BOGOLYUBOV APPROACH TO QUANTUM PLASMA KINETICS

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Starting with the quantum BBGKY-hierarchy for statistical operators, we have obtained a quantum non-Markovian kinetic equation including the dynamical screening of the interaction potential, which exactly takes into account the exchange scattering in a plasma. The collision integral is expressed in terms of the Green function of the linearized Hartree–Fock equation. This quantum kinetic equation corresponds to the law of total energy conservation with account of the polarization and exchange interaction.

1. Introduction

The famous Bogolyubov book "Problems of Dynamic Theory in Statistical Physics" [1] is a basis for the kinetic theory conceptions of gases, fluids, and a plasma. In the work of Bogolyubov and Gurov [2], the kinetic equation for the statistical operator of charged particles was obtained from a chain of equations for the density matrix. In works [3–6], a quantum kinetic equation that differs from the Bogolyubov–Gurov equation was obtained, by taking the medium polarization into account more properly. The corresponding classical equations were derived earlier by Balescu and Lenard [7–8]. In the quantum kinetic equation for weakly coupled polarizable plasmas which was derived by Balescu and Guernsey [5–6], the exchange interaction of particles was retained only in the distribution functions. But it is also necessary to consider the exchange interaction in the scattering amplitude and in the dielectric function. Moreover, the Balescu equation involves the polarization of the system only in the collision integral, while the thermodynamics corresponds to the ideal gas; the dissipative and non-dissipative phenomena are not treated on the equal footing. This discrepancy can be avoided if non-Markovian effects are considered. In the book by Kadanoff and Baym [9], general non-Markovian equations for two-time Green functions were derived. But, in the two-time Green function method, the question arose: which sort of relationship occurs between these functions and and the one-time distribution function? In other words, how should the time argument in the distribution function be chosen? In the non-Markovian case, this question is especially important. Kadanoff and Baym made their choice on the basis of a certain hypothesis (the KB ansatz). Klimontovich [10–11] proceeded from the BBGKY hierarchy for the one-time distribution functions, where no such problem arises. He obtained explicit expressions for non-Markovian Landau collision integrals and Boltzmann kinetic equations. Later on, the KB method and the BBGKY hierarchy were used in deriving non-Markovian quantum equations for weakly coupled systems and neutral particle systems with a very short interaction range [12–15]. In order to describe non-Markovian processes in a polarizable plasma, Klimontovich [16] derived a set of equations for the particle distribution function and the electric field spectral function. However, the equation for the spectral function was not solved, and the collision integral was obtained only in the averaged potential approximation. The non-linear kinetic equation, which generalizes the Balescu–Lenard equation [7–8] for a spatially uniform weakly non-ideal multicomponent classical plasma, has been obtained in [17–18] and, for a non-uniform plasma, in [19]. Starting from the quantum BBGKY-hierarchy for the distribution function, we have solved, in the so-called plasma approximation, the equation for the quantum pair correlation function, the non-Markovian correction being included. The solution to this equation can be expressed in terms of the resolvent of the linearized Hartree–Fock equation. As a result, we obtain a quantum non-Markovian kinetic equation, which involves both the dynamical screening of the interaction potential and the exchange interaction in a non-trivial way. In particular, this equation contains the dielectric function which exactly describes the exchange scattering in a plasma. The quantum kinetic equation derived satisfies the law of total energy conservation with regard for the polarization and the exchange interaction.

2. Quantum Non-Markovian Kinetic Equation

The quantum hierarchy for a multicomponent plasma in the operator techniques takes the form

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b Sp_{(2)}\left[U_{ab}(12), f_{ab}(12)\right], \tag{1}$$

$$\frac{\partial}{\partial t} f_{ab}(12) = [H_{ab}(12), f_{ab}(12)] +$$

$$+\sum_c Sp_{(3)}\left[U_{ac}(13) + U_{bc}(23), f_{abc}(123)\right], \tag{2}$$

where $f_a(1)$ and $f_{ab}(12)$ are one- and two- particle density matrices, $[A, B]$ is the commutator of operators.

$$H_a(1) = \frac{p^2(1)}{2m_a} \text{ is the kinetic energy,} \tag{3}$$

$$H_{ab}(12) = \frac{p^2(1)}{2m_a} + \frac{p^2(2)}{2m_b} + U_{ab}(12) \tag{4}$$

is the two-particle Hamiltonian, and $U_{ab}(12)$ is the two-particle interaction potential.

