FIELDS AND ELEMENTARY PARTICLES

THE LAGRANGIAN AND HAMILTONIAN FORMALISMS FOR THE CLASSICAL RELATIVISTIC ELECTRODYNAMICS MODELS REVISITED

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The work is devoted to studying some new classical electrodynamics models of interacting charged point particles and the aspects of the quantization via the Dirac procedure related to them. Based on the vacuum field theory no-geometry approach developed in [6,7,9], the Lagrangian and Hamiltonian reformulations of some alternative classical electrodynamics models are devised. The Dirac-type quantization procedure for the considered alternative electrodynamics models, based on the obtained canonical Hamiltonian formulations, is developed.

1. Introductory Setting

It is well known [5, 11, 13, 42] that the "*quintessence*" of the classical relativistic electrodynamics in the Minkowski space $\mathbb{M}^4 := \mathbb{E}^3 \times \mathbb{R}$ of a moving charged point particle consists in a successive derivation of the Lorentz force expression

$$F := qE + qu \times B, \tag{1.1}$$

where $q \in \mathbb{R}$ is a particle electric charge, $u \in \mathbb{E}^3$ is its velocity vector (expressed here in the light speed c units),

$$E := -\partial A/\partial t - \nabla \varphi \tag{1.2}$$

is the corresponding external electric field and

$$B := \nabla \times A \tag{1.3}$$

is the corresponding external magnetic field acting on the charged particle, being expressed through the suitable vector $A : \mathbb{M}^4 \to \mathbb{E}^3$ and scalar $\varphi : \mathbb{M}^4 \to \mathbb{R}$ potentials. Writing fields (1.2) and (1.3), we have denoted, by "∇," the standard gradient operation with respect to the spatial variable $r \in \mathbb{E}^3$ and, by "$\times$," the vector product in the three-dimensional Euclidean vector space $\mathbb{E}^3$ naturally endowed with the usual scalar product $\langle \cdot, \cdot \rangle$.

Let the additional Lorentz condition

$$\partial \varphi/\partial t + \langle \nabla, A \rangle = 0 \tag{1.4}$$

for the potentials $(\varphi, A) : \mathbb{M}^4 \to \mathbb{R} \times \mathbb{E}^3$ satisfying the Lorentz-invariant wave field equations

$$\partial^2 \varphi/\partial t^2 - \nabla^2 \varphi = \rho, \partial^2 A/\partial t^2 - \nabla^2 A = J \tag{1.5}$$

be imposed. Here, $\rho : \mathbb{M}^4 \to \mathbb{R}$ and $J : \mathbb{M}^4 \to \mathbb{E}^3$ are, respectively, charge and current densities of the ambient matter satisfying the charge continuity relation

$$\partial \rho/\partial t + \langle \nabla, J \rangle = 0. \tag{1.6}$$

Then the well-known [13, 5, 42, 11] classical electromagnetic Maxwell field equations

$$\nabla \times E + \partial B/\partial t = 0, \quad \langle \nabla, E \rangle = \rho,$$

$$\nabla \times B - \partial E/\partial t = J, \quad \nabla \times B = 0 \tag{1.7}$$

hold for all $(r, t) \in \mathbb{M}^4$ with respect to a chosen reference system $\mathcal{K}$.

We note that the Maxwell equations (1.7) are not directly reduced, via definitions (1.2) and (1.3), to the wave field equations (1.5), if the Lorentz condition (1.4) is not taken into consideration. This fact becomes very important if we change our subject of reasonings to the determining role of the Maxwell equations (1.7) with (1.6) and put the Lorentz condition (1.4) jointly with (1.5) and the continuity relation (1.6) into consideration as governing relations. Concerning the assumptions formulated above, the following proposition holds.

Proposition 1. *The Lorentz-invariant wave equations (1.5) for the potentials $(\varphi, A) : \mathbb{M}^4 \to \mathbb{R} \times \mathbb{E}^3$ considered jointly with the Lorentz condition (1.4) and the charge continuity relation (1.5) are completely equivalent to the Maxwell field equations (1.7).*

PROOF. Really, having substituted the partial derivative $\partial \varphi / \partial t = -\langle \nabla, A \rangle$ following from (1.4) into (1.5), one easily obtains that

$$\partial^2 \varphi / \partial t^2 = -\langle \nabla, \partial A / \partial t \rangle = \langle \nabla, \nabla \varphi \rangle + \rho. \tag{1.8}$$

Relation (1.8) yields the gradient expression

$$\langle \nabla, -\partial A / \partial t - \nabla \varphi \rangle = \rho. \tag{1.9}$$

Taking the electric field definition (1.2) into account, expression (1.9) is reduced to

$$\langle \nabla, E \rangle = \rho, \tag{1.10}$$

entering the first pair of the Maxwell equations (1.7).

Having now applied the operation "$\nabla \times$" to (1.2), we obtain, owing definition (1.3),

$$\nabla \times E + \partial B / \partial t = 0, \tag{1.11}$$

being the next equation entering the first pair of the Maxwell equations (1.7).

Applying now the operation "$\nabla \times$" to (1.3):

$$\nabla \times B = \nabla \times (\nabla \times A) = \nabla \langle \nabla, A \rangle - \nabla^2 A =$$

$$= -\nabla(\partial \varphi / \partial t) - \partial^2 A / \partial t^2 + (\partial^2 A / \partial t^2 - \nabla^2 A) =$$

$$= \frac{\partial}{\partial t}(-\partial \varphi / \partial t - \partial A / \partial t) + J = \partial E / \partial t + J, \tag{1.12}$$

we now obtain the resulting equation

$$\nabla \times B = \partial E / \partial t + J$$

exactly coinciding with that entering the second pair of the Maxwell equations (1.7). The final *"no-magnetic charge"* equation

$$\langle \nabla, B \rangle = \langle \nabla, \nabla \times B \rangle = 0$$

entering (1.7) follows easily from the elementary differential identity $\langle \nabla, \nabla \times \rangle = 0$, thereby finishing the proof. $\square$

The proposition above allows us to consider the potential functions $(\varphi, A) : \mathbb{M}^4 \to \mathbb{R} \times \mathbb{E}^3$ as fundamental ingredients of the ambient vacuum field medium, by means of which we can try to describe the related physical behavior of charged point particles imbedded into the space-time $\mathbb{M}^4$ filled in with the vacuum field medium. This way of reasoning is strongly supported by the next important observation.

Observation. *The Lorentz condition (1.4) means, in reality, the scalar potential field $\varphi : \mathbb{M}^4 \to \mathbb{R}$ continuity relation, whose origin lies in some new field conservation law characterizing the deep intrinsic vacuum field medium structure.*

To make this observation more clear and exact, let us recall the definition [13,42,5,11] of the electric current $J : \mathbb{M}^4 \to \mathbb{E}^3$ in the dynamical form

$$J := \rho v, \tag{1.13}$$

where the vector $v : \mathbb{M}^4 \to \mathbb{E}^3$ is the corresponding electric charge flow velocity understood here [18] in the hydrodynamical sense. Thus, the continuity relation

$$\partial \rho / \partial t + \langle \nabla, \rho v \rangle = 0 \tag{1.14}$$

holds, and it can be easily rewritten [18] as the integral conservation law

$$\frac{d}{dt} \int_{\Omega_t} \rho d^3 r = 0 \tag{1.15}$$

for the charge held inside of any bounded domain $\Omega_t \subset \mathbb{E}^3$ moving in the space-time $\mathbb{M}^4$. Its every intrinsic point $r \in \Omega_t$ changes in time as

$$dr / dt := v \tag{1.16}$$

with respect to the corresponding electric charge flow velocity.

