
N.N. BOGOLYUBOV AND DEVELOPMENT OF PHYSICAL KINETICS

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We present successively the ideas and the results of N.N. Bogolyubov concerning the construction of systems of equations for many-particle distribution functions. A special attention is given to the method of derivation of the kinetic equation for a one-particle distribution function. The results obtained earlier by Boltzmann and N.S. Krylov are discussed in Section 1 in detail. Section 2 presents works by N.N. Bogolyubov on the transition from reversible dynamical equations to the irreversible one for a one-particle distribution function. The Bogolyubov's scenario of a time development of many-particle distribution functions on the kinetic stage of the evolution of a gas and the Bogolyubov's perturbation theory for those functions are given. In particular, the well-known results of N.N. Bogolyubov and K.P. Gurov on the construction of a quantum kinetic equation are discussed. Here, we also present the main results of Peletminsky and Yatsenko in this direction and clarify their weighty contribution to the development of Bogolyubov's ideas in the field of physical kinetics. In Section 3, we survey some achievements of disciples and followers of N.N. Bogolyubov in Russia and in Ukraine. In particular, we consider the results of V.P. Silin on new spectra in a degenerate Fermi-gas and those of Peletminsky and Sokolovskii on the entropy of a gas in higher orders of perturbation theory.

1. Predecessors

Boltzmann and his kinetic equation

As is well known, the foundations of physical kinetics were laid by L. Boltzmann. The main point in his creative life was the derivation of the kinetic equation for the distribution function of atoms of the ideal gas (1872) and the formulation of the H -theorem (1872). While solving these problems, Boltzmann went by an unexplored way. He obtained the following equation:

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{x}} + \vec{F}_0 \frac{\partial f}{\partial \vec{p}} = L. \quad (1.1)$$

Here, f is the distribution function, \vec{F}_0 is the external force, and L is the collision integral, to which a separate section of the Boltzmann's "Lectures on the Theory of Gases" is devoted. It is worth noting that the substantiation of the kinematic part of Eq. (1.1) made by L. Boltzmann is not revised up to now. While introducing the collision integral, Boltzmann used the notions of impact parameter and effective scattering cross-section and the principle of detailed balancing. All these notions are used at present in the textbooks which include the notion of collision integral. In the derivation, Boltzmann made the following assumptions:

1. Hypothesis of molecular chaos. The events, where one finds an atom with a definite momentum (event 1) and with a definite coordinate (event 2) are statistically independent.
2. Collisions occur at a "point". This is the assumption that the interaction radius of atoms is significantly less than the free path length.
3. Collision time is significantly less than the free path time.

From these Boltzmann's hypotheses, central is the hypothesis of molecular chaos. Just it is related to the ideological component of the transition from the mechanical description of phenomena to the statistical one.

The Boltzmann's H -theorem presenting the molecular-kinetic pattern for the second law of thermodynamics entered into the gold fund of physics. In the due time, the intense discussion (Loschmidt, Zermelo) accompanied the H -theorem. It was justly indicated that this theorem contradicts the "full reversibility of mechanics" (Loschmidt). We recall that, in that time period, the results of Lyapunov on the problem of stability of a mechanical motion (1892) were not obtained. Of interest is

the Boltzmann's answer to that criticism: "Try to turn all atoms of a gas backward. Imagine how much time will be required".

At present, after the works of Lyapunov, we understand that a chaos can arise in a number of mechanical systems in the process of their motion. The assertion about the reversibility of a mechanical motion is not true in the general case.

N.S. Krylov and the problem of mechanical stability of the "ideal" gas

We now consider the results obtained by Nikolai Sergeevich Krylov which is an outstanding, in our opinion, predecessor of N.N. Bogolyubov. In his Doctoral dissertation "The Processes of Relaxation in Statistical Systems and the Criterion of Mechanical Instability" (1942), he solved the principal problem posed by Boltzmann, namely, the problem of chaos. He showed that the motion of atoms of a gas is unstable by Lyapunov due to collisions of atoms with one another and calculated the Lyapunov indices and the relaxation time of a gas.

If the interaction energy of atoms with one another is much less than the kinetic energy of an atom, then the relaxation time of the "ideal" gas is determined by the formula

$$t_r \approx \tau \frac{\ln(p(T)/\Delta p_0)}{\ln(l/r_0)}. \quad (1.2)$$

For the time t_r , the uncertainty of the angle between the initially parallel trajectories of atoms attains the value of 2π . This means the termination of the relaxation. In formula (1.2), τ is the free path time, $l = 1/n\sigma$ is the free path, σ is the scattering cross-section of atoms of the gas, r_0 is the atom radius, $p(T)$ is the mean momentum of an atom at a temperature T , and $\Delta p_0 \cong V^{-1/3}$ is the quantum uncertainty of the momentum in the system with volume V .

Omitting the strict proof of formula (1.2), we will present its clear explanation. Let the probing particles approach a fixed atom from a distance of the order of the free path length. Due to the uncertainty of the impact parameter $\Delta\rho \cong h/\Delta p$, the initially parallel trajectories after each collision separate by the angle equal by the order of magnitude to $\Delta\theta \approx r_0/l$. After n collisions, this uncertainty becomes equal to $(r_0/l)^n$. If this value will become equal to 2π , this will mean that the relaxation has happened. By noting that $n \cong t/\tau$, we get formula (1.2). This formula includes the ratio of the logarithms of two large values. It is easy to estimate

that $t_r \approx (3 - 4)\tau$ for a cube 1 cm^3 in volume at the temperature $T \approx 300 \text{ K}$.

In his dissertation, N.S. Krylov discussed also the problem of a variation of the phase volume form under the evolution of a state of the gas. As far as could be judged, N.S. Krylov was the first who advanced the idea of the "subdivision" of a phase volume without change in its connectedness and its value.

In our opinion, such an evolution is accompanied by the transformation of a compact phase volume into a fractal structure. We recall that the fractal structures and their properties were not yet discussed in 1942.