Let us introduce the new operators [20]:

$$f_{ab}(12) = \gamma_{ab}(12) f'_{ab}(12), \tag{5}$$

$$f_{abc}(123) = \gamma_{abc}(123) f'_{abc}(123), \tag{6}$$

where the symmetrization operators are

$$\gamma_{ab}(12) = 1 + \delta_{ab}\eta_a P(12), \tag{7}$$

$$\gamma_{abc}(123) = \gamma_{ab}(12)\left\{1 + \delta_{ac}\eta_a P(13) + \delta_{bc}\eta_b P(23)\right\}, \tag{8}$$

$\eta_a = 1(Bose), -1(Fermi)$; $P(12)$ is the permutation operator. Therefore,

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] +$$

$$+\sum_b Sp_{(2)}\left[U_{ab}(12), \gamma_{ab}(12) f'_{ab}(12)\right], \tag{9}$$

$$\frac{\partial}{\partial t} f'_{ab}(12) = [H_{ab}(12), f'_{ab}(12)] +$$

$$+\sum_c Sp_{(3)}[U_{ac}(13) + U_{bc}(23), (1 + \delta_{ac}\eta_a P(13) +$$

$$+\delta_{bc}\eta_b P(23)) f'_{abc}(123)]. \tag{10}$$

The symmetrization operators (7) and (8) are convenient in that they give the possibility to partially transmit the permutation operator $P(12)$ from the density matrix to the interaction potentials. The density matrices $f_{ab}(12)$ *etc.* possess the quantum symmetry properties: $P(12) f_{ab}(12) = f_{ab}(12) P(12)$ *etc.*, whereas the density matrices $f'_{ab}(12)$ *etc.* possess only the classical symmetry properties: $P(12) f'_{ab}(12) P(12) = f'_{ab}(12)$ *etc.* For the classically symmetric density matrices, the usual conditions for the disentanglement of equations, being the same as those in the classical statistics, hold. Specifically, in the plasma approximation [21], when the triple correlation function is neglected, we have

$$f'_{ab}(12) = f_a(1) f_b(2) + g'_{ab}(12), \tag{11}$$

$$f'_{abc}(123) = f_a(1) f_b(2) f_c(3) +$$

$$+g'_{ab}(12) f_c(3) + g'_{ac}(13) f_b(2) + g'_{bc}(23) f_a(1), \tag{12}$$

where $g'_{ab}(12)$ is the pair correlation function. By substituting Eqs. (5), (6), (11), and (12) into Eqs. (1) and (2),

we obtain a closed set of equations for the one-particle and two-particle statistical operators:

$$\frac{\partial}{\partial t} f_a(1) = [H'_a(1), f_a(1)] +$$

$$+\sum_b Sp_{(2)}\left[U'_{ab}(12), \gamma_{ab}(12) g'_{ab}(12)\right], \tag{13}$$

$$\frac{\partial}{\partial t}g'_{ab}(12)=[H'_a(1)+H'_b(1),g'_{ab}(12)]+A'_{ab}(12)+$$

$$+\sum_c Sp_{(3)}\{[U'_{bc}(23),f_b(2)g'_{ac}(13)]+$$

$$+[U'_{ac}(13),f_a(1)g'_{bc}(23)]\},\qquad(14)$$

where

$$H'_a(1)=\frac{p^2(1)}{2m_a}+U_a(1),\qquad(15)$$

$$U_a(1)=\sum_b Sp_{(2)}\,[U'_{ab}(12),f_b(2)]\equiv U_a^H(1)+U_a^F(1),\qquad(16)$$