Following the same reasonings as above, we can state the next proposition.

Proposition 2. *The Lorentz condition (1.4) is exactly equivalent to the following integral conservation law:*

$$\frac{d}{dt} \int_{\Omega_t} \varphi d^3 r = 0, \tag{1.17}$$

where $\Omega_t \subset \mathbb{R}^3$ is any bounded domain moving according to the evolution equation

$$dr/dt := v, \qquad (1.18)$$

meaning the velocity vector of a local potential field flow inside the domain Ω_t propagating in the space-time $\mathbb{M}^4$. PROOF. Consider, first, the corresponding solutions to the potential field equations (1.5), taking condition (1.13) into account. Owing to the results from [5, 13], one finds that

$$A = \varphi v, \qquad (1.19)$$

giving rise to the following form of the Lorentz condition (1.4):

$$\partial \varphi / \partial t + \langle \nabla, \varphi v \rangle = 0. \qquad (1.20)$$

The latter, obviously, can be equivalently rewritten [18] as the integral conservation law (1.17).

□

The proposition formulated above allows a physically motivated interpretation by means of involving the important notion - *the vacuum potential field* describing the observable interactions between charged point particles. Namely, we can *a priori* endow the ambient vacuum medium with a scalar potential field function $W := q\varphi : \mathbb{M}^4 \to \mathbb{R}$ satisfying the governing vacuum field equations

$$\partial^2 W/\partial t^2 - \nabla^2 W = 0, \quad \partial W/\partial t + \langle \nabla, Wv \rangle = 0, \quad (1.21)$$

taking into account that there are no external sources besides material particles possessing only a virtual capability to disturb the vacuum field medium. Moreover, as the related vacuum potential field function $W : \mathbb{M}^4 \to \mathbb{R}$ allows the natural potential energy interpretation, its origin should be assigned not only to the charged interacting medium, but also to any other medium possessing a virtual capability to interact, including, for instance, material particles interacting through the gravity.

The latter allows us to make the next important step consisting in deriving the equation governing the corresponding potential field $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$ assigned to the vacuum field medium in the vicinity of any particle located at a point $R(t) \in \mathbb{E}^3$ moving with velocity $u \in \mathbb{E}^3$ at time $t \in \mathbb{R}$. As it can be easily enough shown [6, 7, 10], the corresponding evolution equation governing the related potential field function $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$ looks as follows:

$$\frac{d}{dt}(-\bar{W}u) = -\nabla \bar{W}, \qquad (1.22)$$

where, by definition, $\bar{W} := W(r,t)|_{r \to R(t)}, u := dR(t)/dt$ at a point particle location $(R(t), t) \in \mathbb{M}^4$.

Similarly, if we consider two point particles which interact with each other, are located at points $R(t)$ and $R_f(t) \in \mathbb{E}^3$ at time $t \in \mathbb{R}$, and are moving, respectively, with velocities $u := dR(t)/dt$ and $u_f := dR_f(t)/dt$, the corresponding potential field function $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$ related to the particle located at the point $R(t) \in \mathbb{E}^3$, looks as

$$\frac{d}{dt}[-\bar{W}(u - u_f)] = -\nabla \bar{W}. \qquad (1.23)$$

The dynamical potential field equations (1.22) and (1.23) obtained above stimulate us to proceed the further study of their physical properties and to compare them with the already available classical results for suitable Lorentz-type forces described within the electrodynamics of moving charged point particles interacting with an external electromagnetic field.

To realize this program, we, being strongly inspired by works [22–26, 33, 40] and especially by the original works [28, 29] devoted to solving the classical problem of reconciling gravitational and electrodynamical charges within the Mach–Einstein ether paradigm, first reanalyze successively the classical Mach–Einstein relativistic electrodynamics of a moving charged point particle and, second, study the resulting electrodynamic theories associated with our vacuum potential field dynamical equations (1.22) and (1.23), making use of the fundamental Lagrangian and Hamiltonian formalisms. Based on the results obtained, the canonical Dirac-type quantization procedure is applied to the corresponding energy conservation laws related naturally to electrodynamic models considered in the work.

2. Classical Relativistic Electrodynamics Revisited

The classical relativistic electrodynamics of a freely moving charged point particle in the Minkowski space-time $\mathbb{M}^4 := \mathbb{E}^3 \times \mathbb{R}$ is, as well known, based [5, 11, 13, 42] on the Lagrangian formalism assigning the Lagrangian function

$$\mathcal{L} := -m_0(1 - u^2)^{1/2} \qquad (2.1)$$

to it, where $m_0 \in \mathbb{R}$ is the so-called particle rest mass, and $u \in \mathbb{E}^3$ is its spatial velocity in the Euclidean space $\mathbb{E}^3$ expressed here and throughout further in the light speed c units. The least action Fermat principle in the

form

$$\delta S = 0, \quad S := -\int_{t_1}^{t_2} m_0 (1 - u^2)^{1/2} dt \qquad (2.2)$$

for any fixed temporal interval $[t_1, t_2] \subset \mathbb{R}$ gives rise to the well-known relativistic relations for the particle mass

$$m = m_0 (1 - u^2)^{-1/2}, \qquad (2.3)$$

the particle momentum

$$p := mu = m_0 u (1 - u^2)^{-1/2}, \qquad (2.4)$$

and the particle energy

$$\mathcal{E}_0 = m = m_0 (1 - u^2)^{-1/2}. \qquad (2.5)$$

The origin of Lagrangian (2.1) can be extracted, owing to the reasonings from [13,42], from the action expression

$$S := -\int_{t_1}^{t_2} m_0 (1 - u^2)^{1/2} dt = -\int_{\tau_1}^{\tau_2} m_0 d\tau, \qquad (2.6)$$

on the suitable temporal interval $[\tau_1, \tau_2] \subset \mathbb{R}$, where, by definition,

$$d\tau := dt (1 - u^2)^{1/2} \qquad (2.7)$$

and $\tau \in \mathbb{R}$ is the so-called proper temporal parameter assigned to a freely moving particle with respect to the "rest" reference system $\mathcal{K}_r$. Action (2.6) looks from the dynamical point of view slightly controversial, since it is physically defined with respect to the "rest" reference system $\mathcal{K}_r$, giving rise to the constant action $S = -m_0(\tau_2 - \tau_1)$, as the limits of integrations $\tau_1 < \tau_2 \in \mathbb{R}$ were taken to be fixed from the very beginning. Moreover, let us consider this particle as charged with a charge $q \in \mathbb{R}$ and moving in the Minkowski space-time $\mathbb{M}^4$ under the action of an electromagnetic field $(\varphi, A) \in \mathbb{R} \times \mathbb{E}^3$, and let the corresponding classical (relativistic) action functional be chosen (see [5, 11, 13, 26, 42]) as

$$S := \int_{\tau_1}^{\tau_2} [-m_0 d\tau + q\langle A, \dot{r} \rangle d\tau - q\varphi (1 - u^2)^{-1/2} d\tau] \qquad (2.8)$$

