magnitude inclusive are obtained.

It is shown that, in spite of the formal similarity of the constructed formalism to the stationary perturbation theory in quantum mechanics, the perturbation theory of this kind has some fundamental differences. First, the eigenvectors of the unperturbed kinetic matrix are not orthogonal in the general case, in contrast to the quantum-mechanical theory, where the orthogonality of the eigenvectors is ensured by the Hermitian character of the Hamiltonian. Second, the specific character of the structure of the generalized hydrodynamic matrix results in certain peculiarities in the calculation of the unperturbed eigenvectors. As a result, the first correction of perturbation theory to the collective modes of the system will be equal to zero, while the first nontrivial corrections will appear only in the second order.

The proposed perturbation theory is tested by the example of the simplest dynamic models of a simple liquid. It is shown that the results obtained in such a way correctly describe the effects of renormalization of the velocity of sound and the damping coefficients in the hydrodynamic limit, whereas the approximate analytical expressions are reduced to the known results when summing up the corresponding series of perturbation theory.

The obtained results give grounds to believe that there are good prospects to use perturbation theory in the proposed form for studying more complicated models, particularly those of multicomponent liquids.

1. T. Bryk and I. Mryglod, J. Phys. Cond Matt. **16**, L463 (2004).
2. I. Mryglod, T. Bryk, and V. Kuporov, in *Ionic Soft Matter: Modern Trends in Theory and Applications,*

edited by D. Henderson, M. Holovko, and A. Trokhymchuk, NATO Science Series II: Mathematics, Physics and Chemistry, Vol. 206 (Kluwer, Dordrecht, 2005), p. 109.
3. I. Mryglod and V. Kuporov, Ukr. Fiz. Zh. **53**, 908 (2008).
4. G.H. Wegdam, A. Bot, R.P. Schram, and H.M. Schaink, Phys. Rev. Lett. **63**, 2697 (1989).
5. J. Bosse, Phys. Rev. Lett. **57**, 3277 (1986).
6. S.C. Santucci, D. Fioretto, L. Comez, A. Gessini, and C. Masciovecchio, Phys. Rev. Lett. **97**, 225701 (2006).
7. C.-H. Chung, J. Kor. Nucl. Soc. **6**, 171 (1974).
8. C. Morkel and T. Bodensteiner, J. Phys. Cond. Matt. **2**, SA251 (1990).
9. T. Bryk and I. Mryglod, Cond. Matt. Phys. **10**, 481 (2007).
10. F.M. Kuni, *Statistical Physics and Thermodynamics* (Nauka, Moscow, 1981) (in Russian).
11. F.R. Gantmacher, *Theory of Matrices* (Chelsea, New York, 1960).
12. T. Bryk and I. Mryglod, Cond. Matt. Phys. **11**, 139 (2008).
13. J.P. Hansen and I.R. McDonald, *Theory of Simple Liquids* (Academic Press, London, 1986).
14. I. Mryglod, R. Folk, S. Dubyk, and Yu. Rudavskii, Physica A **277**, 389 (2000).
15. T. Bryk and I. Mryglod, Cond. Matt. Phys. **7**, 471 (2004).

Received 15.03.10.
Translated from Ukrainian by H.G. Kalyuzhna

ТЕОРІЯ ЗБУРЕНЬ ДЛЯ КОЛЕКТИВНИХ МОД
У ДИНАМІЦІ ПРОСТИХ ТА СКЛАДНИХ РІДИН

І.М. Мриглод, В.М. Купоров

Р е з ю м е

Формалізм теорії збурень використано для обчислення поправок до узагальнених колективних мод, викликаних слабкими перехресними кореляціями між тепловими та в'язкопружними динамічними процесами в простих та складних рідинах. Збудовано загальне формулювання такої теорії збурень до другого порядку включно. Проведено її апробацію на відомих найпростіших динамічних моделях рідин та виконано аналіз отриманих результатів порівняно з точними результатами для цих моделей. Показано, що результати теорії збурень дозволяють за відповідних умов відтворити відомі результати для цих моделей.