$$U'_{ab}(12)=\gamma_{ab}(12)U_{ab}(12),\qquad(17)$$

$$i\hbar A'_{ab}(12)=[1+\eta_a f_a(1)][1+\eta_b f_b(2)]U_{ab}(12)f_a(1)f_b(2)-$$

$$-f_a(1)f_b(2)U_{ab}(12)[1+\eta_a f_a(1)][1+\eta_b f_b(2)],\qquad(18)$$

$$U_a^H(1)=\sum_b Sp_{(2)}\,[U_{ab}(12),f_b(2)]\qquad(19)$$

is the Hartree field, i.e. the mean self-consistent field and

$$U_a^F(1)=\sum_b Sp_{(2)}\delta_{ab}\eta_a P(12)\,[U_{ab}(12),f_b(2)]\qquad(20)$$

is the Fock field, i.e. the mean field taking only the exchange interaction into account (Pauli's principle). In the plasma approximation [21], the term $[U'_{ab}(12),g'_{ab}(12)]$ in Eq. (13) which describes the direct interaction of two particles (1,2) is not taken into account. Let us consider the homogeneous case. In the Wigner representation, the kinetic equation (13) takes the form

$$\frac{\partial}{\partial t}f_a(\mathbf{p})=J_a(\mathbf{p})=2\hbar^2\sum_b\int d\mathbf{p}'d\mathbf{k}[U_{ab}(\mathbf{k})+$$

$$+\delta_{ab}\eta_a U_{ab}(\mathbf{p}'-\mathbf{p})]g'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k}).\qquad(21)$$

Here, the spin variables are omitted for simplicity. The solution of the equation for the pair correlation function $g'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k})$ in a non-Markovian approximation in the Wigner form can be expressed in the spatially homogeneous case in terms of the resolvent of Eq. (14) and its source (18) as

$$g'_{ab}(p,p',\mathbf{k},t)=(1-\frac{\partial}{\partial t}\frac{\partial}{\partial\Delta})\sum_{a'b'}\int d\mathbf{q}d\mathbf{q}'\times$$

$$\times R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)A'_{a'b'}(\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)\lfloor_{z=0},\qquad(22)$$

$$z=\omega+i\Delta,$$

$$i\hbar A'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k},\mu t)=$$

$$=U_{ab}(\mathbf{k})\{f_a(\mathbf{p})f_b(\mathbf{p}')[1+\eta_a f_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})][1+\eta_b f_b(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2})]-$$

$$-f_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})f_b(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_a(\mathbf{p})][1+\eta_b f_b(\mathbf{p}')]\qquad(23)$$

with the resolvent $R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)$ in Eq. (22), being a product of two resolvents

$$R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},t-t',\mu t')=$$

$$=R_{aa'}(\mathbf{p},\mathbf{q},\mathbf{k},t-t',\mu t')R_{bb'}(\mathbf{p}',\mathbf{q}',\mathbf{k},t-t',\mu t')\qquad(24)$$

which satisfy the linearized Hartree–Fock equation

$$[\hbar z+\Delta_k E_a(\mathbf{p})]R_{aa'}(\mathbf{p},\mathbf{q},\mathbf{k},z,t)=$$

$$=\delta_{aa'}\delta(\mathbf{p}-\mathbf{q})+e_a\Delta_k f_a(\mathbf{p})\sum_c e_c\int d\mathbf{p}'\times$$

$$\times[\Phi(\mathbf{k})+\delta_{ac}\eta_a\Phi(\frac{\mathbf{p}-\mathbf{p}'}{\hbar})]R_{ca'}(\mathbf{p}',\mathbf{q},\mathbf{k},z,t),\qquad(25)$$

where

$$U_{ab}(\mathbf{k})=e_a e_b\Phi(\mathbf{k}),$$

$$\Delta_k E_a(\mathbf{p})=E_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})-E_a(\mathbf{p}-\frac{\hbar\mathbf{k}}{2});$$

$$\Delta_k f_a(\mathbf{p})=f_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})-f_a(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}),\qquad(26)$$

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$$E_a(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_a} + \eta_a \int d\mathbf{p}' U_{aa}(\frac{\mathbf{p}-\mathbf{p}'}{\hbar}) f_a(\mathbf{p}'). \tag{27}$$