with respect to the so-called "rest" reference system parametrized by the Euclidean space-time variables $(r, \tau) \in \mathbb{E}^4$. Here, as before, $\langle \cdot, \cdot \rangle$ is the standard scalar

product in the related Euclidean subspace $\mathbb{E}^3$, and we denoted $\dot{r} := dr/d\tau$ in contrast to the definition $u := dr/dt$. Action (2.8) can be rewritten with respect to the reference system moving with a velocity $u \in \mathbb{E}^3$ as

$$S = \int_{t_1}^{t_2} \mathcal{L} dt, \quad \mathcal{L} := -m_0 (1 - u^2)^{1/2} + q\langle A, u \rangle - q\varphi \qquad (2.9)$$

on the suitable temporal interval $[t_1, t_2] \subset \mathbb{R}$. This gives rise to the [13, 5, 42, 11] dynamical expressions

$$P = p + qA, \qquad p = mu, \qquad (2.10)$$

for the particle momentum and

$$\mathcal{E}_0 = [m_0^2 + (P - qA)^2]^{1/2} + q\varphi \qquad (2.11)$$

for the particle energy. Here, by definition, $P \in \mathbb{E}^3$ means the common momentum of the particle and the ambient electromagnetic field at a space-time point $(r, t) \in \mathbb{M}^4$.

The obtained expression (2.11) for the particle energy $\mathcal{E}_0$ also looks slightly controversial, since the potential energy $q\varphi$, entering additively, has no impact to the particle mass $m = m_0 (1 - u^2)^{-1/2}$. As it was already mentioned [16] by L. Brillouin, the fact that the potential energy has no impact to the particle mass says us that "... any possibility of existing the particle mass related to an external potential energy is completely excluded". This and some other special relativity theory and electrodynamics problems, as is well known, stimulated many other prominent physicists of the past [4, 16, 21, 43, 42] and the present [22–24, 33, 38–41, 44, 45] to make significant efforts aiming to develop alternative relativity theories based on completely different [20, 25, 28, 30–32, 34, 35, 48, 49] space-time and matter structure principles.

There also is another controversial inference from the action expression (2.9). As one can easily show [5, 11, 13, 42], the corresponding dynamical equation for the Lorentz force is given as

$$dp/dt = F := qE + qu \times B, \qquad (2.12)$$

where the operation "$\times$" denotes, as before, the standard vector product, and we put, by definition,

$$E := -\partial A/\partial t - \nabla \varphi \qquad (2.13)$$

for the related electric field and

$$B := \nabla \times A \qquad (2.14)$$

for the related magnetic field acting on the charged point particle q; the operation "∇" is here, as before, the standard gradient. The obtained expression (2.12) means, in particular, that the Lorentz force F depends linearly on the particle velocity vector $u \in \mathbb{E}^3$, giving rise to its strong dependence on the reference system, with respect to which the charged particle q moves. Namely, the attempts to reconcile this and some related controversies [4, 16, 20, 37] forced A. Einstein to devise his special relativity theory and to proceed further to creating his general relativity theory trying to explain the gravity by means of a geometrization of space-time and matter in the Universe. Here, we must mention that the classical Lagrangian function $\mathcal{L}$ in (2.9) is written by means of the mixed combinations of terms expressed by means of both the Euclidean "rest" reference system variables $(r, \tau) \in \mathbb{E}^4$ and the arbitrarily chosen reference system variables $(r, t) \in \mathbb{M}^4$.

These problems were recently analyzed from a completely different "no-geometry" point of view in [6, 7, 9, 10, 20], where new dynamical equations were derived, being free of controversy mentioned above. Moreover, the devised approach allowed one to avoid the introduction of the well-known Lorentz transformations of the space-time reference systems, with respect to which the action functional (2.9) is invariant. From this point of view, there are the very interesting reasonings of work [24], in which the Galilean invariant Lagrangians possessing the intrinsic Poincaré–Lorentz group symmetry are re-analyzed. In what follows, we will reanalyzed the results obtained in [6, 7, 10] from the classical Lagrangian and Hamiltonian formalisms, which will shed a new light on the related physical backgrounds of the vacuum field theory approach to the common study of electromagnetic and gravitational effects.

3. Vacuum Field Theory Electrodynamics Equations: Lagrangian Analysis

3.1. A freely moving point particle – an alternative electrodynamic model

Within the vacuum field theory approach to the common description of the electromagnetism and the gravity devised in [6, 7], the main vacuum potential field function $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$ related to a charged point particle q satisfies, in the case of rested external charged point objects, the dynamical equation (1.21)

$$\frac{d}{dt}(-\bar{W}u) = -\nabla\bar{W}, \qquad (3.1)$$

where, as above, $u := dr/dt$ is the particle velocity with respect to some reference system.

To analyze the dynamical equation (3.1) from the Lagrangian point of view, we will write the corresponding action functional as

$$S := -\int_{t_1}^{t_2} \bar{W} dt = -\int_{\tau_1}^{\tau_2} \bar{W}(1 + \dot{r}^2)^{1/2}\, d\tau \qquad (3.2)$$

in the "rest" reference system $\mathcal{K}_r$. Having fixed the proper temporal parameters $\tau_1 < \tau_2 \in \mathbb{R}$ and using the least action condition $\delta S = 0$, one finds easily that

$$p := \partial\mathcal{L}/\partial\dot{r} = -\bar{W}\dot{r}(1 + \dot{r}^2)^{-1/2} = -\bar{W}u,$$

$$\dot{p} := dp/d\tau = \partial\mathcal{L}/\partial r = -\nabla\bar{W}(1 + \dot{r}^2)^{1/2}. \qquad (3.3)$$

Here, owing to (3.2), the corresponding Lagrangian function

$$\mathcal{L} := -\bar{W}(1 + \dot{r}^2)^{1/2}. \qquad (3.4)$$

Recalling now the definition of particle mass

$$m := -\bar{W} \qquad (3.5)$$

and the relations

$$d\tau = dt(1 - u^2)^{1/2}, \quad \dot{r}d\tau = udt, \qquad (3.6)$$

from (3.3), we easily obtain exactly the dynamical equation (3.1). Moreover, one easily obtains that the dynamical mass defined by expression (3.5) is given as

$$m = m_0(1 - u^2)^{-1/2},$$

which coincides with result (2.3) of the preceding section. Thereby, based on the above-obtained results, one can formulate the following proposition.

