PROGRESS AND PROBLEMS IN QUANTUM GRAVITY

W. KUMMER

Institute f. Theor. Physics, Vienna University of Technology
(Wiedner Hauptstraße 8-10, A-1040 Vienna, Austria)

PACS 04.60.-m

From the point to view of an uncompromising field theorist, quantum gravity is beset with serious technical and, above all, conceptual problems with regard especially for the meaning of genuine "physical" observables. This situation is not really improved by the appearance of recent attempts to reformulate gravity within some novel framework. However, the original aim, a background-independent quantum theory of gravity can be achieved in a particular area, namely 2D dilaton quantum gravity without any assumptions beyond standard quantum field theory. Some important further by-products of the research of the "Kummer's Vienna School" include the introduction of the concept of Poisson-Sigma models, a verification of the "virtual Black Hole" and the extensions to $N = (1, 1)$ and $N = (2, 2)$ 2D-supergravity, for which complete solutions of some old problems have been possible which are relevant for superstring theory.

1. Introduction

The conclusion cannot be avoided that a merging of quantum theory with Einstein's theory of general relativity[1] (GR) is necessitated by consistency arguments. For example, the interaction of a classical gravitational wave with a quantum system inevitably leads to contradictions [3]: If that interaction leads to a collapse of the probability function, momentum conservation breaks down, if it does not, signals must propagate with velocities larger than the speed of light.

When a quantum theory (QT) of gravity is developed along usual lines of quantum field theory (QFT), one is confronted with a fundamental problem, from which many other (secondary) difficulties can be traced. The crucial difference to flat space is the fact that the variables of gravity exhibit a dual role, they are fields living on a manifold which is determined by themselves, "stage" and "actors" coincide. Any separation between those aspects may even be the origin of perturbative non-renormalizability [4]. There exist further problems: the time variable, an object with special properties already in QT, appears in GR on an equal footing with the space coordinates.

In Section 2, we shortly recall the definition of physical variables in QFT and the ensuing ones in quantum gravity (QGR).

Then (Section 3) we turn to those quantities which can be interpreted as "physical observables", both at the classical and at the quantum level. In the latter case, the most serious problems arise in QGR – and are almost completely ignored in the contemporary literature.

In the last , several new approaches to GR and QGR have been introduced which are described in Section 4.

Finally (Section 5), we mention some highlights of the "Vienna approach" to 2D dilaton quantum gravity. In that area which contains also models with physical relevance (e.g., spherically reduced gravity), the application of just the usual concepts of (even nonperturbative!) QFT leads to very interesting consequences [5] which allow physical interpretations in terms of "solid" traditional QFT observables.

2. Field Variables in GR

The field variables of a QFT usually are not directly accessible to experimental measurements. Traditionally, the metric $g_{\mu\nu}$ is the one used in GR. However, from a more fundamental geometric point of view [6], the metric is a "derived" field variable

$$g = e^a \otimes e^b \eta_{ab}, \tag{1}$$

because it is the direct product of the dual basis one forms[2] $e^a = e^a_\mu dx^\mu$ contracted with the flat local Lorentz metric η_{ab} which is used to raise and to lower "flat indices" denoted by Latin letters $\left(\eta = diag\,(1, -1, -1, -1, ...)\,, x^\mu = \left\{x^0, x^i\right\}\right)$. Local Lorentz invariance leads to the "covariant derivative" $D^a_b = \delta^a_b d + \omega^a{}_b$ with a spin connection 1-form $\omega^a{}_b$ as a gauge field. Its antisymmetry $\omega^{ab} = -\omega^{ba}$ implies metricity. Thanks to the Bianchi identities, all covariant tensors relevant for constructing actions in even dimensions can be expressed in terms of e^a, the curvature 2-form $R^{ab} = (D\omega)^{ab}$, and the torsion 2-form

[1] Several reviews of quantum gravity have emerged at the turn of the millennium, cf. e.g. [1, 2].

[2] For details on gravity in the Cartan formulation, we refer to the mathematical literature, e.g. [6].

$T^a = (De)^a$. For nonvanishing torsion, the affine connection $\Gamma_{\mu\nu}{}^{\rho} = E^{\rho}_{a} \, (D_\mu e)^a_\nu$, expressed in terms of components e^a_μ and of its inverse E^ρ_a, also contains, besides the usual Christoffel symbols, a contorsion term in $\Gamma_{(\mu\nu)}{}^{\rho}$, whereas $\Gamma_{[\mu\nu]}{}^{\rho}$ are the components of torsion.