The solution of Eq. (25) takes the form

$$R_{aa'}(\mathbf{p},\mathbf{p}',\mathbf{k},z,t) = \frac{\Gamma_a(\mathbf{p},\mathbf{p}')\delta_{aa'}}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')} +$$

$$+\frac{\Phi(\mathbf{k})}{\varepsilon^{\mathrm{HF}}(\omega,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p})\Psi_{a'}^{(2)}(\mathbf{p}'), \tag{28}$$

where we introduced the notations

$$\Psi_a^{(1)}(\mathbf{p}) = e_a \int d\mathbf{p}'' \frac{\Gamma_a(\mathbf{p},\mathbf{p}'')\Delta_k f_a(\mathbf{p}'')}{\hbar z - \Delta_k E_a(\mathbf{p}'')}, \tag{29}$$

$$\Psi_{a'}^{(2)}(\mathbf{p}') = e_{a'} \int d\mathbf{p}'' \frac{\Gamma_{a'}(\mathbf{p}'',\mathbf{p}')}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')}, \tag{30}$$

and

$$\varepsilon^{\mathrm{HF}}(\omega,\mathbf{k}) = 1 - \Phi(\mathbf{k})\sum_a e_a^2 \int d\mathbf{p}d\mathbf{p}' \frac{\Gamma_a(\mathbf{p},\mathbf{p}')\Delta_k f_a(\mathbf{p}')}{\hbar z - \Delta_k E_a(\mathbf{p}')} \tag{31}$$

is the dielectric function with exchange interaction.

The exchange scattering amplitude $\Gamma_a(\mathbf{p},\mathbf{p}')$ for Eqs. (28)–(31) satisfies an integral equation which contains only the exchange interaction potential,

$$\Gamma_a(\mathbf{p},\mathbf{p}') = \delta(\mathbf{p}-\mathbf{p}')+$$

$$+e_a^2\eta_a \frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \int d\mathbf{p}'' \Phi(\frac{\mathbf{p}-\mathbf{p}''}{\hbar})\Gamma_a(\mathbf{p}'',\mathbf{p}'). \tag{32}$$

The quantity $\Gamma_a(\mathbf{p},\mathbf{p}')$ depends on $\mathbf{k}$ and z as on parameters and is similar to the vertex function well-known in many-particle perturbation theory. Formulae (28)–(32) yield the general expression for the pair correlation function in a non-Markovian approximation with the complete description of the polarization and the exchange interaction of particles:

$$g'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k},t) = -\frac{i}{\hbar}(1-\frac{\partial}{\partial t}\frac{\partial}{\partial \Delta})\Phi(\mathbf{k})\sum_c e_c^2 \int dz d\mathbf{q}\times$$

$$\times f_c(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_c(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]\Big\{\frac{\Psi_a^{(1)}(\mathbf{p})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}[\frac{\Gamma_b(\mathbf{p}',\mathbf{q})\delta_{bc}}{\hbar z - \Delta_k E_c(\mathbf{q})}+$$

$$+\frac{\Phi(\mathbf{k})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}\Psi_b^{(1)}(\mathbf{p}')\Psi_c^{(2)}(\mathbf{q})]^* -$$

$$-\frac{\Psi_b^{*(1)}(\mathbf{p}')}{\varepsilon^{*HF}(z,\mathbf{k})}[\frac{\Gamma_a(\mathbf{p},\mathbf{q})\delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})}+$$

$$+\frac{\Phi(\mathbf{k})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p})\Psi_c^{(2)}(\mathbf{q})]\Big\}. \tag{33}$$