I note that some important results concerning the development of mathematical methods for the solution of the Boltzmann equation and for the derivation of the transfer equations were obtained in the works by Cowling, Enskog, and Chapman.

Such a situation held formally by 1946, when N.N. Bogolyubov published his monograph devoted to the problems of physical kinetics. I used the word "formally" in order to emphasize that the work by N.S. Krylov was not known outside of the physical community of the Leningrad University at that time. The dissertation of N.S. Krylov was published by the initiative of V.A. Fock only in 1950. The propaganda of works by N.S. Krylov started in the 1970–1980s (see, e.g., the book by G.M. Zaslavsky and R.Z. Sagdeev "Introduction to Nonlinear Physics. From a Pendulum to Turbulence and Chaos" (Nauka, Moscow, 1988), p. 115 (in Russian).

After the work by N.S. Krylov, some important problems related to the problem of chaos were clarified, but the further way of development of physical kinetics remained completely unknown. The following urgent questions were out of the discussion: Which form will the kinetic equation acquire in higher approximations of perturbation theory in the weak interaction of atoms with one another? Which stationary solution does the kinetic equation possess in higher approximations? Which fate will be destined for the Boltzmann's H -theorem in higher approximations of perturbation theory?

2. The Break-Through of N.N. Bogolyubov or "Problems of Dynamical Theory in Statistical Physics." The Year 1946

In the hard wartime, Bogolyubov thought, as N.S. Krylov, about the problems of physical kinetics. Naturally, he completely understood the eclectic character of the Boltzmann's approach to the derivation of his kinetic equation. This point was mentioned by Bogolyubov in the introduction to his monograph.

To construct the kinetic equation, Bogolyubov proposed an absolutely new method, in which the equations of the dynamics of a many-body system were regarded as of paramount importance, and all problems related to chaos in one way or another one were transferred to the properties of multiparticle distribution functions. Moreover, the multiparticle distribution functions themselves were introduced into kinetics by N.N. Bogolyubov. These functions are defined as follows:

$$f_s(x_1, \dots, x_s, t) = \frac{1}{(N-s)!} \int dx_{s+1} \dots dx_N D(x_1 \dots x_N, t). \quad (2.1)$$

In this formula, $D(x_1 \dots x_N, t)$ is the probability density of points in the phase space. The sense of this function consists in that the quantity

$$dw = D(x_1 \dots x_N, t) dx_1 \dots dx_N \quad (2.2)$$

determines the probability of the event that the coordinates and momenta of particles at the time moment t belong to the intervals $d^3x_1 d^3p_1, \dots, d^3x_N d^3p_N$.

The functions $f_s(x_1, \dots, x_s, t)$ are normalized in the following way:

$$\int dx_1 \dots dx_s f_s(x_1, \dots, x_s, t) = \frac{N!}{(N-s)!}. \quad (2.3)$$

The first postulate of Bogolyubov

The multiparticle functions $f_s(x_1, \dots, x_s, t)$ satisfy the principle of spatial weakening of correlations. Let us divide the system of s particles into two groups, $s = s' + s''$. Let $R(s')$ and $R(s'')$ be the coordinates of the centers of masses of these groups of atoms. As $R = |R(s') - R(s'')| \rightarrow \infty$, we have

$$f_s(x_1, \dots, x_s, t) \rightarrow f_{s'}(x'_1, \dots, x'_{s'}, t) f_{s''}(x''_1, \dots, x''_{s''}, t). \quad (2.4)$$

This property can be formulated also in terms of correlation functions. Along with multiparticle functions $f_s(x_1, \dots, x_s, t)$, let us introduce correlation functions $g_s(x_1, \dots, x_s, t)$ defined by the equalities

$$f_2(x_1, x_2, t) = f_1(x_1, t) f_1(x_2, t) + g_2(x_1, x_2, t),$$

$$f_3(x_1, x_2, x_3, t) = f_1(x_1, t) f_1(x_2, t) f_1(x_3, t) +$$

$$+ f_1(x_1, t) g_2(x_2, x_3, t) + f_1(x_2, t) g_2(x_1, x_3, t) + f_1(x_3, t) g_2(x_1, x_2, t) + g_3(x_1, x_2, x_3, t). \quad (2.5)$$

By virtue of the principle of spatial weakening of correlations, the correlation function $g_s(x_1, \dots, x_s, t)$ becomes zero if the condition $R = |R(s') - R(s'')| \rightarrow \infty$ is satisfied. If the interaction between atoms is pairwise, then the multiparticle distribution functions satisfy the chain of equations

$$\frac{\partial f_s}{\partial t} = \{H_s, f_s\} + \int dx_{s+1} \{H_{\text{int}}^s, f_{s+1}\}, \quad (2.6)$$

where H_s is the Hamiltonian of the system of s particles, H_{int}^s is the Hamiltonian of the interaction of the $(s+1)$ -th particle with all atoms of the system of s particles, and $\{\dots, \dots\}$ are the Poisson braces. The Hamiltonians H_s and H_{int}^s are

$$H_{\text{int}}^s = \Sigma V_{i,s+1};$$

$$H^s = \Sigma V_{i,j} + \Sigma \left(\frac{p_i^2}{2m} + U(x_i) \right). \quad (2.7)$$

The chain of equations (2.6) is called the chain of the Bogolyubov–Born–Green–Kirkwood–Yvon equations.

The Bogolyubov scenario of the evolution of multiparticle distribution functions or the basic hypotheses of Bogolyubov

For the derivation of the kinetic equation for a one-particle distribution function, Bogolyubov proposed the following scenario of the evolution of multiparticle distribution functions:

1. The defining role in a gas of atoms is played by the characteristic small time τ_0 which is equal by the order to magnitude to the collision time of atoms with one another, $\tau_0 \approx r_0/v$, where r_0 is the radius of an atom, and v is the mean velocity of atoms.

2. At times $0 < t < \tau_0$, the multiparticle distribution functions vary sufficiently rapidly and “remember” their initial values at $t = 0$.