Einstein's gravity in D=4 dimensions postulates vanishing torsion $T^a = 0$ so that $\omega = \omega(e)$. This theory can be derived from the Hilbert action ($G_{\rm N}$ is Newton's constant; the deSitter space with nonvanishing positive cosmological constant Λ results from the replacement $R^{ab} \to R^{ab} - \frac{4}{3}\Lambda e^a \wedge e^b$)

$$L_{(H)} = \frac{1}{16\pi G_n} \int\limits_{\mathcal{M}_4} R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd} + L_{(\text{matter})}, \qquad (2)$$

where a small but definitely nonvanishing value of Λ is suggested by recent astronomical observations [7]. Because of the "Palatini mystery", the independent variation of $\delta\omega$ yields $T^a = 0$, whereas δe produces the Einstein equations (or the Einstein–deSitter ones for$\Lambda \neq 0$).

Instead of working with metric (1), the "new" approaches [8] are based upon a gauge field related to ω^{ab}

$$A^{ab} = \frac{1}{2}\left(\omega^{ab} - \frac{\gamma}{2}\epsilon^{ab}{}_{cd}\omega^{cd}\right). \qquad (3)$$

The Barbero–Immirzi parameter γ [9] is an arbitrary constant. The extension to complex gravity ($\gamma = i$) makes A^a a self-adjoint field and transforms the Einstein theory into the one of an $SU\,(2)$ gauge field

$$A^{\underline{a}}_i = \epsilon^{0\underline{a}}{}_{\underline{b}\,\underline{c}} A^{\underline{b}\,\underline{c}}_i, \qquad (4)$$

where the index $\underline{a}$ =1,2,3. This formulation is the basis of loop quantum gravity and spin foam models (see below).

3. Observables

3.1. Observables in classical GR

At the classical level, the exploration of the global properties of a certain solution of (2), its singularity structure, *etc.*, is only possible by means of the introduction of an additional test field, most simply a test particle with action

$$L_{(\text{test})} = -m_0 \int |ds|,$$

$$ds^2 = g_{\mu\nu}\,(x\,(\tau))\,\frac{dx^\mu}{d\tau}\,\frac{dx^\nu}{d\tau}, \qquad (5)$$

which is another way to incorporate Einstein's old proposal [10] of a "net of geodesics". The path $x^\mu(\tau)$ is parametrized by the affine parameter τ (actually, only the timelike or lightlike $ds^2 \geq 0$ describes the paths of a physical particle).

It is not appreciated always that the global properties of a manifold are *defined* in terms of a specific device like (5). Whereas the usual geodesics derived from (5) depends on $g_{\mu\nu}$ through the Chistoffel symbols only, e.g. in the case of torsion, also the contorsion may contribute ("autoparallels") in the affine connection; spinning particles "feel" the gravimagnetic effect, *etc.* As a consequence, when a field-dependent transformation of the gravity variables is performed (e.g. the conformal transformation from a "Jordan frame" to an "Einstein frame" in the Jordan–Brans–Dicke [11] theory), the action of the device must be transformed in the same way.

3.2. Observables in QFT

In flat QFT, one starts from a Schrödinger equation dependent on field operators and, proceeding through the Hamiltonian quantization to the path integral, the experimentally accessible observables are elements of the S-matrix, or quantities expressible by those. It should be recalled that the properly defined renormalized S-matrix element is obtained by amputation of external propagators in the related Green function, multiplication with polarizations and with a square root of the wave function renormalization constant, taking the mass-shell limit.

It cannot be emphasized too strongly that, in the more general framework of QFT, ordinary quantum mechanics and its Schrödinger equation appear as the nonrelativistic weak coupling limit of the Bethe–Salpeter equation of QFT [12]. Useful notions like eigenvalues of Hermitian operators, collapse of wave functions, *etc.* are *not* basic concepts in this more general frame (cf. footnote 2 in ref. [13]).

In gauge theories, one encounters the additional problem of gauge-dependence, i.e. the dependence on some gauge parameter β introduced by generic gauge fixing. Clearly, the S-matrix elements must be and indeed are [13] independent of β. But other objects, in particular matrix-elements of gauge invariant operators $\mathcal{O}_{\mathcal{A}}$, depend on β. In addition, under renormalization, they mix with operators $\widetilde{\mathcal{O}}_{\widetilde{A}}$ of the same "twist" (dimension minus spin) which depend on Faddeev–Popov ghost fields [14] and are not gauge-invariant:

$$\mathcal{O}^{(\text{ren})} = Z_{AB}\mathcal{O}_B + Z_{A\widetilde{B}}\widetilde{\mathcal{O}}_{\widetilde{B}}$$

$$\widetilde{\mathcal{O}}^{(\text{ren})} = Z_{\widetilde{A}\widetilde{B}}\widetilde{\mathcal{O}}_{\widetilde{B}}. \qquad (6)$$

 ISSN 2071-0194. Ukr. J. Phys. 2009. Vol. 54, No. 8-9

The contribution of such operators to the S-matrix element (sic!) of, e.g., the scaling limit for deep inelastic scattering [15] of leptons on protons [16] occurs only through the anomalous dimensions ($\propto \partial Z_{AB}/\partial\lambda$ for a regularization cut-off λ). And those objects, also thanks to the triangular form of (6), turn out to be indeed gauge-independent!

