FILLING UP THE FERMI SPHERE WITH QUASIPARTICLES OF QUARKS

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The quark behavior while influenced by a strong stochastic gluon field is analyzed. An approximate procedure for calculating the effective Hamiltonian is developed. Considering quarks as the quasiparticles of a model Hamiltonian with four-fermion interaction, we study the response to the process of filling up the Fermi sphere with quarks.

1. Introduction

In the present paper, we analyze the process of filling up the Fermi sphere with quarks modeling the corresponding Slater determinant in the direct way. The quarks are considered as quasiparticles of a model Hamiltonian with four-fermion interaction. In [1], the ground state of this model Hamiltonian was studied in detail, and the singularity of a mean energy functional depending on the current quark mass was found out. In the course of this study, we need to trace back the alterations of respective dressing transformations. It will be seen, from what follows, that this problem is fairly difficult to be treated with the well-developed machinery of the Green functions [2], in which a system is described by the Hamiltonian with additional term controlling the particle number $\mu\ \bar{q}\gamma_4 q$ (μ is the chemical potential of bare quarks). However, the quark chemical potential conception runs with some uncertainties in interpreting a baryon chemical potential [3] and quantitatively produces the arbitrariness of order 50–100 MeV for the phase diagram. Furthermore, each kind of quarks requires its particular chemical potential. Here, we also consider the impact of filling up the Fermi sphere on the process of quasiparticle formation. Actually, the approach proposed should be appended by an analysis of the influence of (meson) bound states. But here we formulate this problem only doing hope to get a justification of such a position while completing our consideration. Let us recall that the conceptual idea of an intricate nature of the QCD vacuum [4] having populated by intensive stochastic gluon fields of nontrivial topological structure forms the basis to explain the quark behavior. Moreover, studying the cooled corresponding lattice configurations gives evidence of this component presence [5], and using the instantons in the singular gauge to fit the data turns out to be very fruitful [6] and allows one to evaluate the ensemble density and the characteristic size of a saturating configuration. Both estimates are in fairly good agreement with the corresponding results of the instanton liquid model [7]. Nevertheless, the keen search of various confining configurations is still going on [8–10] in parallel with collecting the convincing evidences that the construction of a self-consistent ensemble of such configurations is a too complicated problem (see, for example, the estimate for the (anti)instanton ensemble done in [11]). Supposing the high-frequency component of a stochastic ensemble of gluon fields as the dominating contribution, we develop, in fact, an effective theory (which usually encodes the predictions of quantum field theory at low energies), in which all assumptions done in the way to construct it are not of special importance. What is entirely restrictive to fix the effective action at a really low energy (i.e. the low cutoff) up to a few coupling constants, to develop the approximate procedure to analyze the quark interactions and to introduce the corresponding low-energy effective variables is the idea to neglect all the contributions coming from gluon fields generated by the (anti)quarks (quenched approximation). Actually, this means the removal of the corresponding cutoff(s) from consideration; but, by the definition of an effective theory, this operation does not pose itself. Nowadays we know that the mixing of the zero modes is the microscopic mechanism of the sponteneous breakdown of chiral symmetry in the instanton liquid model [12]. In this approach, the quarks are considered in a given gluon background, and the spectrum of the respective Dirac operator is calculated in order to be accompanied then by averaging over the gluon ensemble. It is believed that the zero modes are effectually overlapped at low energy, and the eigenvalues of Dirac operator spread over some range of virtualities. In other words, investigating the behavior of a single quark influenced by an external (stochastic) gluon field, one endeavours to guess how its Green function in the quark ensemble looks like. Clearly, it is a very difficult task to follow starting from the first principles, and many various approximations are developed and adapted. It should be noted also that a great care should be taken in order to obtain the proper thermodynamical limit with nonzero chiral condensate. A lot of that happens to be in striking contrast to some aspects to the Nambu–Jona-Lasinio (NJL) model [13] which is cognate to the instanton liquid model based actually on the similar multifermion interaction. Superficially, the main distinction consists in the appearence of some non-local formfactors instead of the corresponding coupling constant. As to the microscopic consideration, the generation of dynamical quark mass in the NJL model is caused by the reconstruction of the Hamiltonian ground state, and the quarks manifest themselves already as the quasiparticles [14] although the multifermion attractive force should be strong enough, roughly speaking. We will emphasize here that the instanton model and several other models which are based on treating the stochastic ensemble of a strong gluon field become practically identical in many aspects to the NJL model.

2. The Hartree–Fock–Bogolyubov Approximation

We start with briefly recalling how the quasiparticle concept appears in this approach. Let us consider the quark (antiquark) ensemble in the background of a strong stochastic gluon field and suppose this field to be so strong that we could neglect the gluon interchanging processes. The stochastic gluon field is characterized by a correlation function, and its particular form will be discussed and fixed below. The Lagrangian density is

$$\mathcal{L}_E = \bar{q}(i\gamma_\mu D_\mu + im)q, \quad (1)$$

where q and $\bar{q}$ are the quark and antiquark fields with covariant derivative $D_\mu = \partial_\mu - igA^a_\mu t^a$, A^a_μ is the gluon field, $t^a = \lambda^a/2$ are the generators of the color gauge group $SU(N_c)$, m is the current quark mass, and $\mu = 1, 2, 3, 4$. We work in the context of the Euclidean field theory, and γ_μ mean the Hermitian Dirac matrices ($\gamma^+_\mu = \gamma_\mu$, $\{\gamma_\mu, \gamma_\nu\} = 2\,\delta_{\mu\nu}$) in the chiral representation. Then the corresponding Hamiltonian description results from

$$\mathcal{H} = \pi\dot{q} - \mathcal{L}_E, \quad \pi = \frac{\partial \mathcal{L}_E}{\partial \dot{q}} = iq^+, \quad (2)$$

and, in particular, for the noninteracting fields, we have

$$\mathcal{H}_0 = -\bar{q}(i\boldsymbol{\gamma}\nabla + im)q. \quad (3)$$

In the Schrödinger representation, the quark field evolution is determined by the equation for the quark probability amplitude Ψ as

$$\dot{\Psi} = -H\Psi, \quad (4)$$

and the creation and annihilation operators of quarks and antiquarks $a^+, a,\ b^+, b$ have no "time" dependence and consequently look like

$$q_{\alpha i}(\boldsymbol{x}) = \int \frac{d\boldsymbol{p}}{(2\pi)^3} \frac{1}{(2|p_4|)^{1/2}} \times$$

$$\times [a(\boldsymbol{p}, s, c) u_{\alpha i}(\boldsymbol{p}, s, c) e^{i\boldsymbol{px}} +$$

$$+ b^+(\boldsymbol{p}, s, c) v_{\alpha i}(\boldsymbol{p}, s, c) e^{-i\boldsymbol{px}}], \quad (5)$$

where the summation over index s which stands to describe two quark spin polarizations and index c which should play the similar role for a color is implied. Further, we take a specific form of the Dirac conjugated spinor. Fixing a spin polarization can be done, as known, by imposing an additional constraint on the spinor (see below). However, there is no direct analogy with the color polarization, and the particular state should be fixed by the corresponding complete set of diagonal operators which includes the Casimir operators as well. In fact, this complete definition of the spinor color state is unnecessary for us here. All observables are usually expressed by summing up the polarization states of some bilinear spinor combinations as the singlet and octet states, and the singlet component is obviously playing the specific role. The density of interaction Hamiltonian can be presented as

$$\mathcal{V}_S = \bar{q}(\boldsymbol{x}) t^a \gamma_\mu A^a_\mu(t, \boldsymbol{x}) q(\boldsymbol{x}). \quad (6)$$