The expressions for the collision integral and the internal energy take the forms

$$J_a(p) = -2\hbar(1-\frac{\partial}{\partial t}\frac{\partial}{\partial \Delta})\mathrm{Im}i\sum_{bc} e_a e_b e_c^2 \int \Phi(\mathbf{k})d\mathbf{p}'d\mathbf{k}dzd\mathbf{q}\times$$

$$\times[\Phi(\mathbf{k}) + \delta_{ab}\eta_a\Phi(\frac{\mathbf{p}'-\mathbf{p}}{\hbar})]f_c(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_c(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]\times$$

$$\times\Big\{\frac{\Psi_a^{(1)}(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}[\frac{\Gamma_b(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2},\mathbf{q})\delta_{bc}}{\hbar z - \Delta_k E_c(\mathbf{q})}+$$

$$+\frac{\Phi(\mathbf{k})\Psi_b^{(1)}(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2})\Psi_c^{(2)}(\mathbf{q})^*}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}] - \frac{\Psi_b^{*(1)}(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2})}{\varepsilon^{*HF}(z,\mathbf{k})}\times$$

$$\times[\frac{\Gamma_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2},\mathbf{q})\delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})\Psi_a^{(1)}(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})\Psi_c^{(2)}(\mathbf{q})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}]\Big\}, \tag{34}$$

$$U = 2\mathrm{Re}i\sum_{abc}\int \frac{e_a e_b e_c^2}{\hbar} d\mathbf{p}d\mathbf{p}'d\mathbf{k}dzd\mathbf{q}\Phi(\mathbf{k})\times$$

$$\times[\Phi(\mathbf{k})+\delta_{ab}\eta_a\Phi(\frac{\mathbf{p}'-\mathbf{p}}{\hbar})]f_c(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_c(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]\times$$

$$\times\frac{\Psi_b^{*(1)}(\mathbf{p}')}{\varepsilon^{*HF}(z,\mathbf{k})}\Big[\frac{\Gamma_a(\mathbf{p},\mathbf{q})\delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})\Psi_a^{(1)}(\mathbf{p})\Psi_c^{(2)}(\mathbf{q})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}\Big]. \tag{35}$$

The complete description of the exchange interaction of particles is reduced to the solution of the linear integral equation (32).

3. The Law of Total Energy Conservation

We now demonstrate that the derived kinetic equation (34) satisfies the law of total energy conservation taking the potential energy U (35) into account. We have

$$\frac{\partial}{\partial t}(E^{\mathrm{HF}} + U) = 0, \tag{36}$$

where $E^{\rm HF}$ is the sum of the kinetic energy of the system and the self-consistent field energy:

$$E^{\rm HF}=\sum_a\int d\mathbf{p}f_a(\mathbf{p})\frac{\mathbf{p}^2}{2m_a}+$$

$$+\frac{1}{2}\sum_a\int d\mathbf{p}d\mathbf{p}'U_{aa}(\frac{\mathbf{p}-\mathbf{p}'}{\hbar})f_a(\mathbf{p})f_a(\mathbf{p}'). \quad (37)$$

The second term in Eq. (37) describes the energy of the exchange interaction of particles. The self-consistent Hartree field vanishes due to the spatial homogeneity of the system. It is not difficult to see that

$$\frac{\partial}{\partial t}E^{\rm HF}=\sum_a\int d\mathbf{p}E_a(\mathbf{p})\frac{\partial}{\partial t}f_a(\mathbf{p}), \quad (38)$$

where $E_a(\mathbf{p})$ is defined by Eq. (27). Let us multiply Eq. (21) by $E_a(\mathbf{p})$, integrate over the momentum, and sum up over the plasma components. With regard for Eq. (22), we obtain

$$\frac{\partial}{\partial t}E^{\rm HF}=2\hbar^2\sum_{ab}\int d\mathbf{p}d\mathbf{p}'d\mathbf{k}E_a(\mathbf{p})[U_{ab}(\mathbf{k})+$$

$$+\delta_{ab}\eta_aU_{ab}(\mathbf{p}'-\mathbf{p})](1-\frac{\partial}{\partial t}\frac{\partial}{\partial\Delta})\sum_{a'b'}\int d\mathbf{q}d\mathbf{q}'\times$$

$$\times R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)A^{'}_{a'b'}(\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)\lfloor_{z=0}. \quad (39)$$