Proposition 3. *The alternative freely moving point particle electrodynamic model (3.1) allows the least action formulation (3.2) with respect to the "rest" reference system variables, where the Lagrangian function is given by expression (3.4). Its electrodynamics is completely equivalent to that of a classical relativistic freely moving point particle described in Section 2.*

3.2. A moving charged point particle – an alternative electrodynamic model

Proceed now to the case where our charged point particle q moves in the space-time with velocity vector $u \in \mathbb{E}^3$

and interacts with another external charged point particle, moving with velocity vector $u_f \in \mathbb{E}^3$ subject to some common reference system $\mathcal{K}$. As was shown in [6, 7], the corresponding dynamical equation on the vacuum potential field function $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$ is given as

$$\frac{d}{dt}[-\bar{W}(u - u_f)] = -\nabla\bar{W}. \qquad (3.7)$$

As the external charged particle moves in the space-time, it generates the related magnetic field $B := \nabla \times A$, whose magnetic vector potential $A : \mathbb{M}^4 \to \mathbb{E}^3$ is defined, owing to the results of [6, 7, 20], as

$$qA := \bar{W}u_f. \qquad (3.8)$$

Since, owing to (3.3), the particle momentum $p = -\bar{W}u$, Eq. (3.7) can be equivalently rewritten as

$$\frac{d}{dt}(p + qA) = -\nabla\bar{W}. \qquad (3.9)$$

To represent the dynamical equation (3.9) within the classical Lagrangian formalism, we start from the following action functional naturally generalizing functional (3.2):

$$S := -\int_{\tau_1}^{\tau_2} \bar{W}(1 + |\dot{r} - \dot{\xi}|^2)^{1/2} \, d\tau. \qquad (3.10)$$

Here, we denoted $\dot{\xi} = u_f dt/d\tau$, $d\tau = dt(1 - |u - u_f|^2)^{1/2}$, with regard for the relative velocity of our charged point particle q with respect to the reference system $\mathcal{K}_f$, moving with velocity vector $u_f \in \mathbb{E}^3$ subject to the reference system $\mathcal{K}$. In this case, evidently, our charged point particle q moves with the velocity vector $u - u_f \in \mathbb{E}^3$ subject to the reference system $\mathcal{K}_f$, and the external charged particle is, respectively, in rest.

Compute now the least action variational condition $\delta S = 0$, taking into account that, owing to (3.10), the corresponding Lagrangian function is given as

$$\mathcal{L} := -\bar{W}(1 + |\dot{r} - \dot{\xi}|^2)^{1/2}. \qquad (3.11)$$

Thereby, the total momentum of particles

$$P := \partial\mathcal{L}/\partial\dot{r} = -\bar{W}(\dot{r} - \dot{\xi})(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} =$$

$$= -\bar{W}\dot{r}(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} + \bar{W}\dot{\xi}(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} =$$

$$= mu + qA := p + qA, \qquad (3.12)$$

and the dynamical equation is given as

$$\frac{d}{d\tau}(p + qA) = -\nabla\bar{W}(1 + |\dot{r} - \dot{\xi}|^2)^{1/2}. \qquad (3.13)$$

Since $d\tau = dt(1 - |u - u_f|^2)^{1/2}$ and $(1 + |\dot{r} - \dot{\xi}|^2)^{1/2} = (1 - |u - u_f|^2)^{-1/2}$, relation (3.13) yields exactly the dynamical equation (3.9). Thus, we can formulate our result as the next proposition.

Proposition 4. *The alternative classical relativistic electrodynamic model (3.7) allows the least action formulation (3.10) with respect to the "rest" reference system variables, where the Lagrangian function is given by expression (3.11).*

3.3. A moving charged point particle – a dual to the classical alternative electrodynamic model

It is easy to observe that the action functional (3.10) is written in view of the classical Galilean transformations of reference systems. If we now consider both the action functional (3.2) for a charged point particle moving with respect to the reference system $\mathcal{K}_r$ and its interaction with an external magnetic field generated by the vector potential $A : \mathbb{M}^4 \to \mathbb{E}^3$, it can be naturally generalized as

$$S := \int_{t_1}^{t_2} (-\bar{W}dt + q\langle A, dr\rangle) =$$

$$= \int_{\tau_1}^{\tau_2} [-\bar{W}(1 + \dot{r}^2)^{1/2} + q\langle A, \dot{r}\rangle]d\tau, \qquad (3.14)$$

where we accepted that $d\tau = dt(1 - u^2)^{1/2}$.

Thus, the corresponding common particle-field momentum looks as

$$P := \partial\mathcal{L}/\partial\dot{r} = -\bar{W}\dot{r}(1 + \dot{r}^2)^{-1/2} + qA =$$

$$= mu + qA := p + qA \qquad (3.15)$$

and satisfies the equation

$$\dot{P} := dP/d\tau = \partial\mathcal{L}/\partial r = -\nabla\bar{W}(1 + \dot{r}^2)^{1/2} + q\nabla\langle A, \dot{r}\rangle =$$

$$= -\nabla\bar{W}(1 - u^2)^{-1/2} + q\nabla\langle A, u\rangle(1 - u^2)^{-1/2}, \qquad (3.16)$$

where

$$\mathcal{L} := -\bar{W}(1 + \dot{r}^2)^{1/2} + q\langle A, \dot{r}\rangle \qquad (3.17)$$

is the corresponding Lagrangian function. Taking relation $d\tau = dt(1 - u^2)^{1/2}$ into account, one finds easily from (3.16) that

$$dP/dt = -\nabla\bar{W} + q\nabla\langle A, u\rangle. \qquad (3.18)$$

By substituting (3.15) into (3.18) and using the well-known [13] identity

$$\nabla\langle a, b\rangle = \langle a, \nabla\rangle b + \langle b, \nabla\rangle a + b\times(\nabla\times a) + a\times(\nabla\times b), \quad (3.19)$$

where $a, b \in \mathbb{E}^3$ are arbitrary vector functions, we obtain finally the classical expression for the Lorentz force F acting on the moving charged point particle q:

$$dp/dt := F = qE + qu \times B. \qquad (3.20)$$

Here, by definition,

$$E := -\nabla\bar{W}q^{-1} - \partial A/\partial t \qquad (3.21)$$

is the corresponding electric field, and

$$B := \nabla \times A \qquad (3.22)$$

is the corresponding magnetic field.

We formulate the result obtained as the next proposition.

Proposition 5. *The classical relativistic Lorentz force (3.20) allows the least action formulation (3.14) with respect to the "rest" reference system variables, where the Lagrangian function is given by expression (3.17). Its electrodynamics described by the Lorentz force (3.20) is completely equivalent to the classical relativistic moving point particle electrodynamics described by means of the Lorentz force (2.12) in Section 2.*

Concerning the previously obtained dynamical equation (3.13), we can easily observe that it can be equivalently rewritten as

$$dp/dt = (-\nabla\bar{W} - qdA/dt + q\nabla\langle A, u\rangle) - q\nabla\langle A, u\rangle. \quad (3.23)$$

The latter, owing to (3.18) and (3.20), takes finally the following Lorentz-type form

$$dp/dt = qE + qu \times B - q\nabla\langle A, u\rangle \qquad (3.24)$$

earlier found in [6, 7, 20].

Expressions (3.20) and (3.24) are equal to each other up to the gradient term $F_c := -q\nabla\langle A, u\rangle$, which allows to reconcile the Lorentz forces acting on a charged moving particle q with respect to different reference systems. This fact is important for our vacuum field theory approach, since it needs to use no special geometry and makes it possible to analyze both electromagnetic and gravitational fields simultaneously, based on a new definition of the dynamical mass by means of expression (3.5).

4. The Vacuum Field Theory Electrodynamics Equations: Hamiltonian Analysis

It is well known [1, 2, 8, 11, 19] that any Lagrangian theory allows the equivalent canonical Hamiltonian representation via the classical Legendrian transformation. As we have already formulated our vacuum field theory of a moving charged particle q in the Lagrangian form, we proceed now to its Hamiltonian analysis making use of the action functionals (3.2), (3.11), and (3.14).