The obvious dependence on "time" in this Hamiltonian is present in the gluon field only. As is mentioned above, we are planning to work with the stochastic gluon field implying the random process, for which one may define only a probability of realizing some gluon configuration. Such a nature of the gluon field urges (and allows) us to develop the approximate procedure for describing the quark field treating (4) as a probabilistic process. Then the system states are described by the corresponding averages (over a "time"or an ensemble according to the ergodic hypothesis). However, in quantum theory, we are faced with one difficulty in this way, because Ψ is a probability amplitude, and the immediate averaging of $\langle\Psi\rangle$ can be insignificant. Studying a mean probability density $\langle\overset{*}{\Psi}\Psi\rangle$ looks more promising and can be realized by complicating the procedure of continual integration [15]. Adapting these ideas to the gauge theories, we should obviously strive to operate with the gauge invariant quantities which include an ordered exponential, at least. Unfortunately, such a program in what concerns the ensemble consideration is still very far to be realized. However, it is clear that applying the averaging procedure would result in the appearance of a set of corresponding correlation functions $\langle A^2\rangle$, $\langle A^4\rangle$, *etc.* For example, the spontaneous breaking of chiral symmetry is well understood in the instanton liquid model just due to such a trick [16], [17]. It is interesting to note here that the correlation function series summed up is expressed in the highest order of the packing fraction parameter $n\bar{\rho}^4$, where n is the instanton liquid density and $\bar{\rho}$ is the mean size of (anti)instanton, with the covariant derivative in the field of each separate (anti)instanton and includes also the free Green function. Actually, we believe that, when calculating the ground state, it might be informative and reliable to operate with averaging just the probability amplitude $\langle\Psi\rangle$. In the interaction representation, where $\Psi = e^{H_0 t}\Phi$, Eq. (4) can be rewritten as

$$\dot{\Phi} = -V\Phi, \quad V = e^{H_0 t}V_S e^{-H_0 t}. \tag{7}$$

Now the 'time' dependence appears in quark operators as well. After the averaging over the highly oscillating (short-wavelength) component, we obtain the equation

$$\langle\dot{\Phi}(t)\rangle = +\int_0^\infty d\tau\langle V(t)V(t-\tau)\rangle\ \langle\Phi(t)\rangle \tag{8}$$

for the long-wavelength part. (The requirements to validate the factorization of the long-wavelength component are discussed, for example, in [18].) The integration interval in Eq. (8) is extended to the infinite "time" value because of the rapid decrease (supposed) of the correlation function, and its sign is strictly fixed. Within these assumptions, we are also allowed to deal with $\langle\Phi(t)\rangle$ on the right-hand side of equation and to have, as a result, an ordinary differential equation instead of an integro-differential one. Implementing approximation (8) in the quantum field theory models, we run into the trouble at trying to get the most general form of a correlation function if the characteristic quark and gluon correlation times are comparable. Fortunately, if the quark fields are considered to be practically constant on the gluon background, the problem receives the essential simplification. The gluon field contribution may be factorized as a corresponding correlation function $\langle A^a_\mu(x)A^b_\nu(y)\rangle$ [19], and it seems that such a conclusion could be applied for the ground state. Recent lattice measurements of this correlation function provide us with a reasonable arguments to interpret the result as the gluon "mass" generation ($\sim$ 300 – 400 MeV) in the momentum region of order 200 MeV [20]. It is curious to notice that the averaging over ensemble ("time") on the right-hand side of Eq. (8) is performed in both the correlator and $\langle\Phi(t)\rangle$. This means that, by resumming and averaging a certain class of diagrams in the quantum field theory models, one may consider high-order correlator contributions in different ways if the form of the function $\langle\Phi(t)\rangle$ is specified. In addition, the correlation functions in models interesting to us should be translation-invariant, and this implies that the correlator in Eq. (8) has the form

$$\langle V(t)V(t-\tau)\rangle = F(\tau),$$

i.e., for example, an one-dimensional process after having done the integration in Eq. (8) will be described by a constant which characterizes the slow process. In quantum field theory for the problem we are interested in, the correlator connecting two space points

$$\langle\dot{\Phi}(t)\rangle = \int d\boldsymbol{x}\bar{q}(\boldsymbol{x},t)t^a\gamma_\mu q(\boldsymbol{x},t)\int_0^\infty d\tau\times$$

$$\times\int d\boldsymbol{y}\bar{q}(\boldsymbol{y},t-\tau)t^b\gamma_\nu q(\boldsymbol{y},t-\tau)g^2\times$$

$$\times\langle A^a_\mu(t,\boldsymbol{x})A^b_\nu(t-\tau,\boldsymbol{y})\rangle\langle\Phi(t)\rangle$$

appears instead of a constant. Assuming the correlation function is rapidly decreasing with time, we change the "time" $t-\tau$ dependence in the quark fields for t and perform the inverse transformation to the Schrödinger representation (but, in principle, it could be done in a covariant form as well). Then introducing the function $\chi = e^{-H_0 t}\langle\Phi\rangle$, we have [1] the equation

$$\dot{\chi} = -H_{\rm ind}\chi,$$

[1] Let us notice that this form does not coincide in general with $\langle e^{-H_0 t}\Phi\rangle$.

$$\mathcal{H}_{\rm ind} = -\bar{q}(i\gamma\boldsymbol{\nabla} + im)q - \bar{q}t^a\gamma_\mu q\times$$

$$\times\int d\boldsymbol{y}\bar{q}'t^b\gamma_\nu q'\int\limits_0^\infty d\tau g^2\langle A^a_\mu A'^b_\nu\rangle, \quad (9)$$

where $q = q(\boldsymbol{x})$, $\bar{q} = \bar{q}(\boldsymbol{x})$, $q' = q(\boldsymbol{y})$, $\bar{q}' = \bar{q}(\boldsymbol{y})$, $A^a_\mu = A^a_\mu(t,\boldsymbol{x})$, and $A'^b_\nu = A^b_\nu(t-\tau,\boldsymbol{y})$. In order to get the final result, we should fix the form of the correlation function. In this paper, we rely on the stochastic ensemble of (anti)instantons in the singular gauge, and the instanton solution reads as

$$A^a_\mu(x) = \frac{2}{g}4\pi^2 i\rho^2\omega^{ab}\bar{\eta}_{\mu b\nu}\int\frac{dq}{(2\pi)^4}q_\nu\phi(q)e^{iq(x-z)},$$

$$\phi(q) = \frac{1}{q^2}\left(K_2(q\rho) - \frac{2}{q^2\rho^2}\right), \quad (10)$$

where K_2 is the modified Bessel function of imaginary argument, ρ is the instanton size, the matrix ω appoints the pseudo-particle orientation in the color space, z is the coordinate of instanton center, and $\bar{\eta}$ stands for the 't Hooft symbol. In Eq. (9), we imply the correlation function integrated over the "time", for which we obtained, in the highest order in the density n of the (anti)instanton ensemble,

$$\int\limits_0^\infty dx_4\langle A^a_\mu(x)A^b_\nu(y)\rangle = \frac{1}{2}\int\limits_{-\infty}^\infty dx_4\langle A^a_\mu(x)A^b_\nu(y)\rangle =$$

$$= \frac{4(4\pi^2)^2}{g^2}\frac{\delta_{ab}n\rho^4}{N_c^2-1}(\delta_{\mu\nu}\delta_{\alpha\beta} - \delta_{\mu\alpha}\delta_{\nu\beta})\times$$

$$\times\int\frac{dp}{(2\pi)^4}p_\alpha p_\beta e^{ip(x-y)}\phi(-p)\phi(p)\frac{1}{2}2\pi\delta(p_4). \quad (11)$$