In flat QFT, as well as in QGR, the (gauge invariant) "Wilson loop"

$$W_{(\mathcal{C})} = \mathrm{Tr}\, P \,\exp\left(i \oint\limits_{\mathcal{C}} A_\mu dx^\mu\right), \tag{7}$$

parametrized by a path ordered closed curve $\mathcal{C}$, is often assumed to play an important role. In covariant gauges, it is multiplicatively renormalizable with the renormalization constant depending on the length of $\mathcal{C}$, the UV cut-off, and eventual cusp-angles in $\mathcal{C}$ [17]. Still the relation to experimentally observable quantities (should one simply drop the renormalization constant or proceed [13] as for an S-matrix?) is unclear. Worse, for lightlike axial gauges $(n\,A) = 0$ $\left(n^2 = 0\right)$, multiplicative renormalization is not applicable [18]. Then, only for a matrix element of (7) between "on-shell gluons", this type of renormalization is restored. Still the renormalization constant is different from covariant gauge, except for the anomalous dimension derived from it (cf. precisely that feature of operators in deep inelastic scattering of leptons).

In the absence of S-matrix elements, defined as in QFT, nowadays a broad role is attributed to "Dirac observables" defined as quantities which commute with the Hamiltonian of the system. (Just one recent example of this line of argument is [19], where the earlier literature is cited extensively). As the example of the (gauge invariant!) Wilson loops shows, any matrix elements of the related operators in QFT will be gauge-parameter-dependent and hence useless for a description for physical phenomena. No convincing argument is known so far how to extract a *genuine* physical observable from that.

4. Traditional and More Recent Approaches to QGR

"Old" QGR mostly worked with a separation of the two aspects of gravity variables by the decomposition of the metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \tag{8}$$

which consists of a (fixed) classical background $g^{(0)}_{\mu\nu}$ ("stage"), upon which small quantum fluctuations $h_{\mu\nu}$ ("actors") occur. The "observable" (to be tested by a classical device) would be the effective matrix $g^{(eff)}_{\mu\nu} = g^{(0)}_{\mu\nu} + < h_{\mu\nu} >$. Starting computations from action (2), one finds that an ever increasing number of counterterms is necessary. They are different from the terms in the Lagrangian $\mathcal{L} = \sqrt{-g}R/\left(16\pi G_N\right)$ in (2). This is the reason why QGR is called (perturbatively) "nonrenormalizable" [4]. Still, at energies $E \ll \left(G_N\right)^{-1/2}$, i.e. much below the Planck mass scale $m_{Pl} \sim 10^{19} GeV$, such calculations can be meaningful in the sense of an "effective low energy field theory" [20], irrespective of the fact that (perhaps by embedding gravity into string theory) by inclusion of further fields at higher energy scales (Planck scale), QGR may become renormalizable. Of course, such an approach, even when it is modified by iterative inclusion of $\langle h_{\mu\nu}\rangle$ into $g^{(0)}_{\mu\nu}$, *etc.* – which is quite hopeless technically – completely misses inherent background independent effects, i.e. effects when $g^{(0)}_{\mu\nu} = 0$.

One could think also of applying nonperturbative methods developed in numerical lattice calculations for Quantum Chromodynamics (QCD). However, there are problems to define the Euclidean path integral for that, because the Euclidean action is not bounded from below (as it is the case in QCD) [21].

The quantization of gravity which – at least formally – avoids the background dependence is based upon the ADM approach to the Dirac quantization of the Hamiltonian [22]. Space-time is foliated by a sequence of three-dimensional space-like manifolds $\sum_3$, upon which the variables $g_{i\,j} = q_{i\,j}$ and associated canonical momenta $\pi_{i\,j}$ live. The constraints associated to the further variables lapse (N_0) and shift (N_i) in the Hamiltonian density

$$\mathcal{H} = N_0 H^0\left(q,\pi\right) + N_i H^i\left(q,\pi\right) \tag{9}$$

are primary ones. The Poisson brackets of the secondary constraints H^μ close. H^i generates diffeomorphisms on $\sum_3$. In the quantum versions of (9), the solutions of the Wheeler–deWitt equation involving the Hamiltonian constraint