Equation (25) yields the integral equation for resolvent (24):

$$R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)=$$

$$=\frac{1}{\hbar z+\Delta_kE_a(\mathbf{p})+\Delta_kE_b(\mathbf{p}')}\times$$

$$\times\Big\{\delta_{aa'}\delta_{bb'}\delta(\mathbf{p}-\mathbf{q})\delta(\mathbf{p}'-\mathbf{q}')+e_a\Delta_kf_a(\mathbf{p})\sum_ce_c\int d\mathbf{p}_1\times$$

$$\times[\Phi(\mathbf{k})+\delta_{ac}\eta_a\Phi(\frac{\mathbf{p}-\mathbf{p}_1}{\hbar})]R_{cb,a'b'}(\mathbf{p}_1,\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)+$$

$$+e_b\Delta_kf_b(\mathbf{p}')\sum_ce_c\int d\mathbf{p}_2[\Phi(\mathbf{k})+\delta_{bc}\eta_b\Phi(\frac{\mathbf{p}-\mathbf{p}_2}{\hbar})]\times$$

$$\times R_{ac,a'b'}(\mathbf{p},\mathbf{p}_2,\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)\Big\}. \quad (40)$$

The derivative $\frac{\partial}{\partial\Delta}$ of expression (40) gives two terms. For $z=0$, we get

$$\frac{\partial}{\partial\Delta}R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)=$$

$$=-i\hbar\Big(\frac{1}{\Delta_kE_a(\mathbf{p})+\Delta_kE_b(\mathbf{p}')}\Big)^2\Big\{\delta_{aa'}\delta(\mathbf{p}-\mathbf{q})\delta_{bb'}\delta(\mathbf{p}'-\mathbf{q}')+$$

$$+e_a\Delta_kf_a(\mathbf{p})\sum_ce_c\int d\mathbf{p}_1[\Phi(\mathbf{k})+\delta_{ac}\eta_a\Phi(\frac{\mathbf{p}-\mathbf{p}_1}{\hbar})]\times$$

$$\times R_{cb,a'b'}(\mathbf{p}_1,\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)+$$

$$+e_b\Delta_kf_b(\mathbf{p}')\sum_ce_c\int d\mathbf{p}_2[\Phi(\mathbf{k})+\delta_{bc}\eta_b\Phi(\frac{\mathbf{p}-\mathbf{p}_2}{\hbar})]$$

$$R_{ac,a'b'}(\mathbf{p},\mathbf{p}_2,\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)+$$

$$+\frac{1}{\Delta_kE_a(\mathbf{p})+\Delta_kE_b(\mathbf{p}')}\{e_a\Delta_kf_a(\mathbf{p})\sum_ce_c\int d\mathbf{p}_1\times$$

$$\times[\Phi(\mathbf{k})+\delta_{ac}\eta_a\Phi(\frac{\mathbf{p}-\mathbf{p}_1}{\hbar})]\frac{\partial}{\partial\Delta}R_{cb,a'b'}(\mathbf{p}_1,\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)+$$

$$+e_b\Delta_kf_b(\mathbf{p}')\sum_ce_c\int d\mathbf{p}_2[\Phi(\mathbf{k})+\delta_{bc}\eta_b\Phi(\frac{\mathbf{p}-\mathbf{p}_2}{\hbar})]\times$$

$$\times\frac{\partial}{\partial\Delta}R_{ac,a'b'}(\mathbf{p},\mathbf{p}_2,\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)\}. \quad (41)$$

Substituting Eq. (41) into Eq. (40) and symmetrizing the integral, it is not difficult to see that only the first term in Eq. (41) gives a nonzero contribution to Eq. (40), and the right-hand size of Eq. (40) represents itself the derivation of the potential energy, which is determined by Eq. (35). As a result, we obtain the law of total energy conservation (36)

4. Kinetic Equation with Averaged Exchange Interaction

The collision integral (34) and the internal energy (35) are expressed by the amplitude of the scattering interaction $\Gamma_a(\mathbf{p},\mathbf{p}')$ which satisfies the linear integral equation (32). The solution of this equation in case of the Coulomb interaction of particles is difficult and requires an appropriate approximation. The simplest approximation is the replacement of the integrand in (32) by the value averaged over the momentum,

$$\Phi(\mathbf{k})G(z,\mathbf{k}) = \int d\mathbf{p}''\Phi(\frac{\mathbf{p}-\mathbf{p}''}{\hbar})\Gamma_a(\mathbf{p}'',\mathbf{p}'). \quad (42)$$