3. At $\tau_0 \ll t$, the pattern of the evolution of multiparticle distribution functions f_s is essentially changed. For the time of the order of $(4-5)\tau_0$, the distribution functions f_s “forget” their initial values and then vary sufficiently slowly in the course of the time. For the same time, the properties of $f_1(x, t)$ are also changed. It forgets its initial value, forgets the correlation of

the momentum and the coordinate, and becomes a function which determines the probability $dw(\vec{x}, \vec{p}; t) = f_1(\vec{x}, \vec{p}; t)d\vec{x}d\vec{p}$ to find an atom in the interval $(\vec{x}, \vec{x} + d\vec{x})$ with a momentum in the interval $(\vec{p}, \vec{p} + d\vec{p})$.

4. Then the change of f_s is completely determined by the one-particle distribution function: $f_s = f_s(x_1, \dots, x_s, f_1(x, t))$. At great times, the system (functions f_s) remembers only the conservation laws.

5. The second characteristic time in a gas of atoms is the well-known free path time τ_r . It is the time interval, for which the one-particle distribution function $f_1(x, t)$ approaches its equilibrium value (see the result of N.S. Krylov).

The mathematical “design” of a scenario. The systems of kinetic equations of Bogolyubov

The kinetic equation determines a change of the function $f_1(x, t)$ on times t which are significantly greater than τ_0 , but less or of the order of the relaxation time τ_r . The chain of equations (2.6) for the one-particle distribution function yields the equation

$$\frac{\partial f}{\partial t} = \{H^1, f\} + \int dx_2 \{H_{\text{int}}^2, f_2\}. \tag{2.8}$$

This equation is easily transformed to the form

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{x}} + \vec{F}_0 \frac{\partial f}{\partial \vec{p}} = L,$$

$$L = \int dx_2 \{V(\vec{x}_1 - \vec{x}_2), f_2(x_1, x_2; f)\}. \tag{2.9}$$

In order to get a kinetic equation from the chain of equations for the multiparticle distribution functions, it is necessary to find f_2 . If we take f_2 in the form $f_2 = f(x_1; t)f(x_2; t)$, we include the self-consistent “field” in the kinetic equation [3, 4, 7]. In order to construct the collision integral, we need to know f_2 completely.

Bogolyubov found the general solution for multiparticle ($s \geq 2$) distribution functions in the form

$$f_s(x_1, \dots, x_s; f) = \prod_{1 \leq i \leq s} f(x_i) + \int_{-\infty}^0 d\tau S_0^s(\tau) K_s(x_1, \dots, x_s; S_0^1(-\tau) f(x)). \tag{2.10}$$

which is the solution of Eq. (1.8) with the boundary conditions

$$\lim_{\tau \rightarrow -\infty} S_0^s(-\tau) f_s(x_1, \dots, x_s, S_0^1(\tau) f_1(x, t)) =$$

$$= \prod_{1 \leq i \leq s} f_1(x_i, t). \tag{2.11}$$

We call attention to the fact that formula (2.10) includes the operator of evolution of a “free” system, i.e., a system without interaction. Finally, we present the formula for the operator $K_s(x_1, \dots, x_s; f(x))$:

$$K_s(x_1, \dots, x_s; f(x)) = \{V^s, f_s\} + \int dx_{s+1} \times \{H_{\text{int}}^{s+1}, f_{s+1}\} - \int dx \frac{\delta f_s}{\delta f(x; t)} L(x; t). \tag{2.12}$$

Equation (2.9), formula (2.10) for the multiparticle ($s \geq 2$) distribution functions, and formula (2.12) for the operator K_s complete the first, ideological, part of the construction of the equations of physical kinetics on the basis of dynamical equations. The following stage is to demonstrate that the system of equations (2.9), (2.10), and (2.12) gives the well-known expressions: the Boltzmann equation, the Vlasov equation, and the collision integral of Landau. All this was executed by N.N. Bogolyubov and entered his monograph published in 1946.

I wish to express my invariable admiration by the exciting scientific profundity of the system of kinetic equations of Bogolyubov.

In one year after monograph [3], N.N. Bogolyubov and K.P. Gurov published the article on the theory of quantum kinetic equations in Zh. Teor. Eksp. Fiz. [4]. In my opinion, this work has the exceptional meaning not only for kinetic theory, but for general problems of the physics of many-particle systems.

N.N. Bogolyubov and K.P. Gurov (1947). The collision integral in the third order of perturbation theory

In this approximation, the kinetic equation takes the form

$$\frac{\partial}{\partial t} f(x, p) + \frac{\partial E(x, p)}{\partial p} \frac{\partial}{\partial x} f(x, p) -$$

$$- \frac{\partial E(x, p)}{\partial x} \frac{\partial}{\partial p} f(x, p) = L_p^3(f),$$

$$L_p^3(f) = \frac{\pi}{2V^2} \sum |\tilde{\Phi}(p, p_2; p_3, p_4)|^2 \times$$

$$\times \Delta(p + p_2 - p_3 - p_4) \delta(E_p + E_2 - E_3 - E_4) \times$$

$$\times \{f_p f_2(1 + f_3)(1 + f_4) - f_3 f_4(1 + f_p)(1 + f_2)\}, \quad (2.13)$$

where

$$\begin{aligned} \tilde{\Phi}(1, 2; 3, 4) &= \Phi(1, 2; 3, 4) - \frac{i\pi}{2V} \sum \Phi(1, 2; 1', 2') \times \\ &\times \Phi(1', 2'; 1, 2)(1 + f_{1'} - f_{2'}) \delta_-(\varepsilon_3 + \varepsilon_{1'} - \varepsilon_{2'}) - \\ &- \frac{2i\pi}{V} \sum \Phi(1, 1'; 3, 2') \Phi(2, 2'; 4, 1')(f_{1'} - f_{2'}) \times \\ &\times \delta_-(\varepsilon_1 + \varepsilon_{1'} - \varepsilon_3 - \varepsilon_{2'}), \\ E_1 &= \varepsilon_1 + \frac{1}{V} \sum_2 \Phi(1, 2; 1, 2) f_2. \end{aligned} \quad (2.14)$$

I write the kinetic equation in the form given in work [5], where the collision integral was calculated in the third and fourth orders of perturbation theory. It is seen that the kinematic part of the kinetic equation preserves the same form as that for the ordinary kinetic equation. The essential changes occur in the energy of an atom of the gas and the transition amplitude.