$$\int\limits_{\sum_3} H^0\left(q,\, \frac{\delta}{i\delta q}\right) \mid \psi\rangle = 0 \tag{10}$$

formally would correspond to a nonperturbative QGR. Apart from the fact that it is extremely difficult, if not impossible, to find a general solution to (10), there are several basic problems with a quantum theory based upon that equation (e.g. no Hilbert space $\mid \psi\rangle$ can be constructed, no preferred time foliation exists with ensuing inequivalent quantum evolutions [23], problems with usual "quantum causality" exist, the "axiom" of QFT that fields should commute at space-like distances does not hold, *etc.*). A restriction to a finite number of degrees of freedom ("mini superspace") [24] or to a reduced set of an infinite number of degrees of freedom (but still less than that of the original theory – the so-called "midi superspace") [25] has been found to miss essential features.

As all physical states $\mid \psi\rangle$ must be annihilated by the constraint H^μ, a naive Schrödinger equation involving the Hamiltonian constraint H^0,

$$i\hbar \frac{\partial \mid \psi\rangle}{\partial t} = H^0 \mid \psi\rangle = 0, \tag{11}$$

cannot contain a time variable. Actually already in ordinary in quantum mechanics, the time variable is something like an outsider – not associated to any operator, at best defined with reference to a process which evolves in time. Here, the problem even becomes more serious.

A kind of the Schrödinger equation can be produced from (11) by the definition of a "time-function" $T\,(q, \pi, x)$, at the price of an even more complicated formalism [26] with quite ambiguous results – and the problem persists how to connect those with "genuine" observables. All these difficulties are aggravated, when one tries, first, to eliminate constraints by solving them explicitly before quantization. In this way, clearly, a part of the quantum fluctuations is eliminated from the start. As a consequence, different quantum theories constructed in this way are not equivalent.

The "new" gravities (loop quantum gravity, spin foam models) reformulate the quantum theory of space-time by the introduction of novel variables based upon the concept of Wilson loops[3] (7) applied to the gauge field (4).

$$U\,(s_1,\, s_2) = \mathrm{Tr}\, P\, \exp\, \left(i \int_{s_1}^{s_2} ds \frac{dx^i}{ds} A_i \right) \tag{12}$$

defines a holonomy. It is generalized by inserting further invariant operators[4] at intermediate points between s_1 and s_2. From such holonomies, a spin network can be created which represents space-time (in the path integral, it is dubbed "spin foam").

[3] Recall, however, the serious doubts of a quantum field theory regarding the use of such variables!

[4] The same warning should be heeded!

These approaches claim several successes [2]. Introducing diffeomorphism equivalence classes of "labeled graphs" as a basis, a finite Hilbert space can be constructed, and some solutions of the Wheeler–deWitt equation (10) have been obtained. The methods introduce a "natural" coarse graining of space-time which implies a UV cutoff. "Small" gravity around certain states leads in those cases to corresponding linearized Einstein gravity.

However, despite the very active research in this field, a number of very serious open questions persists: the Hamiltonian constructed from spin networks does not lead to massless excitations (gravitons) in the classical limit. The Barbero–Immirzi parameter γ has to be fixed by the requirement of a "correct" Bekenstein–Hawking entropy for the black hole. The most severe problem, however, is the one of observables. By some researchers in this field, it has been claimed that, by "proper gauge fixing" (!), area and volume can be obtained as quantized "observables", which is a contradiction in itself from the point of view of QFT. We must emphasize too that, also in an inherently UV regularized theory (finite), the renormalization remains an issue to be dealt with properly. Also the fate of S-matrix elements, which play such a central role as the proper observables in QFT, is completely unclear in these setups.

Embedding QGR into (super-)string theory [27] does not remove the key problems related to the dual role of the metric. Gravity may well be a string excitation in a string/brane world of 10 or 11 dimensions, possibly a finite theory of everything. Nevertheless, at low energies, Einstein gravity (eventually plus an antisymmetric $B-$field) remains the theory, for which computations must be performed.[5] Unfortunately, the proper choice (let alone the derivation) of a string vacuum in our $D = 4$ space-time is an unsolved problem in the sense that it has too many (billions ?) solutions.

Many other approaches exist, including noncommutative geometry, twistors, causal sets, 3D approaches, dynamical triangulations, Regge calculus, *etc.*, each of which has certain attractive features and difficulties (cf. e.g. [2] and references therein).