Then the dielectric function with regard for the exchange interaction of particles takes the form

$$\varepsilon^{\mathrm{HF}}(z,\mathbf{k}) = 1 - P(z,\mathbf{k})[1 + P(z,\mathbf{k})G(z,\mathbf{k})]^{-1}, \quad (43)$$

where

$$P(z,\mathbf{k}) = \Phi(\mathbf{k})\sum_a e_a^2 \int d\mathbf{p}\frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \quad (44)$$

is the polarization. In the special case of the Hubbard approximation [22], we have

$$G(z,\mathbf{k}) = \frac{1}{2}\frac{k^2}{k^2 + k_f^2}. \quad (45)$$

One form of $G(z,\mathbf{k})$ was found for the equilibrium state using a variation procedure [23–25]:

$$G(z,\mathbf{k}) = \frac{8\pi^2 e^4\hbar^4}{k^2}\frac{1}{P^2(z,\mathbf{k})}\int d\mathbf{p}\int d\mathbf{p}'\frac{\Delta_{\mathbf{k}} f(\mathbf{p})\Delta_{\mathbf{k}} f(\mathbf{p}')}{|\mathbf{p}-\mathbf{p}'|^2}\times$$

$$\times\frac{1}{\hbar z - \Delta_k E_a(\mathbf{p})}[\frac{1}{\hbar z - \Delta_k E_a(\mathbf{p}')} - \frac{1}{\hbar z - \Delta_k E_a(\mathbf{p})}]. \quad (46)$$

Using the expression for the pair correlation function, we find the Markovian collision integral as

$$J_a(\mathbf{p}) = 4\pi^2 e^4\int \Phi^2(\mathbf{k})(1 - G(z,\mathbf{k}))d\mathbf{k}d\mathbf{q}\times$$

$$\times\frac{\delta(\Delta_k E(\mathbf{q}) - \Delta_k E(\mathbf{p}))}{\left|\widetilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar,\mathbf{k})\right|^2}\times$$

$$\times\{f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})f(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})]-$$

$$-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})f(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]\}. \quad (47)$$

The internal energy of particles takes the form

$$U = e^4\int d\mathbf{p}d\mathbf{p}'d\mathbf{k}\frac{\Phi^2(\mathbf{k})}{\Delta_k E(\mathbf{q})-\Delta_k E(\mathbf{p})}\frac{1}{\left|\widetilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar,\mathbf{k})\right|^2}\times$$

$$\times\{f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})f(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2})]-$$

$$-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})f(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2})]\}, \quad (48)$$

where

$$\widetilde{\varepsilon}(z,\mathbf{k}) = 1 - (1 - G(z,\mathbf{k}))P(z,\mathbf{k}). \quad (49)$$

It follows from the expessions for the internal energy of particles (48) and for the collision integral (47) that $\left|\widetilde{\varepsilon}(z,\mathbf{k})\right|^2$ plays the role of the screening of the interaction potential $\Phi(\mathbf{k})$. It is of interest that Eqs. (47) and (48) differ from the corresponding Balescu's expressions by taking the exchange interaction in this screening into account. Moreover, the collision integral (47) contains the additional renormalization of the interaction $(1 - G(z,\mathbf{k}))$. However, $\widetilde{\varepsilon}(z,\mathbf{k})$ does not serve as a linear response function, in contrast to the Hartree–Fock dielectric function $\varepsilon^{\mathrm{HF}}(z,\mathbf{k})$ in Eq. (43).