The first equality is valid due to the symmetry properties of the instanton solution. Then the correlation function can be presented as

$$\widetilde{\langle A^a_\mu A^b_\nu(\boldsymbol{p})\rangle} = \frac{(4\pi^2)^2\ n\rho^4}{g^2}\ \frac{2\ \delta_{ab}}{N_c^2-1}\ [I(p)\ \delta_{\mu\nu} - J_{\mu\nu}(p)]$$

$$I(p) = \boldsymbol{p}^2\phi(-p)\phi(p), \quad J_{ij}(p) = p_i p_j\phi(-p)\phi(p),$$

$$J_{4i} = J_{i4} = J_{44} = 0. \quad (12)$$

In what follows, we suppose that the various stochastic ensembles of gluon fields are characterized by their profile functions $I(p)$ and $J_{\mu\nu}(p)$ and analyze the contribution of a quadratic correlator only. However, this deficiency of fixing the gauge implicitly for the truncated system is compensated, in a sense, by our investigation of the full spectrum of reasonable correlation functions (including the opposite limiting correlators when they are extrapolated even into the peturbative region). With such a form of the induced four-fermion interaction, we are going to search the ground state as the Bogolyubov probe function with vacuum quantum numbers[2] [21]

$$|\sigma\rangle = T|0\rangle,$$

$$T = \Pi_{p,s,c}\exp\{\varphi[a^+(\boldsymbol{p},s,c)b^+(-\boldsymbol{p},s,c)+$$

$$+a(\boldsymbol{p},s,c)b(-\boldsymbol{p},s,c)]\}, \quad (13)$$

which is defined by minimizing the mean energy

$$E = \langle\sigma|H|\sigma\rangle. \quad (14)$$

Here, $\varphi = \varphi(\boldsymbol{p})$, and $|0\rangle$ is the vacuum of the free Hamiltonian, i.e. $a(\boldsymbol{p},s,c)|0\rangle = 0$, $b(\boldsymbol{p},s,c)|0\rangle = 0$. With the dressing T transformation, we introduce the creation and annihilation operators of quasiparticles

$$A = TaT^{-1}, \quad B^+ = Tb^+T^{-1},$$

we present the operator Eq. (5) as

$$q(\boldsymbol{x}) = \int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{1}{(2|p_4|)^{1/2}}\times$$

$$\times[A(\boldsymbol{p},s,c)U(\boldsymbol{p},s,c)e^{i\boldsymbol{px}} + B^+(\boldsymbol{,}p,s,c)V(\boldsymbol{p},s,c)e^{-i\boldsymbol{px}}], \quad (15)$$

where the spinors U and V are defined as

$$U(\boldsymbol{p},s,c) = \cos(\varphi)u(\boldsymbol{p},s,c) - \sin(\varphi)v(-\boldsymbol{p},s,c),$$

$$V(\boldsymbol{p},s,c) = \sin(\varphi)u(-\boldsymbol{p},s,c) + \cos(\varphi)\ v(\boldsymbol{p},s,c). \quad (16)$$

Then the Dirac conjugate spinor takes the form

$$\bar{q}(\boldsymbol{x}) = \int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{1}{(2|p_4|)^{1/2}}\left[A^+(\boldsymbol{p},s,c)\overline{U}(\boldsymbol{p},s,c)e^{-i\boldsymbol{px}}+\right.$$

[2] In order to avoid any misunderstanding, we recall here that fixing a form of the ground state introduces a primary frame.

$$+B(\boldsymbol{p},s,c)\overline{V}(\boldsymbol{p},s,c)e^{i\boldsymbol{px}}\big], \quad (17)$$

with $\overline{U}(\boldsymbol{p},s,c) = U^{+}(\boldsymbol{p},s,c)\gamma_4$ and $\overline{V}(\boldsymbol{p},s,c) = V^{+}(\boldsymbol{p},s,c)\gamma_4$. Now we have to specify the choice of spinors in the Euclidean variables. They obey the Dirac equations

$$(\hat{p}-im)u(p,s)=0, \quad (\hat{p}+im)v(p,s)=0, \quad (18)$$

(with $\hat{p}=p_4\gamma_4+\boldsymbol{p\gamma}$) and the additional constraint which fixes the spinor polarization:

$$i\gamma_5\hat{s}u(p,s)=u(p,s), \quad i\gamma_5\hat{s}v(p,s)=v(p,s). \quad (19)$$

Here, $\gamma_5=-\gamma_1\gamma_2\gamma_3\gamma_4$, and the four-vector s is normalized to 1 and orthogonal to the four-vector p, i.e. $s^2=1$, $(ps)=0$. It could be, for example,

$$s_4=\frac{(\boldsymbol{pn})}{im}, \quad \boldsymbol{s}=\boldsymbol{n}+\frac{(\boldsymbol{pn})\boldsymbol{p}}{im(p_4-im)},$$

where $\boldsymbol{n}$ is an arbitrary unit vector. If the covariant normalization conditions

$$\bar{u}u=2im, \quad \bar{v}v=-2im \quad (20)$$

are satisfied, the spinors are defined with the precision up to the phase factor. All these conditions allow us to formulate the following matrix representation:

$$u(p,s)\bar{u}(p,s)=\frac{\hat{p}+im}{2}(1+i\gamma_5\hat{s}),$$

$$v(p,s)\bar{v}(p,s)=\frac{\hat{p}-im}{2}(1+i\gamma_5\hat{s}). \quad (21)$$

Calculating the mean energy (14), we meet spinors with opposite moments. We introduce the four-vector $q=(p_4,-\boldsymbol{p})$ in order to simplify notations. Using the projection operator (see, for example, [22]), we can express the spinor $v(q,s)$ in terms of the spinor $u(p,s)$:

$$v(q,s)=\alpha\frac{\hat{q}-im}{-2im}\frac{1+i\gamma_5\hat{s}}{2}u(p,s). \quad (22)$$

The coefficient α is fixed by the covariant normalization (20) up to the phase factor as

$$\overset{*}{\alpha}\alpha=-\frac{2\,m^2}{(pq)+m^2}=\frac{m^2}{\boldsymbol{p}^2}, \quad |\alpha|=\frac{m}{|\boldsymbol{p}|}.$$

Then the summation over the spinor states results in

$$\sum_s u(q,s)\bar{v}(p,s)=\alpha\frac{\hat{q}+im}{2im}(\hat{p}-im),$$

$$\sum_s v(p,s)\bar{u}(q,s)=\overset{*}{\alpha}\,(\hat{p}-im)\frac{\hat{q}+im}{2im},$$

$$\sum_s u(p,s)\bar{v}(q,s)=\overset{*}{\alpha}\,(\hat{p}+im)\frac{\hat{q}-im}{2im},$$

$$\sum_s v(q,s)\bar{u}(p,s)=\alpha\frac{\hat{q}-im}{2im}(\hat{p}+im). \quad (23)$$

The polarization, in which the momentum $\boldsymbol{p}$ and the unit polarization vector $\boldsymbol{n}$ are orthogonal $(\boldsymbol{pn})=0$, turns out to be the most convenient for handling. In such a situation, both operators $\hat{p}$ and $\hat{q}$ commute with $\gamma_5\hat{s}$, and the polarization directions of a quark and an antiquark could be taken identical (although, in general case, they should be two different directions). Then the summation over the polarization of quarks and antiquarks is performed separately in the final equations. This allows us not to control the obligatory constraint to have the vacuum quantum numbers of the pairs present in the intermediate calculations. Then, for the spinors with polarizations summed up, we have

$$V\overline{V}=p_4\gamma_4+\cos(\theta)(\boldsymbol{p\gamma}-im)-\frac{\overset{*}{\alpha}+\alpha}{2im}\sin(\theta)(\boldsymbol{p}^2-im\boldsymbol{p\gamma}),$$

$$U\overline{U}=p_4\gamma_4+\cos(\theta)(\boldsymbol{p\gamma}+im)+\frac{\overset{*}{\alpha}+\alpha}{2im}\sin(\theta)(\boldsymbol{p}^2+im\,\boldsymbol{p\gamma}),$$

where $\theta=2\varphi$. In the formulae above, the phase inherent in the sum $\overset{*}{\alpha}+\alpha$ (a spinor is defined up to such a phase) is still indefinite. The direct analysis of the mean energy functional demonstrates that the most preferable value of the phase factor (responsible for the color interaction of quarks) is the value when the coefficient α appears to be a real number. For definiteness, we put $\alpha=+m/|\boldsymbol{p}|$. The curious fact is that the results of summation are not equal ($V\overline{V}(m)=U\overline{U}(-m)$), and they coincide in the chiral limit $m=0$ only. That is, particles and antiparticles formally generate the different contributions. The direct calculations lead to the following result for the mean energy (14) (see [1]):

$$\langle\sigma|H_{\rm ind}|\sigma\rangle=-\int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{2N_c p_4^2}{|p_4|}(1-\cos\theta)-$$

$$-\widetilde{G}\int\frac{d\boldsymbol{p}d\boldsymbol{q}}{(2\pi)^6}\left\{-(3\widetilde{I}-\widetilde{J})\frac{p_4q_4}{|p_4||q_4|}+(4\widetilde{I}-\widetilde{J})\frac{pq}{|p_4||q_4|}\times\right.$$

$$\times\left(\sin\theta-\frac{m}{p}\cos\theta\right)\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)-$$

$$-(2\widetilde{I}\delta_{ij}+2\widetilde{J}_{ij}-\widetilde{J}\delta_{ij})\frac{p_i q_j}{|p_4||q_4|}\times$$

$$\times\left(\cos\theta+\frac{m}{p}\sin\theta\right)\left(\cos\theta'+\frac{m}{q}\sin\theta'\right)\bigg\}. \quad (24)$$