To us, all these "new" approaches appear as – very ingenious – attempts to bypass the technical problems

[5] It should be noted that the now widely confirmed astronomical observations of a positive cosmological constant [7] (if it is a constant and not a "quintessence" field in a theory of the Jordan–Brans–Dicke type [11]) precludes the immediate application of supersymmetry (supergravity) in string theory, because only the anti-deSitter space with $\Lambda < 0$ is compatible with supergravity [28].

of directly applying standard QFT to gravity – without a comprehensive solution of the main problems of QGR being in sight. Thus, the main points of a "minimal" QFT for gravity should be based upon "proven concepts" of QFT with a point of departure characterizing QGR as follows:

(a) QGR at energies $E \ll m_{Pl}$ is an "effective" low-energy theory and therefore need not be renormalizable QGR to all orders.

(b) QGR is based upon classical Einstein's (or Einstein–deSitter) gravity with usual variables (metric or Cartan variables).

(c) At least the quantization of geometry must be performed in a background independent (nonperturbative) way.

(d) Absolutely "safe" quantum observables are only the S-matrix elements $\langle f \mid S \mid i\rangle$ of QFT, where the initial state $\mid i\rangle$ and the final state $\langle f\mid$ are defined only when those states are realized as Fock states of particles in a (at least approximate) flat space environment. In certain cases, it is permissible to employ a semiclassical approach: expectation values of quantum corrections may be added to classical geometric variables and a classical computation is then based on the effective variables obtained in this way.

Clearly, item (d) excludes, by construction, any application to quantum cosmology, where $\mid i\rangle$ would be the (probably nonexistent) infinite past before the Big Bang.

Obviously, the most difficult issue is (c). In the following section, we describe how gravity models in $D = 2$ (e.g. spherically reduced gravity) permit a solution of just that crucial point, leading to novel results.

5. QGR in 1+1 Dimension: the "Vienna School"

5.1. Classical theory: first-order formulation

In the 1990s, the interest in dilaton gravity in D=2 was rekindled by string theory [29], although many results were obtained in a nonsystematic way since the 1980s [30]. For a modern review on dilaton gravity, work [6] may be consulted. The study of dilaton gravity can be motivated briefly from a purely geometrical point of view.

The notation of work [5] is used: e^a is the zweibein one-form, $\epsilon = e^+ \wedge e^-$ is the volume two-form. The one-form ω represents the spin-connection $\omega^a{}_b = \varepsilon^a{}_b\omega$ with the totally antisymmetric Levi-Civita symbol $\varepsilon_{ab}\,(\varepsilon_{01} = +1)$. With the flat metric η_{ab} in light-cone coordinates ($\eta_{+-} = 1 = \eta_{-+}, \eta_{++} = 0 = \eta_{--}$), the torsion 2-form reads $T^\pm = (d \pm \omega) \wedge e^\pm$. The curvature 2-form $R^a{}_b$ can be presented by the 2-form R defined by $R^a{}_b = \varepsilon^a{}_b R, = d \wedge \omega$. Signs and factors of the Hodge $\star$ operation are defined by $\star\varepsilon = 1$.

Since the Hilbert action $\int_{\mathcal{M}_2} R \propto (1-g)$ yields just the Euler number for a surface with genus g, one has to generalize it appropriately. The simplest idea is to introduce a Lagrange multiplier for the curvature, X, also known as "dilaton field", and an arbitrary potential thereof, $V(X)$, in the action $\int_{\mathcal{M}_2} (XR + \epsilon V(X))$. In particular, for $V \propto X$, the Jackiw–Teitelboim model emerges [30]. Having introduced curvature, it is natural to consider torsion as well. By analogy, the first-order gravity action [31]

$$L^{(1)} = \int\limits_{\mathcal{M}_2} (X_a T^a + XR + \epsilon\mathcal{V}(X^a X_a,\, X)) \tag{13}$$

can be motivated, where X_a are the Lagrange multipliers for torsion. It encompasses essentially all known dilaton theories in 2D, also known as Generalized Dilaton Theories (GDT). Spherically reduced gravity (SRG) from D=4 corresponds to $\mathcal{V} = -X^+X^-/\,(2X)$ −const.

Actually, (13) is classically and quantum mechanically equivalent to a more familiar expression for a dilaton theory

$$L^{(\mathrm{GDT})} = \int d^2x\sqrt{-g}\left[\frac{R}{2}X - \frac{U}{2}\,(\nabla X)^2 + V\,(X)\right], \tag{14}$$

where the functions U and V coincide with the ones of a general "potential" quadratic in $X^aX_a = 2X^+X^-$,

$$\mathcal{V} = X^+X^-U + V. \tag{15}$$

In contrast to (13), torsion vanishes in (14), and R is the torsionless curvature scalar. The key advantage of formulation (13) is that it allows an exact (background-independent) solution of the quantum mechanical path integral [32] – although this solution has aspects of a "topological" character. Here, the fact that the Hamiltonian of (13) leads to a constraint algebra just like in a non-Abelian gauge theory and the use of a temporal gauge for the Cartan variables (equivalent to an Eddington–Finkelstein metric at the level of the metric $g_{\mu\nu}$ in (14)) are important ingredients.

