LOWER BOUNDS ON THE MASS OF FERMIONIC DARK MATTER PARTICLES

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UDC 524.8,539.121.7

We constructed new lower bounds on the mass of Dark Matter (DM) particles coming from the analysis of a DM phase-space distribution in different classes of DM-dominated objects (dwarf spheroidal galaxies (dSphs), spiral galaxies, and galaxy groups). For each type of objects, we derived two such bounds. The first, model-independent bound, depends on the information about the current phase space distribution of DM particles only. The stronger, model-dependent bound is quoted for a model of thermal relativistically decoupled DM particles. After that, we discuss possible domains of applicability of the obtained bounds.

1. Introduction

The nature of Dark Matter is one of the most intriguing questions of particle astrophysics. Its solution would have a profound impact on the development of particle physics beyond the Standard Model. It is because the Standard Model of elementary particles does not contain a viable Dark Matter particle candidate – a massive, neutral, and long-lived particle. For example, active neutrinos, which are both neutral and stable, form structures in a top-down fashion [1–5], and thus cannot produce[1] the observed quantity of early-type galaxies (see, e.g., [8, 9]). Therefore, the DM particle hypothesis implies the extension of the Standard Model (SM).

The DM particle candidates may have very different masses (for reviews of DM candidates, see, e.g., [10–12, 14]): massive gravitons with the mass $\sim 10^{-19}$ eV [15], axions with the mass $\sim 10^{-6}$ eV [16], sterile neutrinos having mass in the keV range [17], supersymmetric (SUSY) particles (gravitinos [18], neutralinos [19], axinos [20] with their masses ranging from eV to hundreds GeV, supersymmetric Q-balls [21], WIMPZILLAs with the mass $\sim 10^{13}$ GeV [22, 23], and many others). Thus, the mass of DM particles becomes an important characteristic which may help to distinguish between various DM candidates and, more importantly, may help to differentiate among different models beyond the SM.

If the DM particles are fermions, there is a very robust bound on their mass. Namely, due to the Pauli exclusion principle, there exists the densest "packing" of the fermions in a given region of the phase space. Decreasing the mass of DM particles, one increases their number in a given gravitationally bound object containing DM. The requirement that the phase-space density of DM does not exceed that of the degenerate Fermi gas leads to the *lower mass bound*. For example, for a spherically symmetric DM-dominated object with mass M within the region R, one obtains the lower bound $m_{\rm DEG}$ on the DM mass by demanding that the maximal (Fermi) velocity of the degenerate fermionic gravitating gas of mass M in the volume $\frac{4}{3}\pi R^3$ do not exceed the escape velocity[2] $v_\infty =$

[1]Note that the described analysis does not contradict the fact that the part of Dark Matter is made of active neutrinos. Current restrictions on the active neutrino mass from the joint analysis of the cosmic microwave background and the large scale structure of the Universe imply that the sum of active neutrino masses should be less than 1 eV, see [6, 7].

[2]In this approximation, we neglect the effect of external layers with $r > R$ on the gravitational potential; therefore, this approximation is true only for sufficiently large R.

$\left(\frac{2G_N M}{R}\right)^{1/2}$:

$$\hbar\left(\frac{9\pi M}{2gm_{\rm DEG}^4 R^3}\right)^{1/3} \le \sqrt{\frac{2G_N M}{R}}. \quad (1)$$

This yields

$$m_{\rm DEG}^4 \ge \frac{9\pi\hbar^3}{4\sqrt{2}gM^{1/2}R^{3/2}G_N^{3/2}}. \quad (2)$$

Here and below, g denotes the number of internal degrees of freedom of DM particles, and G_N is the Newton's constant. Such a consideration applied to various DM dominated objects leads to the mass bound which we will call $m_{\rm DEG}$ in what follows (see Table 1 below).[3]

The limit (2) obtained in such a way is very robust, as it is independent of the details of the formation history of the system. The only uncertainties associated with it are those of astronomical nature: systematic errors in the determination of velocity and density distribution. All these issues will be discussed below (Sections 2., 3.).