Here, we designated $p=|\boldsymbol{p}|$, $q=|\boldsymbol{q}|$, $\widetilde{I}=\widetilde{I}(\boldsymbol{p}+\boldsymbol{q})$, $\widetilde{J}_{ij}=\widetilde{J}_{ij}(\boldsymbol{p}+\boldsymbol{q})$, $\widetilde{J}=\sum_{i=1}^{3}\widetilde{J}_{ii}$, $p_4^2+p^2=q_4^2+q^2=-m^2$, $\theta'=\theta(q)$, $\widetilde{G}=(4\pi^2)^2\ n\rho^4$. As a matter of convenience, we singled out the color factor $G'=\frac{2}{N_c^2-1}\widetilde{G}$. To obtain this result, we performed the regularization (subtracting the free Hamiltonian H_0). This results in the presence of unity (together with $-\cos\theta$) in the parentheses of the first integral. Let us also recall that p_4^2 is a negative magnitude in the Euclidean space. Henceforth, we characterize the different stochastic ensembles of the gluon fields by their profile functions $I(p)$ and $J_{\mu\nu}(p)$. The detailed analysis of the functional of mean energy performed in [1] demonstrates that it is discontinuous as a function of the current quark mass. The singularity arises as a result of contributions proportional to $\frac{m}{p}\cos\theta$. At large momenta, the angle of pairing goes to zero $\theta\to 0$, but a decrease of the denominator $\sim 1/p^2$ is not sufficient to compensate p^2 present in the volume integration of the numerator. As a result, the mean energy beyond the chiral limit goes to minus infinity. The same is valid for the chiral condensate:

$$\langle\sigma|\bar{q}q|\sigma\rangle=\frac{i\ N_c}{\pi^2}\int\limits_0^{\infty}dp\ \frac{p^2}{|p_4|}\ (p\sin\theta-m\cos\theta)\ . \quad (25)$$

It is interesting to note that, despite the singular character of the system mean energy and the corresponding quark condensate found, the meson observables are finite, quite well identified, and compatible with experimental energy scale, see [23].

3. Filling up the Fermi Sphere by Quark Quasiparticles

Then the problem of our interest can be formulated in the following way. We need to construct the state filled in by quasiparticles (the Slater determinant)

$$|N\rangle=\prod_{|\boldsymbol{P}|<P_F;S}A^+(\boldsymbol{P};S)|\sigma\rangle, \quad (26)$$

which possesses the minimal mean energy over state $|N\rangle$. The polarization indices run here over all permissible values. The momenta and the polarizations of quasiparticles filling in the Fermi sphere are marked by the capital letters. The free Hamiltonian written in the quasiparticle operators looks like

$$H_0=-\int d\boldsymbol{x}\bar{q}(\boldsymbol{x})(i\boldsymbol{\gamma}\nabla+im)q(\boldsymbol{x})=\int\frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|\times$$

$$\times\left(\cos\theta A^+(\boldsymbol{p};s)A(\boldsymbol{p};s)+\sin\theta A^+(-\boldsymbol{p};s)B^+(\boldsymbol{p};s)+\right.$$

$$\left.+\sin\theta B(-\boldsymbol{p};s)A(\boldsymbol{p};s)-\cos\theta B(\boldsymbol{p};s)B^+(\boldsymbol{p};s)\right)\ . \quad (27)$$

There are two diagonal matrix elements of the free Hamiltonian, and now we are going to consider one of them:

$$\langle N|B(\boldsymbol{p};s)B^+(\boldsymbol{p};s)|N\rangle\sim$$

$$\sim\langle\sigma|A(\boldsymbol{P};S)B(\boldsymbol{p};s)B^+(\boldsymbol{p};s)A^+(\boldsymbol{P};S)|\sigma\rangle.$$

While presenting the matrix element, we demonstrate, on the right-hand side, only one partial contribution constructed by the operator A of some detached sort. The others give, as it can easily be seen, a sum (integral) over states filling the Fermi sphere in. In view of the normalization condition which we assess as $\langle\sigma|A\ A^+|\sigma\rangle=1$ (for operators with coinciding arguments A, A^+), the matrix element leads to the vacuum contribution similar to one known from [1],

$$-\int\frac{d\boldsymbol{p}}{(2\pi)^3}\langle N||p_4|\cos\theta B(\boldsymbol{p};s)B^+(\boldsymbol{p};s)|N\rangle=$$

$$=-\int\frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|\cos\theta.$$

We should also remember that $B|\sigma\rangle=0$ and $A|\sigma\rangle=0$. The partial contribution to the second matrix element is equal to

$$\langle\sigma|A(\boldsymbol{P};S)A^+(\boldsymbol{p};s)A(\boldsymbol{p};s)A^+(\boldsymbol{P};S)|\sigma\rangle=$$

$$=(2\pi)^3\delta(\boldsymbol{p}-\boldsymbol{P})\delta_{sS}\langle\sigma|A(\boldsymbol{P};S)A^+(\boldsymbol{p};s)|\sigma\rangle. \quad (28)$$

Having filled the state, this contribution occurs as

$$\int\frac{d\boldsymbol{p}}{(2\pi)^3}\langle N||p_4|\cos\theta A^+(\boldsymbol{p};s)\ A(\boldsymbol{p};s)|N\rangle=$$

$$= \int^{P_F} \frac{d\boldsymbol{p}}{(2\pi)^3} |p_4| \cos\theta.$$

There are other diagonal matrix elements in the interaction Hamiltonian $\bar{q}t^a\gamma_\mu q\bar{q}'t^b\gamma_\nu q'$ which we are going to designate as 1), 2), α), β), γ), δ). For example, 1) is the matrix element $\langle N|BB^+B'B'^+|N\rangle$, 2) corresponds to $\langle N|BAA'^+B'^+|N\rangle$, α) looks like $\langle N|AA^+A'A'^+|N\rangle$, β) is presented by $\langle N|AA^+B'B'^+|N\rangle$, γ) corresponds to $\langle N|A^+B^+B'A'|N\rangle$, and δ) looks like $\langle N|BB'^+A'^+A'|N\rangle$. Contribution 1) to the interaction term $\langle N|\bar{q}t^a\gamma_\mu q\bar{q}'t^b\gamma_\nu q'|N\rangle$ leads to the spinor form

$$\overline{V}_{\alpha i}(\boldsymbol{p},s)t^a_{ij}\gamma^\mu_{\alpha\beta}V_{\beta j}(\boldsymbol{p},s)\overline{V}_{\gamma k}(\boldsymbol{p}',s')t^b_{kl}\gamma^\mu_{\gamma\delta}V_{\delta l}(\boldsymbol{p}',s').$$