On the other hand, action (13) suggested a generalization to the so-called Poisson-Sigma models (PSM [31])

$$L^{(\mathrm{PSM})} = \int \left(X^A dA_A + \frac{1}{2}P^{AB}A_B \wedge A_A\right). \tag{16}$$

When the Poisson-tensor $P^{AB}\left(X^C\right)$ interpreted as a Schouten bracket $\left\{X^A, X^B\right\}$ fulfills a generalized type of the Jacobi identity, (16) possesses an (on-shell) nonlinear gauge symmetry of the field A_A combined with a certain symmetry transformation in the target space X^A. In the mean-time, PSMs have found applications in string theory [33, 34].

The consideration of graded generalizations of (16), i.e. with anticommuting fields included, has led in recent years to a completely new approach towards 2D-supergravity [35] which just for potentials of type (15) (respectively, prepotentials derived for them) allowed one to find complete classical solutions (including fermions) for $N = (1, 1)$ and $N = (2, 2)$ supergravity, solutions, for which the bosonic part alone [37] was known previously.

The interaction of 2D dilaton gravity (14) with matter, as exemplified in the special case of SRG, for the first time showed that the "virtual Black Hole" introduced into an *ad hoc* manner before [37] naturally appears in a "reliable" S-matrix situation, namely as an intermediate state in the scattering of spherical waves [38]. Also in the particular case of the "stringy" black hole (cf. the last ref. [29]), the quantum correction to the specific heat could be calculated [39] which stabilizes the (to the lowest order) ill-defined value of that quantity.

6. Conclusion and Outlook

The fundamental challenges of quantum gravity – especially if considered as a low-energy ($E \ll m_{Pl}$) theory – are not really situated in the technical domain. Therefore, they are not likely to be mitigated in novel formulations which appeared in the last decades. Rather they are related to the unsolved question how to define genuine physical observables other than the (gauge-independent) S-matrix elements. Of course, the latter come into play only in a very restricted domain of QGR, namely reactions considered in a flat background between infinite early and late times. For any other situation and in particular for questions of quantum cosmology, the explicit dependence on gauge parameters in published results for "observables" signal that we are still far from understanding the basic issues. On the other hand, in a very restricted domain (2D quantum gravity in a flat background) which includes spherically reduced Einstein (-deSitter) gravity, many interesting results can be obtained [5]. They testify to the fact that, at least in such a situation, the uncompromising stance of a quantum field theory can be maintained to its full extent.

The author could have not succeeded in pursuing this program for many years without the large number of collaborators (cf. the references) of the "Vienna Group" and its eventual extensions. He thanks in particular, the Austrian Science Foundation (FWF) for financing several projects related to this field, the last one being P160 30-N08. Among the other forms of support, he is especially grateful to the Austrian Academy of Sciences which, in the framework of the collaboration with the National Academy of Sciences of the Ukraine, co-financed the multilateral research project "Quantum Gravity, Cosmology and Categorification" and which also supported the travel expenses to the Kyiv–Sevastopol Meeting 2005. The talk related to this paper was given by the author at the occasion of the award of the degree of a honorary doctorate by the National Academy of Sciences of the Ukraine.

1. G.T. Horowitz, (1996), gr-qc/9604051; C. Rovelli, Living Rev. Rel. **1**, 1 (1998), gr-qc/9710008: Notes for a brief history of quantum gravity, in *9th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories*, 2000, arXiv:gr-qc/0006061.
2. S. Carlip, Rept. Prog. Phys. **64**, 885 (2001), gr-qc/0108040; L. Smolin, (2003), hep-th/0303185.
3. K. Eppley and E. Hannah, Found. Phys. **7**, 51 (1977).
4. G. 'tHooft and M.J.G. Veltman, Ann. Poincaré Phys. Theor. A **20**, 69 (1974).
5. D. Grumiller, W. Kummer and D.V. Vassilevich, Phys. Rept. **369**, 327 (2002), hep-th/0204253.
6. M. Nakahara, *Geometry, Topology and Physics* (IOP Publishing, Bristol, 1990).
7. A.G. Riess *et al.*, Astrophys. J. **560**, 49 (2001), astro-ph/0104455; A.G. Riess, The Publications of the Astronomical Society of the Pacific **112**, 1284 (2000); Supernova Search Team, A.G. Riess et al., Astron. J. **116**, 1009 (1998), astro-ph/9805201; Supernova Cosmology Project, S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999), astro-ph/9812133.
8. A. Sen, Phys. Lett. B **119**, 89 (1982); A. Ashtekar, Phys. Rev. Lett. **57**, 2244 (1986); Phys. Rev. D **36**, 1587 (1987); C. Rovelli, Class. Quant. Grav. **8**, 1613 (1991).
9. J.F. Barbero, Phys. Rev. D **51**, 5507 (1995), gr-qc/9410014; G. Immirzi, Class. Quant. Grav. **14**, L177 (1997), gr-qc/9612030; Nucl. Phys. Proc. Suppl. **57**, 65 (1997), gr-qc/9701052.