Let us take, first, the Lagrangian function (3.4) and momentum (3.3) to define the corresponding Hamiltonian function

$$H := \langle p, \dot{r}\rangle - \mathcal{L} =$$

$$= -\langle p, p\rangle\bar{W}^{-1}(1 - p^2/\bar{W}^2)^{-1/2} + \bar{W}(1 - p^2/\bar{W}^2)^{-1/2} =$$

$$= -p^2\bar{W}^{-1}(1 - p^2/\bar{W}^2)^{-1/2} + \bar{W}^2\bar{W}^{-1}(1 - p^2/\bar{W}^2)^{-1/2} =$$

$$= -(\bar{W}^2 - p^2)(\bar{W}^2 - p^2)^{-1/2} = -(\bar{W}^2 - p^2)^{1/2}. \quad (4.1)$$

As a result, we easily obtain [1,2,8,11] that the Hamiltonian function (4.1) is a conservation law of the dynamical field equation (3.1). That is, for all $\tau, t \in \mathbb{R}$,

$$dH/dt = 0 = dH/d\tau, \qquad (4.2)$$

which naturally allows one to interpret it as the energy expression. Thus, we can write that the particle energy

$$\mathcal{E} = (\bar{W}^2 - p^2)^{1/2}. \qquad (4.3)$$

The suitable Hamiltonian equations equivalent to the vacuum field equation (3.1) look as

$$\dot{r} := dr/d\tau = \partial H/\partial p = p(\bar{W}^2 - p^2)^{-1/2}$$

$$\dot{p} := dp/d\tau = -\partial H/\partial r = \bar{W}\nabla\bar{W}(\bar{W}^2 - p^2)^{-1/2}. \quad (4.4)$$

Thereby, based on the above-obtained results, one can formulate the following proposition.

Proposition 6. *The alternative freely moving point particle electrodynamic model (3.1) allows the canonical Hamiltonian formulation (4.4) with respect to the "rest" reference system variables, where the Hamiltonian function is given by expression (4.1). Its electrodynamics is completely equivalent to the classical relativistic freely moving point particle electrodynamics described in Section 2.*

Based now on the Lagrangian expression (3.11), one can construct, in the same way as above, the Hamiltonian function for the dynamical field equation (3.9) describing the motion of a charged particle q in an external electromagnetic field in the canonical Hamiltonian form:

$$\dot{r} := dr/d\tau = \partial H/\partial P, \quad \dot{P} := dP/d\tau = -\partial H/\partial r. \quad (4.5)$$

Here,

$$H := \langle P, \dot{r}\rangle - \mathcal{L} =$$

$$= \langle P, \dot{\xi} - P\bar{W}^{-1}(1 - P^2/\bar{W}^2)^{-1/2}\rangle + \bar{W}[\bar{W}^2(\bar{W}^2 - P^2)^{-1}]^{1/2} =$$

$$= \langle P, \dot{\xi}\rangle + P^2(\bar{W}^2 - P^2)^{-1/2} - \bar{W}^2(\bar{W}^2 - P^2)^{-1/2} =$$

$$= -(\bar{W}^2 - P^2)(\bar{W}^2 - P^2)^{-1/2} + \langle P, \dot{\xi}\rangle =$$

$$= -(\bar{W}^2 - P^2)^{1/2} - q\langle A, P\rangle(\bar{W}^2 - P^2)^{-1/2}. \quad (4.6)$$

We took into account that, owing to definitions (3.8) and (3.12),

$$qA := \bar{W}u_f = \bar{W}d\xi/dt =$$

$$= \bar{W}\frac{d\xi}{d\tau} \cdot \frac{d\tau}{dt} = \bar{W}\dot{\xi}(1 - |u - v|^2)^{1/2} =$$

$$= \bar{W}\dot{\xi}(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} =$$

$$= -\bar{W}\dot{\xi}(\bar{W}^2 - P^2)^{1/2}\bar{W}^{-1} = -\dot{\xi}(\bar{W}^2 - P^2)^{1/2}, \quad (4.7)$$

or

$$\dot{\xi} = -qA(\bar{W}^2 - P^2)^{-1/2}, \quad (4.8)$$

where $A : \mathbb{M}^4 \to \mathbb{R}^3$ is the related magnetic vector potential generated by the moving external charged particle.

Thereby, we can state that the Hamiltonian function (4.6) satisfies the energy conservation conditions

$$dH/dt = 0 = dH/d\tau \quad (4.9)$$

for all $\tau, t \in \mathbb{R}$. That is, the suitable energy expression

$$\mathcal{E} = (\bar{W}^2 - P^2)^{1/2} + q\langle A, P\rangle(\bar{W}^2 - P^2)^{-1/2} \quad (4.10)$$

holds. Result (4.10) essentially differs from that obtained in [13] which makes use of the well-known Einsteinian Lagrangian for a moving charged point particle

q in an external electromagnetic field. Thereby, our result can be formulated as follows.

Proposition 7. *The alternative classical relativistic electrodynamic model (3.7) allows the Hamiltonian formulation (4.5) with respect to the "rest" reference system variables, where the Hamiltonian function is given by expression (4.6).*

To make this difference more clear, we will analyze below the Lorentz force (3.20) from the Hamiltonian point of view based on the Lagrangian function (3.17). Thus, we obtain that the corresponding Hamiltonian function

$$H := \langle P, \dot{r}\rangle - \mathcal{L} = \langle P, \dot{r}\rangle + \bar{W}(1 + \dot{r}^2)^{1/2} - q\langle A, \dot{r}\rangle =$$

$$= \langle P - qA, \dot{r}\rangle + \bar{W}(1 + \dot{r}^2)^{1/2} =$$

$$= -\langle p, p\rangle\bar{W}^{-1}(1 - p^2/\bar{W}^2)^{-1/2} + \bar{W}(1 - p^2/\bar{W}^2)^{1/2} =$$

$$= -(\bar{W}^2 - p^2)(\bar{W}^2 - p^2)^{-1/2} = -(\bar{W}^2 - p^2)^{1/2}. \quad (4.11)$$

Since $p = P - qA$, expression (4.11) takes the final "*no interaction*" [13, 42, 46, 47] form

$$H = -[\bar{W}^2 - (P - qA)^2]^{1/2}, \quad (4.12)$$

being conservative with respect to the evolution equations (3.15) and (3.16), i.e.,

$$dH/dt = 0 = dH/d\tau \quad (4.13)$$

for all $\tau, t \in \mathbb{R}$. The latter are simultaneously equivalent to the following Hamiltonian system:

$$\dot{r} = \partial H/\partial P = (P - qA)[\bar{W}^2 - (P - qA)^2]^{-1/2},$$

$$\dot{P} = -\partial H/\partial r = (\bar{W}\nabla\bar{W} - \nabla\langle qA, (P - qA)\rangle)\times$$

$$\times[\bar{W}^2 - (P - qA)^2]^{-1/2}, \quad (4.14)$$

that can be easily checked by direct calculations. Really, the first equation

$$\dot{r} = (P - qA)[\bar{W}^2 - (P - qA)^2]^{-1/2} = p(\bar{W}^2 - p^2)^{-1/2} =$$

$$= mu(\bar{W}^2 - p^2)^{-1/2} = -\bar{W}u(\bar{W}^2 - p^2)^{-1/2} = u(1 - u^2)^{-1/2} \quad (4.15)$$

holds, owing to the condition $d\tau = dt(1 - u^2)^{1/2}$ and definitions $p := mu$ and $m = -\bar{W}$ postulated from the very beginning. Similarly, we obtain

$$\dot{P} = -\nabla\bar{W}(1 - p^2/\bar{W}^2)^{-1/2} + \nabla\langle qA, u\rangle(1 - p^2/\bar{W}^2)^{-1/2} =$$

$$= -\nabla \bar{W}(1 - u^2)^{-1/2} + \nabla \langle qA, u \rangle (1 - u^2)^{-1/2}, \quad (4.16)$$

exactly coinciding with Eq. (3.18) subject to the evolution parameter $t \in \mathbb{R}$. We now formulate our result as the next proposition.