Bearing in mind the completeness property of the spinor basis, there appears the unit matrix while summing up all color polarizations, for example, $\sum_c V_i(c)\overline{V}_j(c) = \delta_{ij}$. Therefore, contribution 1) as was mentioned in [1] turns out to be zero. The partial contribution 2) looks like

$$\langle\sigma|A(\boldsymbol{P};S)B(\boldsymbol{p};s)\ A(\boldsymbol{q};t)A^+(\boldsymbol{p}';s')B^+(\boldsymbol{q}';t')\times$$

$$\times A^+(\boldsymbol{P};S)|\sigma\rangle = (2\pi)^6\left[\delta(\boldsymbol{q}-\boldsymbol{p}')\delta_{ts'}\langle\sigma|A(\boldsymbol{P};S)\times\right.$$

$$\times A^+(\boldsymbol{P};S)|\sigma\rangle - \delta(\boldsymbol{q}-\boldsymbol{P})\delta_{tS}\langle\sigma|A(\boldsymbol{P};S)\times$$

$$\left.\times A^+(\boldsymbol{p}';s')|\sigma\rangle\right]\delta(\boldsymbol{p}-\boldsymbol{q}')\delta_{st'}.$$

One can obtain from this expression that it contributes to the matrix element $\langle N|\bar{q}t^a\gamma_\mu q\bar{q}'t^b\gamma_\nu q'|N\rangle$ (if the normalization condition of one single state is taken into account) with the result

$$-\overline{V}_{\alpha i}(\boldsymbol{p};s)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{P};S)\overline{U}_{\gamma k}(\boldsymbol{P};S)t^a_{kl}\gamma^\nu_{\gamma\delta}V_{\delta l}(\boldsymbol{p};s)+$$

$$+\overline{V}_{\alpha i}(\boldsymbol{p};s)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{q};t)\overline{U}_{\gamma k}(\boldsymbol{q};t)t^a_{kl}\gamma^\nu_{\gamma\delta}V_{\delta l}(\boldsymbol{p};s).$$

The calculation of matrix elements β), γ), and δ) demonstrates that they have a similar structure, but their contributions cancel one another.

The matrix element α) deserves the special discussion. Unlike the aforementioned examples, the major partial contribution here should be searched for the pair of quasiparticles

$$\langle\sigma|A(\boldsymbol{Q};T)A(\boldsymbol{P};S)A^+(\boldsymbol{p};s)A(\boldsymbol{q};t)A^+(\boldsymbol{p}';s')A(\boldsymbol{q}';t')\times$$

$$\times A^+(\boldsymbol{P};S)A^+(\boldsymbol{Q};T)|\sigma\rangle.$$

When the momenta $\boldsymbol{Q}$ and $\boldsymbol{P}$ coincide, as known (see, for example, [2]), the next term in the $1/V$ decomposition (V is the volume occupied by the system) appears. But if we are interested in knowing how one quasiparticle modifies the dressing transformation, it is necessary to consider its matrix element with $|1\rangle = A^+(\boldsymbol{P};S)|\sigma\rangle$. Omitting the intermediate calculations, we present the contribution of the scattering term α) as

$$-\overline{U}_{\alpha i}(\boldsymbol{Q};T)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{P};S)\overline{U}_{\gamma k}(\boldsymbol{P};S)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{Q};T)+$$

$$+\overline{U}_{\alpha i}(\boldsymbol{P};S)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{P};S)\overline{U}_{\gamma k}(\boldsymbol{Q};T)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{Q};T)+$$

$$+\overline{U}_{\alpha i}(\boldsymbol{Q};T)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{p};s)\overline{U}_{\gamma k}(\boldsymbol{p};s)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{Q};T)+$$

$$+\overline{U}_{\alpha i}(\boldsymbol{Q};T)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{Q};T)\overline{U}_{\gamma k}(\boldsymbol{P};S)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{P};S)+$$

$$-\overline{U}_{\alpha i}(\boldsymbol{P};S)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{Q};T)\overline{U}_{\gamma k}(\boldsymbol{Q};T)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{P};S)+$$

$$+\overline{U}_{\alpha i}(\boldsymbol{P};S)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{p};s)\overline{U}_{\gamma k}(\boldsymbol{p};s)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{P};S), \tag{29}$$

The second and fourth terms give zero contribution for the same reasons which were given for contribution 1). The first and fifth terms, up to the re-designations, are equal to each other. The third and sixth terms presents the contribution of two states with momenta $\boldsymbol{P}$ and $\boldsymbol{Q}$. It is not difficult to understand that, for the ensemble, we get simply the integral over the Fermi sphere (see below). When one quasiparticle is considered, the matrix element unlike (29) is proportional to

$$\overline{U}_{\alpha i}(\boldsymbol{P};S)t^a_{ij}\gamma^\mu_{\alpha\beta}U_{\beta j}(\boldsymbol{p};s)\overline{U}_{\gamma k}(\boldsymbol{p};s)t^a_{kl}\gamma^\nu_{\gamma\delta}U_{\delta l}(\boldsymbol{P};S). \tag{30}$$

As a result, for the traces which we are interested in, we obtain

$$\left.\begin{array}{l}\mathrm{Tr}(V\overline{V}\gamma_\mu t^a U'\overline{U}'\gamma_\nu t^a)\\ \mathrm{Tr}(U\overline{U}\gamma_\mu t^a U'\overline{U}'\gamma_\nu t^a)\end{array}\right\} = 4\frac{N_c^2-1}{2}\times$$

$$\times\left\{p_4q_4g_{\mu\nu} \pm m^2\delta_{\mu\nu}\left(\cos\theta - \frac{\overset{*}{\alpha}+\alpha}{2}\frac{p^2}{m^2}\sin\theta\right)\times\right.$$

$$\times\left(\cos\theta' - \frac{\overset{*}{\alpha}{}'+\alpha'}{2}\frac{q^2}{m^2}\sin\theta'\right) + (\delta_{4\mu}\delta_{i\nu}+\delta_{4\nu}\delta_{i\mu})\times$$

$$\times\left[\left(\cos\theta+\frac{\overset{*}{\alpha}+\alpha}{2}\sin\theta\right)q_4p_i+\right.$$

$$\left.+\left(\cos\theta'+\frac{\overset{*}{\alpha}{}'+\alpha'}{2}\sin\theta'\right)p_4q_i\right]+$$

$$+\left(\delta_{i\mu}\delta_{j\nu}-\delta_{ij}\delta_{\mu\nu}+\delta_{i\nu}\delta_{j\mu}\right)p_iq_j\times$$

$$\left.\times\left(\cos\theta+\frac{\overset{*}{\alpha}+\alpha}{2}\sin\theta\right)\left(\cos\theta'+\frac{\overset{*}{\alpha}{}'+\alpha'}{2}\sin\theta'\right)\right\}, \quad (31)$$

where $p=|\boldsymbol{p}|$, $q=|\boldsymbol{q}|$ and $\theta'=\theta(q)$.

$$\langle A^a_\mu A^b_\nu\rangle(\boldsymbol{x}-\boldsymbol{y})=\delta^{ab}\,\widetilde{G}\frac{2}{N_c^2-1}\times$$

$$\times[I(\boldsymbol{x}-\boldsymbol{y})\delta_{\mu\nu}-J_{\mu\nu}(\boldsymbol{x}-\boldsymbol{y})]. \quad (32)$$

Here, the second term is spanned onto the vector of relative distance.