In the equilibrium state, expression (48) satisfies the fluctuation-dissipation theorem since

$$\mathrm{Im}\frac{P(z,\mathbf{k})}{\left|\widetilde{\varepsilon}(z,\mathbf{k})\right|^2} = \mathrm{Im}\frac{P(z,\mathbf{k})}{\varepsilon^{\mathrm{HF}}(z,\mathbf{k})}. \quad (50)$$

5. Conclusion

Using the operator technique within the BBGKY hierarchy, we have obtained a closed set of equations for the one- and two-particle density matrices, referring to the plasma approximation which considers also the exchange interaction. The equation for the pair correlation function is solved with the help of the resolvent of the Hartree–Fock equation. The expression obtained for the pair correlation function takes the non-Markovian correction and the exchange interaction into account. The latter is described by the scattering amplitude which obeys the integral equation formulated above. The expression for the time-dependent non-local collision integral and the internal energy are obtained with regard for the exchange interaction and the polarization. The quantum kinetic equation derived satisfies the law of total energy conservation involving the polarization and the exchange interaction. The Hubbard approximation was generalized to the non-equilibrium case.

1. N.N. Bogolyubov, *Problems of Dynamic Theory in Statistical Physics* (FITTL, Moscow-Leningrad, 1946) (in Russian).
2. N.N. Bogolyubov and K.P. Gurov, Sov. Phys. JETP **17**, 614 (1947).
3. O.V. Konstantinov and V.L. Perel, Sov. Phys. JETP **39**, 861 (1960).
4. V.P. Silin, Sov. Phys. JETP **40**, 1768 (1961).
5. R. Balescu, Phys. of Fluids **4**, 94 (1961).
6. R.L. Guernsey, Phys. Rev. Lett. **127**, 1446 (1962).
7. R. Balescu, Phys. of Fluids **3**, 52 (1960).
8. A. Lenard, Ann. Phys. (NY) **10**, 390 (1960).
9. L.P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1964).
10. Yu.L. Klimontovich, Sov. Phys. JETP **30**, 1352 (1970).
11. Yu.L. Klimontovich, *Kinetic Theory of Nonideal Gases and Nonideal Plasma* (Academic Press, New York, 1975).
12. Yu.L. Klimontovich, and W. Ebeling, Sov. Phys. JETP **36**, 476 (1973).
13. K. Morawetz, and G. Röpke, Phys. Rev. E **51**, 4246 (1995).
14. Th. Bornath, D. Kremp, and M. Schlanges, Phys. Rev. E **54**, 3274 (1996).
15. D. Kremp, M. Bonitz, W.D. Kraeft, M. Schlanges, Ann. of Phys. (NY) **258**, 320 (1997).
16. Yu.L. Klimontovich, Sov. Phys. JETP **35**, 920 (1972).
17. V.V. Belyi, Yu.A. Kukharenko, and J. Wallenborn, Phys. Rev. Lett. **76**, 3554 (1996).
18. V.V. Belyi, Yu.A. Kukharenko, and J. Wallenborn, J. Plasma Phys. **59**, 657 (1998).
19. V.V. Belyi, Yu.A. Kukharenko, and J. Wallenborn, Contrib. Plasma Phys. **42**, 3 (2002).
20. N.N. Bogolyubov, J. Phys. (Moscow) **11**, 23 (1947).
21. R. Balescu, *Statistical Mechanics of Charged Particles* (Wiley-Interscience, New York, 1963).
22. J. Hubbard, Proc. Roy. Soc. A **243**, 336 (1958).
23. F. Brosens, L.M. Lemmens, and J.T. Devrees, Phys. Stat. Sol. (b) **74**, 45 (1976).
24. F. Brosens, J.T. Devrees, and L.M. Lemmens, Phys. Stat. Sol. (b) **80**, 99 (1977).
25. F. Brosens, L.M. Lemmens, and J.T. Devrees, Phys. Stat. Sol. (b) **81**, 551 (1977).

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БОГОЛЮБОВСЬКИЙ ПІДХІД ДО КІНЕТИКИ КВАНТОВОЇ ПЛАЗМИ

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Р е з ю м е

Методом М.М. Боголюбова одержано немарківське кінетичне рівняння для квантової системи заряджених частинок з точним врахуванням поляризації та обмінної взаємодії частинок. Інтеграл зіткнень виражено через функції Гріна лінеаризованого рівняння Хартрі–Фока. Квантове кінетичне рівняння відповідає закону збереження повної енергії із врахуванням поляризації та обмінної взаємодії частинок.