10. A. Einstein, Annalen Phys. **49**, 769 (1916).
11. M. Fierz, Helv. Phys. Acta **29**, 128 (1956); P. Jordan, Z. Phys. **157**, 112 (1959); C. Brans and R.H. Dicke, Phys. Rev. **124**, 925 (1961).
12. E.E. Salpeter and H.A. Bethe, Phys. Rev. **84,** 1232 (1951); N. Nakanishi, Prog. Theor. Phys. Suppl. **43**, 1 (1969); R. Barbieri and E. Remiddi, Nucl. Phys. B **141**, 413 (1978); W.E. Caswell and G.P. Lepage, Phys. Rev. A **18**, 810 (1978).
13. W. Kummer, Eur. Phys. J. C **21**, 175 (2001), hep-th/0104123.
14. J.A. Dixon and J.C. Taylor, Nucl. Phys. B **78**, 552 (1974); J.A. Dixon, Nucl. Phys. B **99**, 420 (1975); H. Kluberg-Stern and J.B. Zuber, Phys. Rev. D **12**, 467 (1975); W. Konetschny and W. Kummer, Nucl. Phys. B **124**, 145 (1977); S.D. Joglekar and B.W. Lee, Ann. Phys. **97**, 160 (1976).
15. J.I. Friedman, H.W. Kendall, and R.E. Taylor, Rev. Mod. Phys. **63,** 573 (1991).
16. D.J. Gross and F. Wilczek, Phys. Rev. D **8**, 3633 (1973); Phys. Rev. D **9**, 980 (1974).
17. A.M. Polyakov, Nucl. Phys. B **164**, 171 (1980); V.S. Dotsenko and S.N. Vergeles, Nucl. Phys. B **169**, 527 (1980); R.A. Brandt, F. Neri, and M.-A. Sato, Phys. Rev. D **24**, 879 (1981); J. Frenkel and J.C. Taylor, Nucl. Phys. B **246**, 231 (1984).
18. A. Andrasi, Eur. Phys. J. C **18**, 601 (2001), hep-th/9912138.
19. B. Dittrich, *Partial and Complete Observables for Canonical General Relativity*, gr-qc/0507106.
20. J.F. Donoghue, Phys. Rev. D **50**, 3874 (1994), gr-qc/9405057.
21. *Euclidean Quantum Gravity*, edited by G.W. Gibbson and S.W. Hawking (World Scientific, Singapore, 1993).
22. R. Arnowitt, S. Deser, and C.W. Misner, in *Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962).
23. C.G. Torre and M. Varadarajan, Class. Quant. Grav. **16**, 2651 (1999), hep-th/9811222.
24. B.S. DeWitt, Phys. Rev. **160**, 1113 (1967); C.W. Misner, Phys. Rev. **186**, 1319 (1969).
25. K. Kuchař, Phys. Rev. D **4**, 955 (1971)
26. P. Bergmann and A. Komar, in *Recent Developments in General Relativity* (Pergamon, New York, 1962); C. Teitelboim, Phys. Lett. B **56**, 376 (1975); K. Kuchař, in *Quantum gravity 2: an Oxford Symposium*, edited by C. Isham, R. Penrose, and D. Sciama (Clarendon Press, Oxford, 1975).
27. J. Polchinski, *String Theory* (Cambridge University Press, 1998), Vol. 1: An Introduction to the Bosonic String; Vol. 2: Superstring Theory and Beyond.
28. P. Van Nieuwenhuizen, Phys. Rept. **68**, 189 (1981).
29. G. Mandal, A.M. Sengupta, and S.R. Wadia, Mod. Phys. Lett. A **6**, 1685 (1991); S. Elitzur, A. Forge and E. Rabinovici, Nucl. Phys. B **359**, 581 (1991); E. Witten, Phys. Rev. D **44**, 314 (1991); R. Dijkgraaf, H. Verlinde, and E. Verlinde, Nucl. Phys. B **371**, 269 (1992); C.G. Callan, Jr., S.B. Giddings, J.A. Harvey and A. Strominger, Phys. Rev. D **45**, 1005 (1992), hep-th/9111056.