Proposition. *Model (3.20) dual to the classical relativistic electrodynamics model allows the Hamiltonian formulation (4.14) with respect to the "rest" reference system variables, where the Hamiltonian function is given by expression (4.12).*

5. The Quantization of Electrodynamics Models within the Vacuum Field Theory no-Geometry Approach

5.1. Statement of the problem

In our recent works [6, 7], there was devised a new regular no-geometry approach to deriving, from the first principles, the electrodynamics of a moving charged point particle q in an external electromagnetic field. This approach has, in part, to reconcile the existing mass-energy controversy [16] within the classical relativistic electrodynamics. Based on the vacuum field theory approach proposed in [6, 7, 20], we have reanalyzed this problem in the sections above both from the Lagrangian and Hamiltonian points of view, having derived crucial expressions for the corresponding energy functions and Lorentz-type forces acting on moving charge point particle q.

Since all of our electrodynamics models were represented here in the canonical Hamiltonian form, they are suitable for applying the Dirac-type quantization procedure to them [3, 12, 15] and for obtaining the related Schrödinger-type evolution equations. The following section is devoted namely to this problem.

5.2. Free point particle electrodynamics model and its quantization

The charged point particle electrodynamics models discussed in Sections 2 and 3 in detail were also considered in [7] from the dynamical point of view, where an attempt to apply the quantization Dirac-type procedure to the corresponding conserved energy expressions was done. Nevertheless, within the canonical point of view, the true quantization procedure should be based on the suitable canonical Hamiltonian formulation of the models which looks in the case under consideration as (4.4), (4.5), and (4.14).

In particular, consider the free charged point particle electrodynamics model (4.4) governed by the Hamilto-

nian equations

$$dr/d\tau := \partial H/\partial p = -p(\bar{W}^2 - p^2)^{-1/2},$$

$$dp/d\tau := -\partial H/\partial r = -\bar{W}\nabla\bar{W}(\bar{W}^2 - p^2)^{-1/2}, \quad (5.1)$$

where we denoted, as before, the corresponding vacuum field potential characterizing a medium field structure by $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$, the standard canonical coordinate-momentum variables by $(r, p) \in \mathbb{E}^3 \times \mathbb{E}^3$, and the proper "rest" reference system $\mathcal{K}_r$ time parameter related to our moving particle by $\tau \in \mathbb{R}$. The notation $H : \mathbb{E}^3 \times \mathbb{E}^3 \to \mathbb{R}$ stands for the Hamiltonian function

$$H := -(\bar{W}^2 - p^2)^{1/2} \quad (5.2)$$

expressed here and throughout further in the light speed units. The "rest" reference system $\mathcal{K}_r$ parametrized by the variables $(r, \tau) \in \mathbb{E}^4$ is related to any other reference system $\mathcal{K}$, subject to which our charged point particle q moves with a velocity vector $u \in \mathbb{E}^3$, and which is parametrized by the variables $(r, t) \in \mathbb{M}^4$ via the Euclidean infinitesimal relation

$$dt^2 = d\tau^2 + dr^2 \quad (5.3)$$

which is equivalent to the Minkowskian infinitesimal relation

$$d\tau^2 = dt^2 - dr^2. \quad (5.4)$$

The Hamiltonian function (5.2) satisfies, evidently, the energy conservation conditions

$$dH/dt = 0 = dH/d\tau \quad (5.5)$$

for all $t, \tau \in \mathbb{R}$. This means that the suitable energy value

$$\mathcal{E} = (\bar{W}^2 - p^2)^{1/2} \quad (5.6)$$

can be treated by means of the Dirac-type quantization scheme [12] to obtain, as $\hbar \to 0$, (or the light speed $c \to \infty$) the governing Schrödinger-type dynamical equation. To do this, similarly to [6, 7], we need to make canonical operator replacements $\mathcal{E} \to \hat{\mathcal{E}} := -\frac{\hbar}{i}\frac{\partial}{\partial\tau}$, $p \to \hat{p} := \frac{\hbar}{i}\nabla$, as $\hbar \to 0$, in the energy-determining expression

$$\mathcal{E}^2 := (\hat{\mathcal{E}}\psi, \hat{\mathcal{E}}\psi) = (\psi, \hat{\mathcal{E}}^2\psi) = (\psi, \hat{H}^+\hat{H}\psi). \quad (5.7)$$

Here, by definition, owing to (5.6),

$$\hat{\mathcal{E}}^2 = \bar{W}^2 - \hat{p}^2 = \hat{H}^+\hat{H} \quad (5.8)$$

is a suitable operator factorization in the Hilbert space $\mathcal{H} := L_2(\mathbb{R}^3; \mathbb{C})$, and $\psi \in \mathcal{H}$ is the corresponding normalized quantum vector state. Since the elementary identity

$$\bar{W}^2 - \hat{p}^2 = \bar{W}(1 - \bar{W}^{-1}\hat{p}^2\bar{W}^{-1})^{1/2}(1 - \bar{W}^{-1}\hat{p}^2\bar{W}^{-1})^{1/2}\bar{W} \tag{5.9}$$

holds, we can set, by definition, following (5.8) and (5.9), the operator

$$\hat{H} := (1 - \bar{W}^{-1}\hat{p}^2\bar{W}^{-1})^{1/2}\bar{W}. \tag{5.10}$$

Having calculated the operator expression (5.10) as $\hbar \to 0$ up to operator accuracy $O(\hbar^4)$, we obtain easily

$$\hat{H} = \frac{\hat{p}^2}{2m(u)} + \bar{W} := -\frac{\hbar^2}{2m(u)}\nabla^2 + \bar{W} \tag{5.11}$$

with regard for the dynamical mass definition $m(u) := -\bar{W}$ (in the light speed units). Thereby, based now on (5.7) and (5.11), we obtain, up to operator accuracy $O(\hbar^4)$, the Schrödinger-type evolution equation

$$i\hbar\frac{\partial\psi}{\partial\tau} := \hat{\mathcal{E}}\psi = \hat{H}\psi = -\frac{\hbar^2}{2m(u)}\nabla^2\psi + \bar{W}\psi \tag{5.12}$$

with respect to the "rest" reference system $\mathcal{K}_r$ evolution parameter $\tau \in \mathbb{R}$. Concerning the related evolution parameter $t \in \mathbb{R}$ parametrizing a reference system $\mathcal{K}$, Eq. (5.12) takes the form

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2 m_0}{2m(u)^2}\nabla^2\psi - m_0\psi. \tag{5.13}$$

Here we took into account that, owing to (5.6), the classical mass relation

$$m(u) = m_0(1 - u^2)^{-1/2} \tag{5.14}$$

holds, where $m_0 \in \mathbb{R}_+$ is the corresponding rest mass of our point particle q.