Now we have to collect together all the results obtained for the one quasiparticle. For the matrix element of the interaction Hamiltonian, we have

$$\langle 1|\bar{q}t^a\gamma_\mu q\bar{q}'t^a\gamma_\nu q'|1\rangle\sim\frac{N_c^2-1}{2}\frac{1}{4|p_4||q_4|}\times$$

$$\times\mathrm{Tr}\left[-V(\boldsymbol{p})\overline{V}(\boldsymbol{p})\gamma_\mu U(\boldsymbol{P})\overline{U}(\boldsymbol{P})\gamma_\nu+\right.$$

$$V(\boldsymbol{p})\overline{V}(\boldsymbol{p})\gamma_\mu U(\boldsymbol{q})\overline{U}(\boldsymbol{q})\gamma_\nu+$$

$$\left.+U(\boldsymbol{P})\overline{U}(\boldsymbol{P})\gamma_\mu U(\boldsymbol{p})\overline{U}(\boldsymbol{p})\gamma_\nu\right]. \quad (33)$$

The polarization indices are omitted here. It is interesting to notice that the factors spanned onto the tensors $g_{\mu\nu}$, $(\delta_{4\mu}\delta_{i\nu}+\delta_{4\nu}\delta_{i\mu})$, $(\delta_{i\mu}\delta_{j\nu}-\delta_{ij}\delta_{\mu\nu}+\delta_{i\nu}\delta_{j\mu})$, (see Eq. (31)) cancel each other in the first and third terms. They survive only in the second term which describes the pure vacuum contribution (considered in [1], see also discussion below). Let us define the partial energy density per one quark degree of freedom, as

$$w=\frac{\mathcal{E}}{2N_c},\quad \mathcal{E}=E/V,$$

where E is the total energy of the ensemble. Collecting all contributions together, we have, for the state with the single quasiparticle,

$$w_1=|p_4|\cos\theta+2G\int\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{pq}{|p_4||q_4|}\left(\sin\theta-\frac{m}{p}\cos\theta\right)\times$$

$$\times\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)(I-J/4)+w_{\rm vac}, \quad (34)$$

where $I=\widetilde{I}(\boldsymbol{p}+\boldsymbol{q})$, $J_{ij}=J_{ij}(\boldsymbol{p}+\boldsymbol{q})$, $J=\sum_{i=1}^3 J_{ii}$. The term $w_{\rm vac}$ describes the vacuum contribution and is shown below. It is convenient to pick out the color index $G=\frac{2\widetilde{G}}{N_c}$. The energy of the state develops a minimal value if the condition

$$\frac{dw_1}{d\theta}=0$$

is satisfied. Neglecting the modifications in the vacuum contribution $w_{\rm vac}$, we have

$$-|p_4|\sin\theta+2G\frac{p}{|p_4|}\left(\sin\theta-\frac{m}{p}\cos\theta\right)\int\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\times$$

$$\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)(I-J/4)=0, \quad (35)$$

which is equivalent, with the precision up to the terms $g_{\mu\nu}$, $(\delta_{i\mu}\delta_{j\nu}-\delta_{ij}\delta_{\mu\nu}+\delta_{i\nu}\delta_{j\mu})$, to the condition of minimum of the mean vacuum energy $\frac{dw_{\rm vac}}{d\theta}=0$, see [1]. That is, in the situation of a single quasiparticle, the dressing transformation in this approximation is not changed as compared with the vacuum one.

For the matrix element of the interaction Hamiltonian for the ensemble of quasiparticles, we have

$$\langle N|\bar{q}t^a\gamma_\mu q\bar{q}'t^a\gamma_\nu q'|N\rangle\sim\frac{N_c^2-1}{2}\frac{1}{4|p_4||q_4|}\times$$

$$\times\mathrm{Tr}\left[-V(\boldsymbol{p})\overline{V}(\boldsymbol{p})\gamma_\mu U(\boldsymbol{P})\overline{U}(\boldsymbol{P})\gamma_\nu+\right.$$

$$+V(\boldsymbol{p})\overline{V}(\boldsymbol{p})\gamma_\mu U(\boldsymbol{q})\overline{U}(\boldsymbol{q}\gamma_\nu+U(\boldsymbol{P})\overline{U}(\boldsymbol{P})\gamma_\mu U(\boldsymbol{p})\overline{U}(\boldsymbol{p})\gamma_\nu+$$

$$\left.+2U(\boldsymbol{P})\overline{U}(\boldsymbol{P})\gamma_\mu U(\boldsymbol{Q})\overline{U}(\boldsymbol{Q})\gamma_\nu\right], \quad (36)$$

(see Eq. (33) to compare). It is pertinent to mention the coefficient of 2 in front of the last term. We present the matrix element implying some ordering of

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the momenta, for instance, $|\boldsymbol{Q}| < |\boldsymbol{P}|$. But, in the expressions which are averaged over the state $|N\rangle$, it is convenient to present the result by integrating over the whole Fermi sphere, without taking into account the ordering. It is evident then that we should take the contribution by a factor of two smaller than that in the interaction term, see the formula below. In the last term similar to the 'vacuum' matrix element (the second term there), the contributions spanned on the tensors $g_{\mu\nu}$, $(\delta_{i\mu}\delta_{j\nu}-\delta_{ij}\delta_{\mu\nu}+\delta_{i\nu}\delta_{j\mu})$ survive. For simplicity, we consider, in this paper, only the situation where the second correlator equals zero $J_{\mu\nu}=0$. In addition, we neglect all the distinctions provoked by the tensors $g_{\mu\nu}$, $(\delta_{i\mu}\delta_{j\nu}-\delta_{ij}\delta_{\mu\nu}+\delta_{i\nu}\delta_{j\mu})$. Collecting all the contributions together for the mean partial energy up to an insignificant constant, we have

$$\langle N|w|N\rangle = \int^{P_F} \frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|\cos\theta + 2G\int^{P_F}\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\times$$

$$\times\left(\sin\theta-\frac{m}{p}\cos\theta\right)\int\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)I-$$

$$-G\int^{P_F}\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\left(\sin\theta-\frac{m}{p}\cos\theta\right)\int^{P_F}\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\times$$

$$\times\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)I+\int\frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|(1-\cos\theta)-$$

$$-G\int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\left(\sin\theta-\frac{m}{p}\cos\theta\right)\times$$

$$\times\int\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)I. \qquad (37)$$

Performing the transposition of integration and changing the variables as $\boldsymbol{p}\to\boldsymbol{q}$, the interaction term can be rewritten in the following conventional form: $2\int^{P_{\rm F}}d\boldsymbol{p}\int d\boldsymbol{q}-\int^{P_{\rm F}}d\boldsymbol{p}\int^{P_{\rm F}}d\boldsymbol{q}-\int d\boldsymbol{p}\int d\boldsymbol{q} = -\int_{P_{\rm F}}\boldsymbol{p}\int_{P_{\rm F}}d\boldsymbol{q}$. Finally, we obtain the following expression for the partial energy

$$\langle N|w|N\rangle = \int^{P_{\rm F}}\frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|+\int_{P_{\rm F}}\frac{d\boldsymbol{p}}{(2\pi)^3}\,|p_4|\,(1-\cos\theta)-$$

$$-G\int_{P_F}\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\left(\sin\theta-\frac{m}{p}\cos\theta\right)\times$$

$$\times\int_{P_{\rm F}}\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)I. \qquad (38)$$

It has quite interesting interpretation. Compared to the vacuum mean energy[3], see [1],

$$w_{\rm vac}=\langle\sigma|w|\sigma\rangle=\int\frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|(1-\cos\theta)-$$

$$-G\int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\left(\sin\theta-\frac{m}{p}\cos\theta\right)\times$$

$$\times\int\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)I.$$

It is easy to see that, in the considered symmetric case where we neglect all contributions generated by the tensors $g_{\mu\nu}$, $(\delta_{i\mu}\delta_{j\nu}-\delta_{ij}\delta_{\mu\nu}+\delta_{i\nu}\delta_{j\mu})$ for the state with the filled Fermi sphere, the angles of pairing could be defined by the condition of functional minimum (38) only for the momenta larger than the Fermi momentum $P_{\rm F}$ (see [24] for comparison). The quarks forming the Fermi sphere look like the free (non-interacting) ones, see the first term.