Formulas (2.13) and (2.14) were first derived by N.N. Bogolyubov and K.P. Gurov in 1947. Their consideration allows one to conclude the following (conclusions by Bogolyubov–Gurov).

1. On the kinetic stage of evolution, the energy of a particle becomes a functional of the nonequilibrium one-particle distribution function. The correction to the energy coincides with that given by the formula of quantum mechanics at the occupation numbers equal to f .

2. The transition probability $\tilde{\Phi}(1, 2; 3, 4)$ also becomes a functional of the nonequilibrium one-particle distribution function. The correction to the transition probability coincides with that given by the formula of quantum mechanics at the occupation numbers equal to f .

Let us now cite the work by Bogolyubov and Gurov. “The collective effects are revealed, e.g., in the dependence of the transition probability and the energy on the occupation numbers. Equation (50) [Eqs. (2.13) and (2.14) of this text] could be interpreted, nevertheless, by the indicated scheme if we refer “collisions” not to individual molecules, but to some “elementary excitations”, for which the dependence on the momentum is presented by the formula $E(p) = T(p) + \frac{s}{V} \int v(|p - p'|) w(p') d^3 p'$.” Our calculation [5] showed that the conclusions by Bogolyubov–Gurov about the renormalization of the energy of an atom and the transition amplitude are completely valid.

3. Development of Bogolyubov's Ideas by His Disciples and Followers

Nikolai Nikolaevich Bogolyubov created a powerful scientific school. If we add his “out-of-sight” disciples and followers to the direct ones, the school becomes unbounded. It is impossible to enumerate all scientists influenced by N.N. Bogolyubov. I mention only scientists working in the field of physical kinetics and in some branches of the physics of condensed matter, though I am well acquainted and in friendly relations with such outstanding disciples of Bogolyubov as V.G. Kadyshchenskii, A.N. Sisakyan, and A.N. Tavkhelidze.

In this section, I will follow two principles: 1) chronology and 2) the significance of results. It is natural that I will present my subjective estimates.

For a number of decades, the significant contributions to the development of Bogolyubov's ideas in the field of physical kinetics and the physics of condensed matter were given by N.N. Bogolyubov (jr.), V.V. Belyi, V.L. Bonch-Bruевич, A.G. Zagorodny, D.N. Zubarev, Yu.L. Klimontovich, S.V. Peletminsky, N.M. Plakida, G. Röpke, Yu.G. Rudoi, A.A. Rukhadze, V.P. Silin, S.V. Tyablikov, A.V. Shelest, Yu.A. Tserkovnikov, I.R. Yukhnovsky, A.A. Yatsenko.

Each of these physicists enriched science by results of paramount importance. Many of them formed the own scientific schools. The survey of their achievements can be found in the article by N.M. Plakida, V.A. Zagrebnoy, Yu.G. Rudoi, and E.E. Tareeva “Nikolai Nikolaevich Bogolyubov and statistical mechanics” [6]. Here, we will consider the works of only five of the above-mentioned scientists.

I should like to mention a great influence of Nikolai Nikolaevich Bogolyubov (jr.), Sergei Vladimirovich Tyablikov, and Dmitrii Nikolaevich Zubarev on my scientific work. The scientific influence of D.N. Zubarev and S.V. Tyablikov is related to the Green's functions. At once after the publication of the article by N.N. Bogolyubov and S.V. Tyablikov concerning the Green's functions, I applied their method to the calculation of properties of many-sublattice ferrites. I say the same about the remarkable survey by D.N. Zubarev on the Green's functions in *Uspekhi Fiz. Nauk* in 1960. Equally important for me were the works by D.N. Zubarev on nonequilibrium statistical operators [8] which are close by a number of ideas to the activity of S.V. Peletminsky and his school in Kharkov. I have also a lot of discussions with Nikolai Nikolaevich Bogolyubov (jr.) on the problem of the connection of a symmetry of Hamiltonians with

properties of the energy spectra of elementary excitations.

Let us now return to the work by Bogolyubov–Gurov.

The Bogolyubov’s results concerning quasiparticles were to a great extent developed by V.P. Silin (see works [9–13]. In work [9] which is based on the Bogolyubov–Gurov article, Klimontovich and Silin advanced the idea of the calculation of dispersion laws for quasiparticles in a gas of fermions or bosons caused by the interaction of atoms of the gas, by applying a kinetic equation. I note that Vlasov showed earlier (in 1952) that the frequencies of plasma waves can be obtained from the equation with a self-consistent interaction between electrons and ions. Klimontovich and Silin started from the following kinetic equation:

$$\begin{aligned} & \frac{\partial}{\partial t} f - \frac{\vec{p}}{m} \frac{\partial}{\partial \vec{q}} f + \frac{i}{(2\pi)^3} \int d\vec{q}' d\vec{p}'' d\vec{p}' d\vec{k} \times \\ & \times \exp[i\vec{k}(\vec{q}'' - \vec{p})] f(\vec{q}', \vec{p}') f(\vec{q}, \vec{p}'') \left[U(|\vec{q} - \vec{q}' + \right. \\ & \left. + \left(\frac{1}{2}\right) \vec{k}|) - U(|\vec{q} - \vec{q}' - \left(\frac{1}{2}\right) \vec{k}|) \right] = I(f). \end{aligned} \quad (3.1)$$