For particular DM models (with the known primordial velocity dispersion) and under certain assumptions about the evolution of the system which led to the observed final state, this limit can be strengthened [26–33]. The argument is based on the *Liouville theorem* (see, e.g., [29, 34]) and assumes that the collapse of the system is dissipationless and collisionless. The Liouville theorem states that the phase-space distribution function $f(t,x,v)$ does not change in the course of dissipationless collisionless dynamics. The consequence of the Liouville theorem is that the function $f(t,x,v)$ "moves" in the phase-space, according to the Hamiltonian flow, and, therefore, its maximum (over the phase space) remains unchanged. Thus, if one could determine the characteristics of a phase-space distribution function from astronomically observed quantities, the Liouville theorem allows one to connect the measured values and properties of the initial phase distribution of DM particles.

One such characteristics of the phase-space distribution is its *maximum*. Any physical measurement can probe only the phase-space distribution averaged over some phase-space region - a *coarse-grained* phase-space density (PSD) (as opposed to exact or *fine-grained* PSD). Such a coarse-grained PSD, averaged over phase-space cells $\Delta\Pi(x,v)$ centered around points (x,v) in the phase space, is defined via

$$\bar{f}(t,x,v) = \frac{1}{\mathrm{vol}(\Delta\Pi)} \int\limits_{\Delta\Pi(x,v)} d\Pi'\, f(t,x',v') \quad (3)$$

(here $\mathrm{vol}(\Delta\Pi)$ is the volume of a phase-space cell).

From definition (3), it is clear that the maximal (over the whole phase space) value of the coarse-grained PSD $\bar{f}_{\max}(t)$ cannot exceed the maximal value of the corresponding fine-grained PSD. On the other hand, as a consequence of the Liouville theorem, the maximum of the fine-grained PSD $f_{\max}$ does not change in time. Thus, one arrives at the inequality

$$\bar{f}_{\max}(t) \le f_{\max}. \quad (4)$$

Inequality (4) allows one to relate the properties of DM at the present time t with its primordial properties encoded in $f_{\max}$.

For example, let us assume that DM particles possess initially a relativistic Fermi–Dirac distribution function with some temperature $T_{\rm FD}$ (relativistically decoupled thermal relics),

$$f_{\rm FD}(p) = \frac{g}{(2\pi\hbar)^3}\frac{1}{e^{p/T_{\rm FD}}+1}. \quad (5)$$

If we recover from astronomical measurements that the coarse-grained PSD of the system is described in the final state by the a pseudoisothermal sphere (see, e.g., [34]) with a core radius r_c and a 1D velocity dispersion σ, whose maximum is given by

$$\bar{f}_{\rm iso,max} = \frac{9\sigma^2}{4\pi G_N(2\pi\sigma^2)^{3/2}r_c^2}, \quad (6)$$

the comparison of the maximum of the coarse-grained PSD (6) with its primordial (fine-grained) value leads to the so-called Tremaine–Gunn mass bound [26]:

$$m_{\rm FD} \ge m_{\rm TG}, \quad m_{\rm TG}^4 \equiv \frac{9(2\pi\hbar)^3}{(2\pi)^{5/2}gG_N\sigma\, r_c^2}. \quad (7)$$

For the case of initial distribution (5), this bound is stronger than that based on the Pauli exclusion principle by a factor of $2^{1/4}$ [26]. For different primordial DM distributions, this difference can be significant (as we will demonstrate below). We would like to stress, though,

[3] The spatially homogeneous DM distribution is only an approximation. In reality, one should consider a self-gravitating degenerate fermionic gas. It is possible to show that, under some external conditions, the system of weakly interacting fermions undergoes a first-orger phase transition to a nearly degenerate "fermion star" [24]. The existence of such objects may also have insteresting astrophysical applications [25].

that these stronger bounds make assumptions about the evolution of the phase-space density, while the one based on the Pauli exclusion principle does not assume anything about either the primordial velocity distribution of particles or the formation history of the observed object and simply compares the measured phase-space density with the maximally allowed one for fermions.