30. B.M. Barbashov, V.V. Nesterenko, and A.M. Chervyakov, Teor. Mat. Fiz. **40**, 15 (1979); J. Phys. A **13**, 301 (1979); Theor. Math. Phys. **40**, 572 (1979); E. D'Hoker and R. Jackiw, Phys. Rev. D **26**, 3517 (1982); C. Teitelboim, Phys. Lett. B **126**, 41 (1983): E. D'Hoker, D. Freedman and R. Jackiw, Phys. Rev. D **28**, 2583 (1983); E.D'Hoker and R. Jackiw, Phys. Rev. Lett. **50**, 1719 (1983).
31. N. Ikeda and K.I. Izawa, Prog. Theor. Phys. **90**, 237-246 (1993), hep-th/9304012; N. Ikeda, Ann. Phys. **235,** 435-464 (1994), hep-th/9312059; P. Schaller and T. Strobl, Mod. Phys. Lett. A **9,** 3129-3136 (1994), hep-th/9405110.
32. W. Kummer and D.J. Schwarz, Nucl. Phys. B **382**, 171 (1992); F. Haider and W. Kummer, Int. J. Mod. Phys. A **9**, 207 (1994); W. Kummer, H. Liebl, and D.V. Vassilevich, Nucl. Phys. B **493**, 491 (1997), gr-qc/9612012; Nucl. Phys. B **544**, 403 (1999), hep-th/9809168.
33. V. Schomerus, JHEP **06**, 030 (1999), hep-th/9903205.
34. N: Seiberg and E. Witten, JHEP **09**, 032 (1999), hep-th/9908142.
35. M. Ertl, W. Kummer, and T. Strobl, JHEP **01**, 042(2001), hep-th/0012219; L. Bergamin and W. Kummer, JHEP **0305**, 074 (2003), hep-th/0209209; L. Bergamin and W. Kummer, Phys. Rev. D **68**, 1004005/1 (2003), hep-th/0306217; L. Bergamin, D. Grumiller, and W. Kummer, J. Phys. A **37**, 3881 (2004), hep-th/0310006; L. Bergamin and W. Kummer, Eur. Phys. J. C **39,** 541 (2005), hep-th/0402138; L. Bergamin and W. Kummer, Eur. Phys. J. C **39,** s53 (2005), hep-th/0411104.
36. Y.C. Park and A. Strominger, PRD **47**, 1569 (1993), hep-th/9210017.
37. S.W. Hawking, Phys. Rev. D **53**, 3099 (1996), hep-th/9510029.
38. D. Grumiller, W. Kummer, and D.V. Vassilevich, Nucl. Phys. B **580**, 438 (2000), gr-qc/0001038; P. Fischer, D. Grumiller, W. Kummer, and D.V. Vassilevich, Phys. Lett. B **521**, 357 (2001), gr-qc/0105034, Erratum ibid. B **532,** 373 (2002); D. Grumiller, Int. J. Mod. Phys. A **17**, 989 (2001), hep-th/0111138; Class. Quant. Grav. **19**, 997 (2002), gr-qc/0111097.
39. D. Grumiller, W. Kummer, and D.V. Vassilevich, JHEP **07,** 009 (2003), hep-th/0305036.

Received 01.07.09

УСПІХИ ТА ПРОБЛЕМИ КВАНТОВОЇ ГРАВІТАЦІЇ

В. Куммер

Р е з ю м е

З огляду безкомпромісного фізика-теоретика квантова теорія гравітації пов'язана із серйозними технічними і, перш за все, концептуальними проблемами, особливо в сенсі істинних "фізичних" спостерігачів. Ця ситуація суттєво не поліпшилася навіть із появою останніх спроб в контексті новітніх переформулювань гравітації. Незважаючи на це, основна мета щодо побудови основ квантової гравітації незалежної від будь-яких припущень за межами стандартної квантової теорії поля може бути досягнута для часткового випадку, а саме двовимірної ділатонної квантової гравітації. Подальшими побічними результатами досліджень "Віденської школи Куммера" є розширення двовимірної ділатонної квантової гравітації до $N = (1,1)$ і $N = (2,2)$ двовимірної супергравітації, для якої стало можливим знаходження повних розв'язків деяких старих задач, що мають безпосереднє відношення до теорії суперструн, включення до неї концепції σ-моделей Пуассона та встановлення існування "віртуальних чорних дірок".