Here, $U(|\vec{q} - \vec{q}'|)$ is the interaction energy of atoms, m is the atom mass, and $I(f)$ is the collision integral. Into this equation, let us substitute the distribution function in the form

$$f = f_0(\vec{p}) + \varphi(\vec{q}, \vec{p}; t), \quad (3.2)$$

where $f_0(\vec{p})$ is the equilibrium homogeneous distribution function, $I(f_0) = 0$, and $\varphi(\vec{q}, \vec{p}; t)$ describes weakly excited states of a gas. If the relaxation time is much more than the period of oscillations of elementary excitations corresponding to deviations from the equilibrium state, we can set $I(\varphi)$ to zero. In this case, it is easy to obtain the dispersion equation for elementary excitations. It takes the form [9]

$$\begin{aligned} & 1 - W(k) \int \left(f_0\left(\vec{p} + \left(\frac{1}{2}\right) \vec{k}\right) - f_0\left(\vec{p} - \left(\frac{1}{2}\right) \vec{k}\right) \right) \times \\ & \times \left[\frac{\vec{k}\vec{p}}{m} + \omega + \gamma \right]^{-1} d\vec{p} = 0. \end{aligned} \quad (3.3)$$

Here, $W(k)$ is the Fourier-transform of the interaction between atoms. The quantity $\gamma = 1/\tau$ models the action of the collision integral and serves for the removal of the uncertainty while calculating the integral

in formula (3.3). In the final results, the relaxation time $\tau \rightarrow \infty$. In [9], the authors obtained the Bogolyubov spectrum for a Bose gas of interacting atoms at $T = 0$ and the spectrum of collective oscillations at the zero temperature in the degenerate Fermi gas of neutral or charged particles.

Then V.P. Silin considered the spectra of excitations in the Bose and Fermi gases at finite temperatures, found their damping, calculated the parameters of collective waves for an electron gas, predicted the existence of spin waves in nonferromagnetic metals, and determined their frequencies and damping.

The results obtained by V.P. Silin present a significant development of both the theory of quantum kinetic equations and the ideas of the interaction of atoms and electrons with one another.

By highlighting the history of the region of the kinetic theory, I should like to say that V.P. Silin has played the extremely important role in the derivation of a series of new specific results on the basis of the Bogolyubov–Gurov kinetic equation in the approximation of self-consistent field, in the development of their result, and in its popularization.

Let us now return to monograph [3]. In the last section of the monograph, N.N. Bogolyubov analyzed the situation as for the collision integral in the case of the Coulomb interaction in a plasma. He noted that the cutting-off of a divergent integral in the formula for the collision integral was introduced by Landau “from the outside”. “It is clear that such a breaking of the integration which does not follow from the very derivation of the kinetic equation and is imposed “from the outside” cannot be considered satisfactory, and the question about the construction of an adequate kinetic equation for systems with the Coulomb interaction remains open” (N.N. Bogolyubov).

This problem was completely solved by V.P. Silin and A.A. Rukhadze in 1961 [13] who generalized the earlier results of Balescu (1960) and Lenard (1960). Silin and Rukhadze demonstrated that the result obtained by Landau must be corrected by means of the replacement of the Rutherford scattering cross-section by the “screened” scattering cross-section

$$d\sigma = d\sigma_{\text{poz}} |\varepsilon(qV, q)|^{-2}. \quad (3.4)$$

In this formula, $\varepsilon(\omega, \vec{q})$ is the high-frequency dielectric permittivity of a plasma with regard for the spatial dispersion, q is the momentum transferred at a collision, and V is the velocity of the center of inertia of the colliding particles. This is an absolutely nontrivial result involving not the static, but high-frequency dielectric

permittivity with regard for the spatial dispersion (see also [23]).

In the same 1961 year, V.P. Silin and A.A. Rukhadze described fluctuations in a nonequilibrium plasma and showed that the collision integral can be given in terms of the cross-section scattering of the electromagnetic field on fluctuations of a plasma [16]. Just this open a natural way for the introduction of the dielectric permittivity $\varepsilon(\omega, k)$ in the Landau collision integral. Within this approach, the authors made conclusion about the acceleration of relaxation processes near the regions of instability of thermodynamic phases (the regions of critical opalescence).

The important contribution to the development of the Bogolyubov's ideas in the field of physical kinetics was made by S.V. Peletminsky and his school. In 1967, the article by S.V. Peletminsky and A.A. Yatsenko "To the quantum theory of kinetic and relaxation processes" was published in Zh. Eksper. Teor. Fiz. [17] (see also [18]). This work is exceptionally rich by ideas and applications. Here, I present the main ideas of the work by Peletminsky and Yatsenko and the most significant results. We now consider a system with the Hamiltonian

$$H = H_0 + V. \quad (3.5)$$

Let there exist the collection of physical quantities in the system and the operators corresponding to them which are invariant under the commutation with the basic Hamiltonian:

$$[H_0, \hat{\gamma}] = a_{\alpha\beta} \hat{\gamma}_\beta. \quad (3.6)$$

As such quantities, I indicate the operator of the number of particles $a_p^+ a_p$ with momentum p , the operator of energy $\varepsilon(p) a_p^+ a_p$ of particles with momentum p , the operator creation (annihilation) of a pair of particles $a_p^+ a_p^+$ ($a_p a_p$), etc. An important specific feature of relation (3.6) is the possibility to determine the dependence of the operators on the time in the basic approximation (due to the operator H_0):

$$\hat{\gamma} = \exp[-i\hat{a}t] \hat{\gamma}(0). \quad (3.7)$$

It is assumed that a quasi-Gibbsian distribution ρ^0 corresponding to the given parameters γ is established on the first stage of the relaxation:

$$\exp[-iH_0\tau] \rho \exp[iH_0\tau]_{\tau \rightarrow \infty} \rightarrow \rho^0,$$

$$\rho^0 = \exp[\Omega(\gamma) - Y_\alpha(\gamma) \hat{\gamma}_\alpha]. \quad (3.8)$$

This distribution satisfies the conditions of normalization and the given values of the parameters γ :

$$\text{Sp} \rho^0 = 1, \quad \text{Sp} \rho^0 \hat{\gamma}_\alpha = \gamma_\alpha. \quad (3.9)$$