Because we cannot directly measure the final value of the coarse-grained phase-space density, we should make some assumptions to find its maximal value. The conservative way is to use the "maximally coarse-grained distribution". It is based on the simple fact that the average value of a function cannot exceed its maximal value. Therefore, the value of the coarse-grained phase-space density, averaged over the whole phase-space volume of the system, can be used as a robust estimate of $\bar{F}_{\max}$ independently of the assumptions about the real form of a phase-space density distribution.

2. The Case of Maximal Coarse-Graining for a Spherically Symmetric Shape of Dark Matter Halo

To this end, we consider an (approximately spherically symmetric) gravitating system (bearing a dwarf spheroidal galaxy in mind), that has mass $M(R)$ confined within radius R. The phase-space volume occupied by DM particles forming such a system can be approximated by

$$\Pi_\infty = \left(\frac{4}{3}\pi\right)^2 R^3 v_\infty^3, \tag{8}$$

where we have introduced the *escape velocity* v_∞. The "coarsest" PSD is such that the averaging of (3) goes over the whole phase-space volume, $\Delta\Pi = \Pi_\infty$:

$$\bar{F} = \frac{M}{\Pi_\infty} = \frac{9}{16\pi^2}\frac{M}{R^3 v_\infty^3} = \frac{3\bar{\rho}}{4\pi v_\infty^3}. \tag{9}$$

Below, we consider the particular examples of such systems.

2.1. Dwarf spheroidal galaxies (dSphs)

As an example, dwarf spheroidal galaxies (dSphs) are the compact (around ~ 1 kpc) and gravitationally bound systems with low surface brightness and high velocity dispersion. To explain the latter, one needs to introduce the mass-to-luminosity value which is hundreds of times more than the value observed in usual galaxies (see, e.g., [46]). The possible reason for such a huge velocity dispersion is the disturbance of the cental part of dSphs with the tidal forces of our Galaxy. However, the observable features of tidal disturbance – the so-called "tidal tails" formed from stars which are going out of dSph – were found in single dSphs (Sagitarius, Ursa Major II). Therefore, the most realistic model by the moment for a *majority* of dSphs is their domination with Dark Matter. As a result, it is assumed that the dSphs are the *most compact* objects dominated with Dark Matter.

As an estimate for R, we take *half-light radius* r_h (i.e. the radius, where the surface brightness profile falls to 1/2 of its maximal value). Neglecting a possible influence of the ellipticity of stellar orbits (c.f. Appendix) and assuming a constant DM density within r_h and the isothermal distribution of stars [50], we obtain the following estimate of the average DM density within r_h:

$$\bar{\rho} = \frac{3\log 2}{2\pi}\frac{\sigma^2}{G_N r_h^2}. \tag{10}$$

Assuming the isotropic velocity distributions,[4] the escape velocity v_∞ of DM particles is related to the velocity dispersion σ via $v_\infty \simeq \sqrt{6}\sigma$. In such a way, we obtain the averaged PSD $\bar{F}$:

$$\bar{F} = \frac{M}{\Pi_\infty} = \frac{\bar{\rho}}{8\pi\sqrt{6}\sigma^3} \approx \frac{3\log 2}{16\sqrt{6}\pi^2 G_N \sigma r_h^2}, \tag{11}$$

which coincides with its maximal value (being flat).

As a consequence of Eq. (4), this "coarse-grained" PSD $\bar{F}$ is smaller than the $f_{\max}$ – the maximum value of fine-grained PSD equal to its primordial value

$$\bar{F} \le f_{\max}. \tag{12}$$

Equation (12) relates the observed properties of the dSphs (l.h.s.) to the microscopic quantity on the r.h.s. of the inequality which depends on the production mechanism of DM.

[4]This assumption seems to be correct for DM particles, since numerical simulations of DM structures of different scales show that the velocity anisotropy $\beta(r) \equiv 1 - \frac{\sigma_\theta^2+\sigma_\phi^2}{2\sigma_r^2}$ tends to be zero toward the central region [36–40]. It is not clear whether β equals zero for stars in dSphs. The assumption of the isotropy of stellar velocities leads to the *cored* density profiles [41, 42]; therefore, our estimate for $\bar{\rho}$ tends to be robust. This is confirmed by comparison of estimate (10) with those based on [43–45], where DM density profiles were obtained under the assumptions of different anisotropic distributions of stars in dSphs.