Now let us calculate the quark chemical potential which, by definition, is an energy necessary for adding (removing) one quasiparticle to (from) a system $\mu=\frac{\partial E}{\partial N}$, where $N=2N_cV\int^{P_F}\frac{d\boldsymbol{p}}{(2\pi)^3}=\frac{N_c}{3\pi^2}VP_{\rm F}^3$ is the total number of particles in the volume V (see [25]). Redefining the chemical potential as $\mu=\frac{2\pi^2}{P_{\rm F}{}^2}\frac{\partial w}{\partial P_{\rm F}}$, we consider the model with a correlation function behaving itself as the δ-function in the coordinate space. It is easy to see that we come to the Nambu–Jona-Lasinio model [13] in this approach. The regularization is required to obtain an intelligent result in this model. We adjust the NJL model for the parameter set given by [26] and limit the integration interval over the momentum in Eq. (37) with the quantity $|\boldsymbol{p}|<\Lambda$ ($\Lambda=631$ MeV). Then functional (38) can be written in the form (inessential terms contributing to the constant values are omitted)

$$w=w_0+\int_{P_{\rm F}}^{\Lambda}\frac{d\boldsymbol{p}}{(2\pi)^3}\left[|p_4|\,(1-\cos\theta)-G\frac{p}{|p_4|}\times\right.$$

$$\left.\times\left(\sin\theta-\frac{m}{p}\cos\theta\right)\int_{P_{\rm F}}^{\Lambda}\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\left(\sin\theta'-\frac{m}{q}\cos\theta'\right)\right],$$

[3] Just this expression is in the last line of Eq. (37).

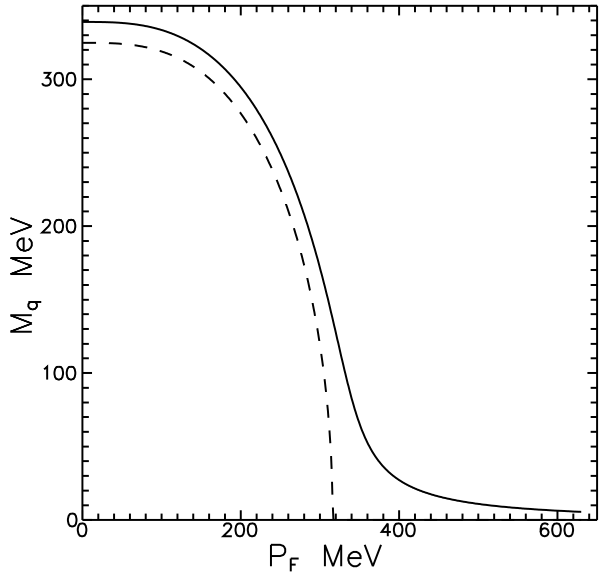

Fig. 1. Dynamical quark mass ($|M_q|$) as a function of the Fermi momentum for the NJL model. The solid line corresponds to the current quark mass $m = 5.5$ MeV. The dashed one shows the behavior in the chiral limit

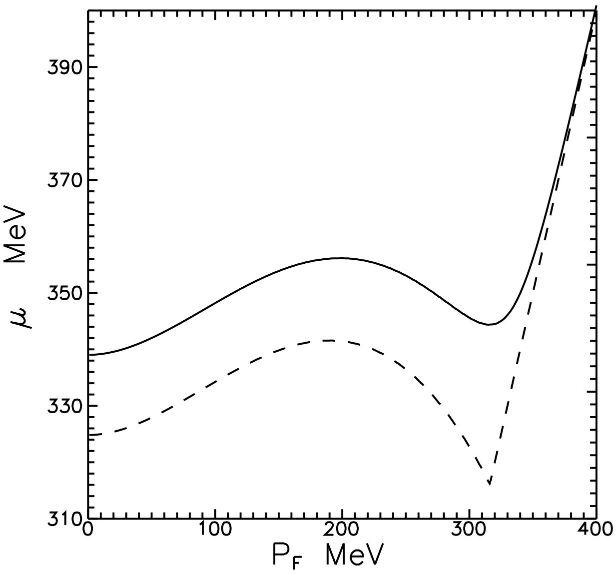

Fig. 2. Quark chemical potential as a function of the Fermi momentum for the NJL model. The solid line corresponds to the current quark mass $m = 5.5$ MeV and the dashed one shows the behavior in the chiral limit

(39)

where $w_0 = \int^{P_F} \frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|$ is the contribution coming from the free quarks, $m = 5.5$ MeV. The equation to calculate the equilibrium angle θ reads as

$$(p^2 + m^2)\sin\theta - M_q\,(p\cos\theta + m\sin\theta) = 0, \tag{40}$$

where

$$M_q = 2G\int\limits_{P_{\rm F}}^{\Lambda} \frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\left(\sin\theta - \frac{m}{p}\cos\theta\right). \tag{41}$$

It allows us to obtain the well-known self-consistent gap equation for the dynamical quark mass. For the parameters used, the dynamical quark mass at zero Fermi momentum is $M_q = -335$ MeV. For the quark condensate,

$$\langle N|\bar{q}q|N\rangle = \frac{iN_c}{\pi^2}\int\limits_{P_{\rm F}}^{\Lambda} dp\frac{p^2}{|p_4|}(p\sin\theta - m\cos\theta), \tag{42}$$

and $\langle\sigma|\bar{q}q|\sigma\rangle = -i(247\ \text{MeV})^3$. The constant characterizing the four-fermion interaction strength was taken as $G\Lambda^2/(2\pi^2) = 1.34$. In Fig. 1, the dynamical quark mass as a function of the Fermi momentum is depicted. For comparison, the data are presented for the current quark mass $m = 5.5$ MeV (the solid line), and the dashed line corresponds to the calculation in the chiral limit. For the NJL model, in particular, the quark chemical potential equals

$$\mu = |P_4^{\rm F}|\cos\theta_{\rm F} + M_q\frac{P_F\sin\theta_F - m\cos\theta_{\rm F}}{|P_4^{\rm F}|}. \tag{43}$$

When the Fermi momentum reaches zero value, the chemical potential coincides with the dynamical quark mass $\mu(0) = |M_q| - M_q m/|M_q| = |M_q| + m$. In Fig. 2, the quark chemical potential is depicted as a function of the Fermi momentum for the configurations analogous to the ones shown in Fig. 1. The dependence of the chemical potential on the Fermi momentum could be interpreted as the effect of a rapid decrease of the dynamical quark mass with increase in the Fermi momentum. Then, using Eq. (40), the chemical potential can be presented as

$$\mu = \frac{M_q P_{\rm F}}{|P_4^{\rm F}|\sin\theta_{\rm F}}.$$

In view of the identity

$$(|P_4|^2 - M_q m)^2 + M_q^2P^2 = [P^2 + (m - M_q)^2]|P_4|^2, \tag{44}$$

we come to the noteworthy definition of the chemical potential

$$\mu = [P_{\rm F}^2 + (m - M_q)^2]^{1/2}.$$

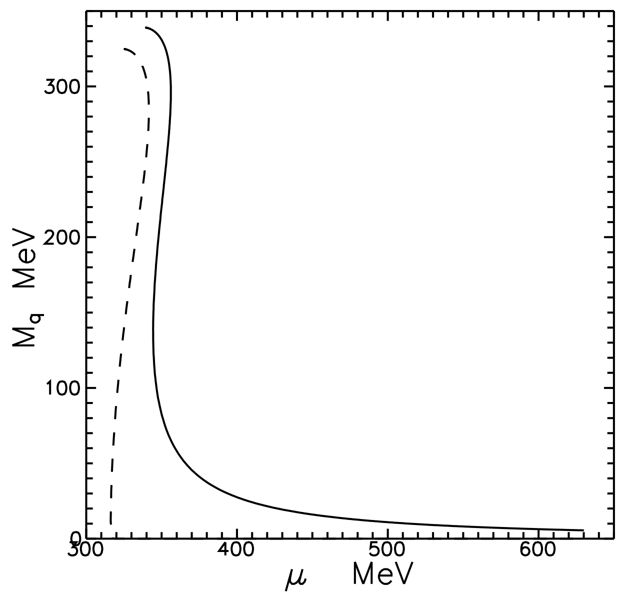

Fig. 3. Dynamical quark mass ($|M_q|$) as a function of the chemical potential. The solid line corresponds to the current quark mass $m = 5.5$ MeV. The dashed one shows the behavior in the chiral limit