Further, Peletminsky and Yatsenko followed the Bogolyubov scenario. Namely, it was assumed that there exists a sufficiently small time τ_0 , after which the multiparticle distribution functions are simplified, and the coarsened density matrix $\sigma(\gamma(t, \rho))$ is formed. The temporal dependence of this coarsened density matrix is determined by that of the parameters γ . In the same way as in Section 1, the boundary condition for the coarsened operator $\sigma(\gamma(t, \rho))$ is formulated, and the integral equation

$$\sigma(\gamma) = \rho^0(\gamma) - \int_{-\infty}^0 d\tau e^{\varepsilon\tau} \exp[iH_0\tau] f(e^{-i\alpha\tau}) \exp[-iH_0\tau] \quad (3.10)$$

is obtained. In the last formula, the following notations are introduced:

$$\varepsilon \rightarrow +0, \quad f = [V, \sigma(\gamma)] - i \left(\frac{\delta}{\delta\gamma_\alpha} \sigma(\gamma) \right) L_\alpha;$$

$$L_\alpha(\gamma) = i \text{Sp} \sigma(\gamma) [V, \gamma_\alpha]. \quad (3.11)$$

The change of the parameters γ with time is described by the equation

$$\frac{\partial \gamma_\alpha}{\partial t} = i a_{\alpha\beta} \gamma_\beta + L_\alpha(\gamma). \quad (3.12)$$

In order to obtain the kinetic equation, it is necessary to take the operators $a_1^+ a_2$ as a collection of the quantities γ . Here, indices "1,2" serve for the notation of the momenta of atoms $1 \leftrightarrow p_1$ (the full collection of quantum numbers). The nondiagonal operator of the number of particles is considered by two reasons. In order to obtain the one-particle distribution function $f(p, x)$, it is necessary to have the Wigner matrix $f_{p-\frac{k}{z}, p+\frac{k}{z}}$. The second reason is related to the circumstance that we had learnt the technique of N.N. Bogolyubov by the recommendation of V.P. Silin and A.A. Rukhadze in connection with the problem of the relaxation in plasma in a strong magnetic field. This problem was posed for us by our teacher Aleksandr Il'ich Akhiezer. As a mechanism of relaxation, we took the mechanism of emission and absorption of

quanta of the electromagnetic field with the Larmor frequency ω_H by electrons. The process of emission and absorption of quanta $h\omega_H$ is described by nondiagonal elements of the distribution function. Omitting the results of calculations (I do not say “simple”, because they are not simple really), I will give the answer for a gas of bosons (fermions) in the second order of perturbation theory. I will follow works [5, 18]. In that works, we calculated the collision integral for a one-particle nondiagonal distribution function, the collision integral in the third order by perturbation theory, and the collision integral in the fourth order of perturbation theory for a homogeneous distribution function.

The collision integral in the second order of perturbation theory

$$\begin{aligned} \frac{\partial}{\partial t} f_{12} &= i(\varepsilon_1 - \varepsilon_2) f_{12} + \frac{\pi}{2V^2} \sum \Phi(1'', 2''; 3'', 4'') \times \\ &\times \Phi(1', 2'; 3', 4') \delta_-(\varepsilon_{1''} + \varepsilon_{2''} - \varepsilon_{3''} - \varepsilon_{4''}) \times \\ &\times \{ f_{4''2'} f_{3''1'} (\delta_{3'1''} + f_{3'1''}) (\delta_{12''} + f_{12''}) - \\ &- f_{3'1''} f_{12''} (\delta_{1'3''} + f_{1'3''}) (\delta_{4''2'} + f_{4''2'}) \} \delta_{4'2} + \text{h.c.} \end{aligned} \quad (3.13)$$

This equation is written for bosons. For fermions, it is necessary to make replacement $\delta_{12} + f_{12} \rightarrow (\delta_{12} - f_{12})$. The kinetic equation (3.13) else reminds slightly the ordinary kinetic equation for fermions or bosons. In order to pass to the standard form of this equation, it is necessary to multiply it by $\exp(-ikx) f_{p-k/2, p+k/2}$ and to integrate over k . As a result, we obtain

$$\begin{aligned} \frac{\partial}{\partial t} f(x, p) + \frac{\partial \varepsilon(x, p)}{\partial p} \frac{\partial}{\partial x} f(x, p) - \\ - \frac{\partial \varepsilon(x, p)}{\partial x} \frac{\partial}{\partial p} f(x, p) &= L_p^2(f), \\ L_p^2(f) &= \frac{\pi}{2V^2} \sum |\Phi(p, p_2; p_3, p_4)|^2 \Delta(p + p_2 - p_3 - p_4) \times \\ &\times \delta(\varepsilon_p + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \{ f_p f_2 (1 + f_3) (1 + f_4) - \\ &- f_3 f_4 (1 + f_p) (1 + f_2) \}. \end{aligned} \quad (3.14)$$

By expanding the collision integral in k , we can find the corrections to the dissipative terms in the transfer equations caused by the gradients of the distribution function. The kinetic equation in the third approximation in the weak interaction for a homogeneous distribution has the form (2.13), (2.14). I note that the main stages of the calculation of the collision integral and the kinetic equation (3.14) coincide with those in the above-mentioned work by Bogolyubov and Gurov (1947).

After our work which was published in 1968, we understood that the method of Bogolyubov allows one to derive a kinetic equation and a coarsened multiparticle distribution function in any order of perturbation theory. The following questions arose at once: Which are the entropy and the Green’s functions on the kinetic stage of the evolution? Which are the poles of Green’s functions, and how do these poles correspond to the results for the energy of particles obtained within the method of kinetic equations? These questions were formulated by S.V. Peletminsky, and he with his disciples obtained their brilliant solution.