As $\hbar/c \to 0$, the obtained linear Schrödinger equation (5.13) coincides really with the well-known equation [13, 12, 5] from classical quantum mechanics.

5.3. Classical charged point particle electrodynamics model and its quantization

We start here from the first vacuum field theory reformulation of the classical charged point particle electrodynamics considered in Section 3 and based on the conserved Hamiltonian function (4.12)

$$H := -[\bar{W}^2 - (P - qA)^2]^{1/2}. \tag{5.15}$$

Here, $q \in \mathbb{R}$ is the particle charge, $(\bar{W}, A) \in \mathbb{R} \times \mathbb{E}^3$ is the corresponding electromagnetic field potentials, and $P \in \mathbb{E}^3$ is the common particle-field momentum defined as

$$P := p + qA, \quad p := mu \tag{5.16}$$

and satisfying the well-known classical Lorentz force equation. Here, $m := -\bar{W}$ is the observable dynamical mass related to our charged particle, and $u \in \mathbb{E}^3$ is its velocity vector with respect to a chosen reference system $\mathcal{K}$, being all expressed here, as before, in the light speed units.

As our electrodynamics based on (5.15) is canonically Hamiltonian, the Dirac-type quantization scheme

$$P \to \hat{P} := \frac{\hbar}{i}\nabla, \quad \mathcal{E} \to \hat{\mathcal{E}} := -\frac{\hbar}{i}\frac{\partial}{\partial\tau} \tag{5.17}$$

should be applied to the suitable energy expression

$$\mathcal{E} := [\bar{W}^2 - (P - qA)^2]^{1/2} \tag{5.18}$$

following from the conservation conditions

$$dH/dt = 0 = dH/d\tau \tag{5.19}$$

satisfied for all $\tau, t \in \mathbb{R}$.

Passing now the same way as above, we can factorize the operator $\hat{E}^2$ as follows:

$$\bar{W}^2 - (\hat{P} - qA)^2 = \bar{W}[1 - \bar{W}^{-1}(\hat{P} - qA)^2\bar{W}^{-1}]^{1/2}\times$$

$$\times[1 - \bar{W}^{-1}(\hat{P} - qA)^2\bar{W}^{-1}]^{1/2}\bar{W} := \hat{H}^+\hat{H}.$$

Here, by definition (here as $\hbar/c \to 0$, $\hbar c = \text{const}$),

$$\hat{H} := \frac{1}{2m(u)}(\frac{\hbar}{i}\nabla - qA)^2 + \bar{W} \tag{5.20}$$

up to operator accuracy $O(\hbar^4)$. Thereby, the related Schrödinger-type evolution equation in the Hilbert space $\mathcal{H} = L_2(\mathbb{R}^3; \mathbb{C})$ looks as

$$i\hbar\frac{\partial\psi}{\partial\tau} := \hat{\mathcal{E}}\psi = \hat{H}\psi = \frac{1}{2m(u)}(\frac{\hbar}{i}\nabla - qA)^2\psi + \bar{W}\psi \tag{5.21}$$

with respect to the rest reference system $\mathcal{K}_r$ evolution parameter $\tau \in \mathbb{R}$. The corresponding Schrödinger-type evolution equation with respect to the evolution parameter $t \in \mathbb{R}$ looks, respectively, as

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{m_0}{2m(u)^2}(\frac{\hbar}{i}\nabla - qA)^2\psi - m_0\psi. \tag{5.22}$$

The Schrödinger-type evolution equation (5.21) (as $\hbar/c \to 0$, $\hbar c = \text{const}$) completely coincides [14, 12] with that well known from the classical quantum mechanics.

5.4. Modified charged point particle electrodynamics model and its quantization

Consider now, within the canonical point of view, the true quantization procedure of the electrodynamics model, which looks as (3.13) and whose Hamiltonian function (4.6) is

$$H := -(\bar{W}^2 - P^2)^{1/2} - q\langle A, P\rangle(\bar{W}^2 - P^2)^{-1/2}. \quad (5.23)$$

This means that the suitable energy function

$$\mathcal{E} := (\bar{W}^2 - P^2)^{1/2} + q\langle A, P\rangle(\bar{W}^2 - P^2)^{-1/2}, \quad (5.24)$$

where, as before,

$$P := p + qA, \quad p := mu, \quad m := -\dot{W}, \quad (5.25)$$

is a conserved quantity for (3.13), which we will canonically quantize via the Dirac procedure (5.17). To make this, let us consider the quantum condition

$$\mathcal{E}^2 := (\hat{\mathcal{E}}\psi, \hat{\mathcal{E}}\psi) = (\psi, \hat{\mathcal{E}}^2\psi), \quad (\psi, \psi) := 1, \quad (5.26)$$

where, by definition, $\hat{\mathcal{E}} := -\frac{\hbar}{i}\frac{\partial}{\partial t}$, and $\psi \in \mathcal{H} = L_2(\mathbb{R}^3; \mathbb{C})$ is a suitable normalized quantum state vector. Making now use of the energy function (5.24), one can easily obtain

$$\mathcal{E}^2 = \bar{W}^2 - (P - qA)^2 +$$

$$+q^2[\langle A, A\rangle + \langle A, P\rangle(\bar{W}^2 - P^2)^{-1}\langle P, A\rangle] \quad (5.27)$$

which transforms upon the canonical Dirac-type quantization $P \rightarrow \hat{P} := \frac{\hbar}{i}\nabla$ into the symmetrized operator expression

$$\hat{\mathcal{E}}^2 = \bar{W}^2 - (\hat{P} - qA)^2 +$$

$$+q^2\langle A, A\rangle + q^2\langle A, \hat{P}\rangle(\bar{W}^2 - \hat{P}^2)^{-1}\langle \hat{P}, A\rangle. \quad (5.28)$$

Having factorized operator (5.28) in the form $\hat{\mathcal{E}}^2 = \hat{H}^+\hat{H}$, we obtain, up to operator accuracy $O(\hbar^4)$ (as $\hbar/c \rightarrow 0$, $\hbar c = \text{const}$),

$$\hat{H} := \frac{1}{2m(u)}(\frac{\hbar}{i}\nabla - qA)^2 -$$

$$-\frac{q^2}{2m(u)}\langle A, A\rangle - \frac{q^2}{2m^3(u)}\langle A, \frac{\hbar}{i}\nabla\rangle\langle\frac{\hbar}{i}\nabla, A\rangle, \quad (5.29)$$

where we put, as before, $m(u) = -\bar{W}$ in the light speed units. Thus, owing to (5.26) and (5.29), the resulting Schrödinger evolution equation takes the form

$$i\hbar\frac{\partial\psi}{\partial\tau} := \hat{H}\psi = \frac{1}{2m(u)}(\frac{\hbar}{i}\nabla - qA)^2\psi -$$

$$-\frac{q^2}{2m(u)}\langle A, A\rangle\psi - \frac{q^2}{2m^3(u)}\langle A, \frac{\hbar}{i}\nabla\rangle\langle\frac{\hbar}{i}\nabla, A\rangle\psi \quad (5.30)$$

with respect to the "rest" reference system proper evolution parameter $\tau \in \mathbb{R}$. Similarly, one obtains also the related Schrödinger-type evolution equation with respect to the time parameter $t \in \mathbb{R}$, on which we will not stop here. Result (5.30) essentially differs from the corresponding classical Schrödinger evolution equation (5.21), which forces us, thereby, to reanalyze more thoroughly the main physically motivated principles put into the backgrounds of the classical electrodynamic models described by the Hamiltonian functions (5.15) and (5.23) giving rise to different Lorentz-type force expressions. We plan to do this analysis in a next work under preparation in detail.