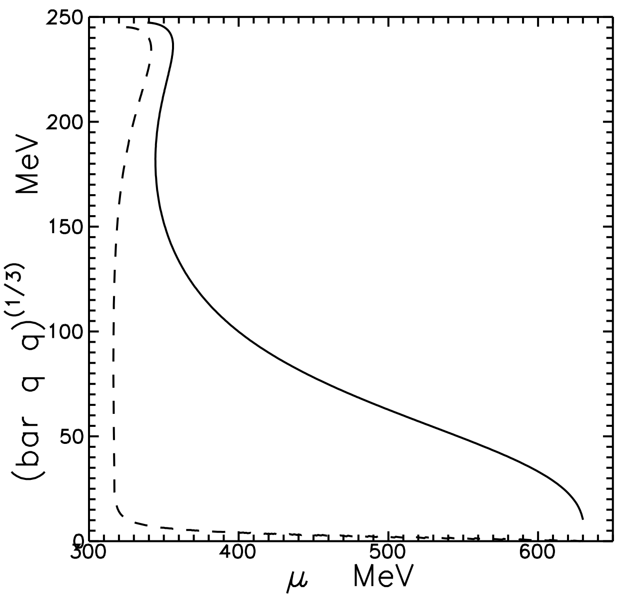

Fig. 4. Quark condensate ($|\langle\bar{q}q\rangle^{1/3}|$) as a function of the chemical potential. The solid line corresponds to the current quark mass $m = 5.5$ MeV. The dashed one shows the behavior in the chiral limit

We recall that the chemical potential for the free fermion gas increases monotonically with the Fermi momentum. The curious feature of the NJL model is the appearance of a state almost degenerate with the vacuum state, while the process of filling up the Fermi sphere reaches the momenta close to the dynamical quark mass value (the similar value is characteristic of the momentum of a quark inside a baryon, see, for example, [3]). This state density with a factor of 3 (which expresses the relation between baryonic and quark degrees of freedom) absorbed corresponds to a normal nuclear density ($n \sim 0.12/\mathrm{fm}^3$), and the chiral condensate could be estimated as $|\langle\bar{q}q\rangle^{1/3}| \sim 100$ MeV. In the chiral limit, the chemical potential near the discussed point is even smaller than the vacuum one. The full coincidence of the chemical potentials occurs at the current quark mass around 2 MeV. In fact, Fig. 2 shows that the u quark bond looks stronger than that of the d quark. For clarity, Fig. 3 shows the dynamical quark mass as the function of chemical potential (Fig. 4 for the quark condensate). The pressure of the quark ensemble

$$P = -\frac{dE}{dV} = -\frac{\partial E}{\partial V} + \frac{P_\mathrm{F}}{3V}\,\frac{\partial E}{\partial P_\mathrm{F}} = -\mathcal{E} + \mu n, \tag{45}$$

is depicted in Fig. 5 as a function of the Fermi momentum, where $n = N/V$ is the quark density. The quark pressure at the values of the Fermi momentum close to the value of dynamical quark mass is approximately degenerate with the vacuum pressure (slightly lower than the vacuum one). The vacuum density is of order of 40–50 $\mathrm{MeV/fm}^3$ and corresponds well to the value extracted from the bag models, see, for example, [3]. Actually, all the NJL results could be obtained in the mean field approximation because the trigonometric terms in the mean energy definition (39) can be rewritten (using Eq. (44) again) as the functions of the dynamical and current quark masses in the form

$$\frac{p\sin\theta - m\cos\theta}{|p_4|} = \frac{M_q - m}{[p^2 + (M_q - m)^2]^{1/2}},$$

$$|p_4|\cos\theta = \frac{p^2 + m(m - M_q)}{[p^2 + (M_q - m)^2]^{1/2}}.$$

In order to trace back the dependence of all results on the formfactor form, we consider the model (in a sense, opposite to the NJL model) with the formfactor behaving itself as a δ-function in the momentum space $I(p) = (2\pi)^3\delta(\boldsymbol{p})$. This limit is an analog of the Keldysh model which is well known in the condensed matter physics [27], and the mean energy functional (38) develops the following form:

$$w = \int^{P_\mathrm{F}} \frac{d\boldsymbol{p}}{(2\pi)^3}|p_4| + \int_{P_\mathrm{F}} \frac{d\boldsymbol{p}}{(2\pi)^3}|p_4|(1 - \cos\theta) -$$

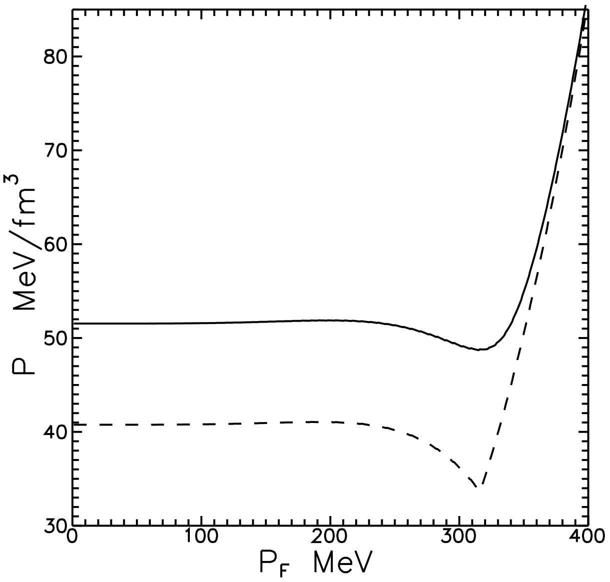

Fig. 5. Pressure of the quark ensemble as a function of the Fermi momentum. The solid line for the current quark mass $m = 5.5$ MeV. The dashed one shows the behavior in the chiral limit

$$-G \int\limits_{P_{\rm F}} \frac{d\boldsymbol{p}}{(2\pi)^3} \frac{p^2}{|p_4|^2} (\sin\theta - \frac{m}{p}\cos\theta)^2. \quad (46)$$

The chemical potential is defined in this approach as

$$\mu = |P_4^{\rm F}| \cos\theta_{\rm F} + G \frac{(P_{\rm F}\sin\theta_{\rm F} - m\cos\theta_{\rm F})^2}{|P_4^{\rm F}|^2}. \quad (47)$$

It follows from this definition that, at low Fermi momenta, the chemical potential goes to $\mu \to m + G$. In [1], the constant G was taken of order of the dynamical quark mass in NJL. As the Fermi momentum increases, the chemical potential remains approximately constant and starts to increase, when the Fermi momentum exceeds G. The vacuum density ($P_F = 0$) turns out to be singular, and, for the pressure difference, we obtain

$$P - P(0) = \frac{2\ N_c}{2\pi^2} \left[\frac{P_{\rm F}^3}{3}\mu - \int\limits_0^{P_{\rm F}} dp p^2 \cos\theta |p_4| - \right.$$

$$\left. -G \int\limits_0^{P_{\rm F}} dp \frac{p^4}{|p_4|^2} \left(\sin\theta - \frac{m}{p}\cos\theta\right)^2 \right]. \quad (48)$$

This function is slowly monotonically growing till the Fermi momentum value of G. In summary, we would like to emphasize our main result which, as we believe,is quite transparent. Our estimate of the effects responding to the process of filling up the Fermi sphere demonstrates the parallels between this quasiparticle picture and the conventional bag model but with one new essential element. It is just the presence of instability region $dP/dP_{\rm F} < 0$. Then states (26) could be considered as a natural 'building' material for baryons. In principle, one of the ways to construct the corresponding bound state could follow the Walecka model ideas [28] with utilizing the information on the behavior of scalar and vector fields and the respective constants of their interaction with quarks which have been gained up to now [24].

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ЗАПОВНЕННЯ ФЕРМІ-СФЕРИ КВАЗІЧАСТИНКАМИ КВАРКІВ

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Р е з ю м е

Проаналізовано поведінку кварків під дією сильного стохастичного глюонного поля. Розвинуто наближену процедуру для розрахунку ефективного гамільтоніана. Розглядаючи кварки як квазічастинки у модельному гамільтоніані з чотириферміонною взаємодією, вивчено наслідки заповнення фермісфери кварками.