Entropy on the kinetic stage of the evolution. S.V. Peletminsky, A.I. Sokolovskii (1974) [19]

After the formulation of the equation for the coarsened density matrix (3.10) and the kinetic equation (2.8) and after the definition of a collision integral, it would seem that the calculation of the entropy on the kinetic stage of the evolution is a simple problem. For its calculation, we must take the coarsened density matrix σ and use the Neumann formula

$$\tilde{S} = -\text{Sp} \sigma(f) \text{Log} \sigma(f). \quad (3.15)$$

In the zero approximation in the interaction, the entropy is given by the formula

$$S_0 = -\text{Sp} \rho^0(f) \text{Log} \rho^0(f) \quad (3.16)$$

and coincides with the well-known combinatorial distribution. In a sufficiently simple way, it is possible to show that entropy (3.15) contains no terms linear in interaction. On the kinetic stage of the evolution, the contribution to the entropy starts from the terms of the second order in the interaction. It is determined by the formula

$$\tilde{S}^2 = -\text{Sp} \sigma^1(f) (\text{Log} \sigma(f))^1. \quad (3.17)$$

In this formula, $\sigma^1(f)$ and $(\text{Log} \sigma(f))^1$ can be determined from the equation for the coarsened density matrix as

$$\sigma^1(f) = i \int_{-\infty}^0 d\tau e^{\varepsilon\tau} [\rho^0(f), V(\tau)];$$

$$(\text{Log}\sigma(f))^1 = i \int_{-\infty}^0 d\tau e^{\varepsilon\tau} [\text{Log}\rho^0(f), V(\tau)]. \quad (3.18)$$

These formulas jointly with formula (3.17) allow one seemingly to start the calculation of the entropy on the kinetic stage of the evolution in the second order in the interaction. However, we will meet, on this way, the problem of the divergence of integrals. Let us pay attention to formulas (3.18). These formulas include the integration over the time with the “decoding” given by the factor $\exp[\varepsilon\tau]$, $\varepsilon \rightarrow +0$. As a result, there appears the generalized functions $\delta_{\pm}(x)$. Their multiplication leads to the appearance of a factor $(\varepsilon^2 + x^2)^{-1}$, whose integration leads to a singularity. This situation means that formula (3.17) requires the regularization in the sense of the Bogolyubov–Parasiuk theorem. Such a regularization was performed in the work by Peletminsky and Sokolovskii, where the authors obtained the following formula for the entropy:

$$\begin{aligned} \tilde{S} &= S^0 + S^2, \\ S^0 &= \sum [(1 + f_1)\text{Log}(1 + f_1) - f_1\text{Log}f_1], \\ S^2 &= \frac{\pi}{8V^2} \text{Im} \sum |\Phi(1, 2; 3, 4)|^2 \delta'_-(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times \\ &\times [f_1 f_2 (1 + f_3)(1 + f_4) - (1 + f_1)(1 + f_2) f_3 f_4] \times \\ &\times \text{Log} \frac{(1 + f_1)(1 + f_2) f_3 f_4}{f_1 f_2 (1 + f_3)(1 + f_4)}. \end{aligned} \quad (3.19)$$

Peletminsky and Sokolovskii got also a formula for the coarsely structural entropy in higher orders of perturbation theory in the weak interaction. The general formula takes the form

$$S(f) = -R \text{Sp}\sigma(f) \text{Log}\sigma(f), \quad (3.20)$$

where R is the Bogolyubov–Parasiuk operation of regularization. This relation satisfies all the requirements. It coincides with the Gibbs equilibrium entropy and corresponds to the entropy maximum in the equilibrium state. Thus, we have

$$S = -\text{Sp}w \text{Log}w = S_{\text{eq}},$$

$$\left(\frac{\partial}{\partial f_1} S(f) - \beta \frac{\partial}{\partial f_1} H(f) + \beta \mu \right) \Big|_{f=n_1} = 0.$$

Here,

$$H(f) = \text{Sp}\sigma(f)H, \quad n_1 = \text{Sp}w a_1^\dagger a_1, \quad (3.21)$$

and w is the Gibbs equilibrium statistical operator.

It was proved in [19] that the entropy increases in the course of the time in any order of perturbation theory in the weak interaction. The exact theorem on the growth of the entropy in the process of evolution on the kinetic stage is one of the most significant results developing the Bogolyubov’s ideas.

Kinetic equation for a gas of solitons

In the consideration of this question, I will follow works [20–23] by I.V. Bar’yakhtar, V.G. Bar’yakhtar, and E.N. Economou.

Above, we considered the kinetic equations for a gas of atoms or ions and for electrons of a plasma. In collisions of atoms, a change in their coordinates Δx is significantly less than the distance, at which the one-particle distribution function varies, $\Delta x \ll l$, where l is the free path length. A change of the momentum in a collision is more or of the order of the mean (thermal) value. For this reason, an increase of the entropy of the gas of atoms is caused by the “mixing” in the momentum space. I.V. Bar’yakhtar noticed that the situation in collisions of solitons is opposite. In collisions, the momenta of solitons are invariable. Only the coordinates of solitons are changed. This circumstance was a push for three authors to construct the kinetics of of solitons.

We now recall the main properties of solitons. They are particle-like solutions of some nonlinear equations. For definiteness, let us consider the sine-Gordon equation

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} + m^2 \sin \varphi = 0. \quad (3.22)$$

This equation has the following solutions:

$$\begin{aligned} \varphi_k &= \pm 4 \arctg \left[\exp \left(\frac{x - v_k t - x_0}{\delta_k} \right) \right], \\ \varphi_b &= 4 \arctg \left[\frac{\omega_1 \sin[\omega_b t - k_b x - \varphi_0]}{\omega_2 \text{ch}[\delta_b^{-1}(x - v_b t - x_{0b})]} \right]. \end{aligned} \quad (3.23)$$

The first and second of these formulas give solutions named, respectively, a kink and a breather. In these formulas, δ_k and δ_b are linear sizes of a kink and a breather, v_k and v_b are their velocities, x_0 and x_{0b} are their initial coordinates, and ω_1 and ω_2 are the parameters of a

breather ($\omega_1^2 + \omega_2^2 = 1$). The energies of these objects are

$$E_k = \frac{8m}{\sqrt{1 - v_k^2}}; \quad E_b = \frac{16m\omega_2}{\sqrt{1 - v_b^2}}. \quad (3.24)$$