6. Conclusion

Based on the results obtained, we can claim that all of the electrodynamical field equations discussed above are equivalent to canonical Hamiltonian flows with respect to the corresponding proper "rest" reference systems parametrized by suitable time parameters $\tau \in \mathbb{R}$. Owing to the passing to the laboratory reference system $\mathcal{K}$ parametrized with the time parameter $t \in \mathbb{R}$, the related Hamiltonian structures appear to be naturally lost, giving rise to a new interpretation of the real charged particle motion as such one having the absolute sense only with respect to the proper "rest" reference system and being completely relative with respect to all other reference systems. Concerning the Hamiltonian expressions (4.1), (4.6), and (4.12) obtained above, one observes that all of them depend strongly on the vacuum potential field function $\bar{W} : \mathbb{M}^4 \rightarrow \mathbb{R}$, thereby dissolving the mass problem of the classical energy expression, before pointed out [16] by L. Brillouin. We mention here that, subject to the canonical Dirac-type quantization procedure, it can be applied only to the corresponding dynamical field systems considered with respect to their proper "rest" reference systems.

Remark 9. *Some comments can be also made concerning the classical relativity principle. Namely, we*

have obtained our results completely without using the Lorentz transformations of reference systems but only the natural notion of the "rest" reference system and its suitable parametrization with respect to any other moving reference systems. It looks reasonable, since the true state changes of a moving charged particle q are exactly realized in reality only with respect to its proper "rest" reference system. Thereby, the only question still here left open is that about the physical justification of the corresponding relation between the time parameters of the moving and "rest" reference systems.

This relation, being accepted throughout this work, looks as

$$d\tau = dt(1 - u^2)^{1/2}, \tag{6.1}$$

where $u := dr/dt \in \mathbb{E}^3$ is the velocity vector, with which the "rest" reference system $\mathcal{K}_r$ moves with respect to another arbitrarily chosen reference system $\mathcal{K}$. Expression (6.1) means, in particular, that the equality

$$dt^2 - dr^2 = d\tau^2 \tag{6.2}$$

holds, and it exactly coincides with the classical infinitesimal Lorentz invariant. Its appearance is, evidently, not casual here, since all our dynamical vacuum field equations were successively derived [6, 7] from the governing equations on the vacuum potential field function $W : \mathbb{M}^4 \to \mathbb{R}$ in the form

$$\partial^2 W/\partial t^2 - \nabla^2 W = \rho,$$

$$\partial W/\partial t + \nabla(vW) = 0, \quad \partial\rho/\partial t + \nabla(v\rho) = 0, \tag{6.3}$$

being *a priori* Lorentz invariant, where we denoted the charge density by $\rho \in \mathbb{R}$ and the suitable local velocity of vacuum field potential changes by $v := dr/dt$. Thereby, the dynamical infinitesimal Lorentz invariant (6.2) reflects this intrinsic structure of Eqs. (6.3). Being rewritten in the nonstandard Euclidean form

$$dt^2 = d\tau^2 + dr^2, \tag{6.4}$$

it gives rise to a completely other time relation between reference systems $\mathcal{K}$ and $\mathcal{K}_r$:

$$dt = d\tau(1 + \dot{r}^2)^{1/2}, \tag{6.5}$$

where we denoted, as earlier, the related particle velocity with respect to the "rest" reference system by $\dot{r} := dr/d\tau$.

Thus, we observe that all our Lagrangian analysis completed in Section 2 is based on the corresponding functional expressions written in these "Euclidean" space-time coordinates and with respect to which the least action principle was applied. So, we see that there exist two alternatives – the first is to apply the least action principle to the corresponding Lagrangian functions expressed in the Minkowski-type space-time variables with respect to an arbitrary chosen laboratory reference system $\mathcal{K}$, and the second is to apply the least action principle to the corresponding Lagrangian functions expressed in the space-time Euclid-type variables with respect to the "rest" reference system $\mathcal{K}_r$.

As a slightly amusing but exciting inference, following from our analysis in this work, is the fact that all of the classical special relativity results related to the electrodynamics of charged point particles can be obtained one-to-one, by making use of our new definitions of the dynamical particle mass and the least action principle with respect to the associated Euclid-type space-time variables parametrizing the "rest" reference system.

An additional remark is needed concerning the quantization procedure of the proposed electrodynamics models. If the dynamical vacuum field equations are expressed in the canonical Hamiltonian form, only technical problems are left to quantize them and to obtain the corresponding Schrödinger-type evolution equations in suitable Hilbert spaces of quantum states. There exists still another important inference from the approach devised in this work. It consists in the complete lost of the essence of the well-known Einsteinian equivalence principle [4, 5, 13, 37, 42] becoming superfluous for our vacuum field theory of electromagnetism and gravity.

Based on the canonical Hamiltonian formalism devised in this work, concerning the alternative charged point particle electrodynamics models, we succeeded in treating their Dirac-type quantization. The obtained results were compared with classical ones, but the physically motivated choice of a true model is left for the future studies. Another important aspect of the developed vacuum field theory no-geometry approach to combining the electrodynamics with the gravity consists in singling out the decisive role of the related "rest" reference system $\mathcal{K}_r$. Namely, with respect to the "rest" reference system evolution parameter $\tau \in \mathbb{R}$, all of our electrodynamics models allow both the Lagrangian and Hamiltonian formulations suitable for the canonical quantization. The physical nature of this fact remains, by now, not enough understood. There is, by now [13, 30–32, 37, 40–42], no physically reasonable explanation of this decisive role of the "rest" reference system, except for the very in-

teresting reasonings by R. Feynman who argued in [5] that the relativistic expression for the classical Lorentz force (2.12) has physical sense only with respect to the "rest" reference system variables $(r, \tau) \in \mathbb{E}^4$. In the sequel of our work, we plan to analyze the quantization scheme in more details and make a step toward the vacuum quantum field theory of infinite-many-particle systems.

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ПЕРЕФОРМУЛЮВАННЯ ЛАГРАНЖЕВОГО
ТА ГАМІЛЬТОНОВОГО ФОРМАЛІЗМІВ ДЛЯ МОДЕЛЕЙ
КЛАСИЧНОЇ РЕЛЯТИВІСТСЬКОЇ ЕЛЕКТРОДИНАМІКИ

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Р е з ю м е

Розглянуто деякі нові моделі класичної електродинаміки взаємодіючих заряджених точкових частинок та деякі аспекти квантування за методом Дірака. У межах негеометричного підходу в теорії поля вакууму, розвинутого в роботах [6, 7, 9], запропоновано лагранжеве та гамільтонове переформулювання деяких альтернативних моделей класичної електродинаміки та розвинуто процедуру квантування діраківського типу для цих моделей на основі запропонованого канонічного гамільтонового формалізму.