The momenta are connected with velocities by ordinary relativistic formulas. In collisions, the energies and momenta of solitons are conserved separately. In addition, the quantity named the “angular momentum”, $K = xE$, is conserved. Thus, the conservation laws under a collision of two kinks have the form

$$E_1 = E'_1, \quad E_2 = E'_2, \quad (3.25)$$

$$x_1 E_1 + x_2 E_2 = x'_1 E'_1 + x'_2 E'_2.$$

As for the coordinate of one of the kinks, its change due to the collision is

$$\Delta x_1 = x'_1 - x_1 = \frac{\delta_{1k}}{2} \text{Sig}(v) \text{Log} \left| \frac{1 + \sqrt{1 - v^2}}{1 - \sqrt{1 - v^2}} \right|,$$

$$v = \frac{v_1 - v_2}{1 - v_1 v_2}. \quad (3.26)$$

We now present the kinetic equation for kinks in the case where their density is small:

$$\frac{\partial f}{\partial t} + [v + \delta v] \frac{\partial f}{\partial x} = D \frac{\partial^2 f}{\partial x^2}. \quad (3.27)$$

Here,

$$\delta v = \int |v| \Delta x_{12} f(2, t) d2,$$

$$D = \frac{1}{2} \int |v| (\Delta x)_{12}^2 f(2, t) d2, \quad (3.28)$$

and v is the relative velocity of solitons.

We now comment upon the obtained kinetic equation and the renormalization of a velocity. Due to the specificity of the problem (the energy conservation law for each soliton in a collision), the Bogolyubov renormalization of the energy is absent. Instead, the velocity of a kink becomes a functional of the distribution function. In our case, this is an analog of the Bogolyubov–Gurov result. The result of the renormalization of a velocity admits the following obvious interpretation. In the collision of a probing kink with a slower kink, the former makes “a jump” along the motion direction. But if the

kink collides with a faster one, the former recoils backward, against the initial direction of its motion. For this reason, the velocity of the probing kink becomes a function depending on the distribution of solitons in space. Equation (3.27) has form of the diffusion equation. It is easy to see that the collision integral leads to the increase in the entropy of solitons

$$S = \int f(1 - \text{Log} f) d1. \quad (3.29)$$

Namely, we have

$$\frac{dS}{dt} = \int d1 d2 |v_{12}| (f_2/f_1) \left(\Delta x_{12} \frac{\partial f(1)}{\partial x_1} \right)^2 > 0. \quad (3.30)$$

I emphasize that the production of the entropy for solitons is related only to the “leveling” of inhomogeneities in the coordinate space. It is easy to prove that the most probable distribution in the momentum space is the Maxwell one. But the dynamics of solitons does not involve the mechanism of exchange by energy. A single known way to switch on it is to break the exact integrability.

For demonstration, I have considered specific features of the collision integral only for solitons of the kink type. In our works, we also analyzed the collision integrals for all three types of solitary nonlinear waves following from the sine-Gordon equation (kinks, breathers, and phonons). We also considered the collision integrals and the kinetic equations for solitons of a nonlinear Schrödinger equation [23], constructed the transfer equations, and calculated the coefficients of diffusion and heat conduction.

In conclusion, I mention two important, in my opinion, results obtained in Kyiv. First of all, I mean the results of A.G. Zagorodny on the physics of a dusty plasma consisting of electrons, ions, neutral atoms, and finely divided solid particles (dusts). By continuously absorbing electrons and ions from a plasma, dusts accumulate a great electric charge, whose value depends on the state of the surrounding plasma. The microscopic state of a dust is determined not only a coordinate and a momentum, but also its charge, which allows one to consider the charge of a dust as a dynamical variable. The use of the methods developed by N.N. Bogolyubov allowed to give a generalization of the BBGKY hierarchy for a dusty plasma and to propose the kinetic equations for plasma particles and dusts with regard for the absorption of electrons and ions by the dusty component. On the basis of such equations, the effective interaction potentials of dusts were calculated, and the

new effects related to the destruction of the screening of dusts and the appearance of negative friction were discovered.

The second result was obtained by V.F. Los'. In the work "Nonlinear generalized controlling equations and the account of initial correlations," he proved the matching of multiparticle distribution functions to the one-particle one for systems with "mixing". This work will be published in the jubilee issue of the journal "Teoret. i Matemat. Fizika" devoted to the centenary of the birthday of N.N. Bogolyubov.

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М.М. БОГОЛЮБОВ І РОЗВИТОК ФІЗИЧНОЇ КІНЕТИКИ

В.Г. Бар'якhtar

Резюме

Послідовно викладено ідеї та результати М.М. Боголюбова, що стосуються рівнянь для багаточастинкових функцій розподілу. Особлива увага надається методу побудови кінетичного рівняння для одночастинкової функції розподілу. У розділі 1 обговорено результати, отримані Л. Больцманом і М.С. Криловим. Розділ 2 присвячено роботам М.М. Боголюбова щодо переходу від оборотних динамічних рівнянь до необоротного динамічного рівняння для одночастинкової функції розподілу. Викладено сценарій Боголюбова для зміни багаточастинкових функцій розподілу у часі на кінетичній стадії еволюції газу та теорію збурень Боголюбова для цих функцій. Обговорено відому роботу М.М. Боголюбова і К.П. Гурова, де побудовано квантове кінетичне рівняння. Викладено також результати Пелетмінського і Яценка в цьому напрямі, що суттєво розвинули ідеї М.М. Боголюбова в галузі фізичної кінетики. В розділі 3 викладено досягнення учнів та послідовників М.М. Боголюбова в Росії і Україні. Зокрема, розглянуто роботи В.П. Сіліна, де отримано спектри виродженого фермі-газу, і роботи Пелетмінського і Соколовського щодо ентропії газу у вищих порядках теорії збурень.