In this paper, we are interested in the relativistic Fermi–Dirac distribution of particles (5), with its $f_{\max}$ being equal to

$$f_{\max,\mathrm{FD}} = \frac{g\, m_{\mathrm{FD}}^4}{2(2\pi\hbar)^3} \qquad (13)$$

(we fix the overall normalization of the phase-space distribution function by the relation $M = \int d^3x\, d^3v\, f(t,x,v)$, where M is the total mass of the system).

Do obtain the lower mass bound, we use the data on three dSphs (Leo IV, CVnII, Coma Berenices), being recently analyzed in [51]. Several factors contribute to the errors of σ and r_h.

First of all, as σ is the dispersion of measured velocities, it has the statistical error (which can be quite large for the ultra-faint dSphs, where the number of stars can be rather small ($\sim$ 10–100, c.f. [51, Table 3]). However, the systematic error is much larger. The authors of [51] found the systematic error on their determination of the velocity dispersion to be 2.2 km/ s. We add this error in quadratures to the statistical errors found in [51, Table 3]. The results are shown in column 4 in Table 1.

The half-light radius r_h is a derived quantity, and there are several contributions to its errors. First of all, the surface brightness profile is measured in angular units, and their conversion to parsecs requires the knowledge of the distance toward the object. These distances are generally known with uncertainties of about 10% (see [35] and references therein).

Another uncertainty comes from the method of determination of r_h. The surface brightness profile gets fit to various models to determine this quantity. For several dSphs: Coma Berenices, Canes Venatici II, Hercules, and Leo IV, the authors used two different profiles (Plummer and exponential) for evaluating the annular half-light radius [52]. We use their results to estimate the systematic error on r_h at 20% for our dSphs. The results of the determination of r_h are shown in column 3 of Table 1. The obtained values of $\bar{F}_{\mathrm{dSph}}$ are presented in column 5 of Table 1.

2.2. Spiral galaxies

In this section, we study the restrictions on the phase-space density coming from the analysis of the inner parts of spiral galaxies. The presence of Dark Matter can be derived from the analysis of the so-called rotational curves showing the dependence of a rotational velocity of stars around the galaxy center as a function of the distance to the galaxy center. The important property of such rotational curves is their behavior at large distances (halo region), where the brightness of luminous matter is much less than that of central parts of the galaxy. In the halo region, the values of rotational velocities usually tend to be constant. To explain this behavior, one needs the additional Dark Matter component in the halo region.

As a result, the Dark Matter density in a galaxy can be found with the help of the relation

$$\bar{\rho}_{\mathrm{gal}} = \frac{3}{4\pi G_N}\frac{v_h^2(r)}{r^2}, \qquad (14)$$

where $v_h(r)$ – part of the rotational velocity contributed to Dark Matter[5] at distance r from the galaxy center. As the velocity of DM particles in a compact halo does not exceed $v_\infty = \sqrt{2}v_h(r)$ (otherwise, DM particles would have escaped from the halo), we obtain the DM phase-space density estimate (see also (9))

$$\bar{F}_{\mathrm{gal}} = \frac{3\bar{\rho}_{\mathrm{gal}}}{4v_\infty^3} = \frac{9}{32\pi^2\sqrt{2}G_N v_h(r_{\mathrm{gal}})r_{\mathrm{gal}}^2}, \qquad (15)$$

where r_{gal} is the *inner* radius of the halo.

T a b l e 1. Parameters of selected dSphs with the largest values of $\bar{F}_{\mathrm{dSph}}$ obtained from [51, 55] (columns 1–5) and derived values of lower mass bound for different Dark Matter types (columns 6 and 7). m_{DEG} denotes the model-independent bound coming from the Pauli principle (2), m_{FD} – model-dependent bound for DM particles with the momentum distribution (5)

dSphs (1)	Reference (2)	r_h, pc (3)	σ, km/s (4)	$\bar{F}_{\mathrm{dSph}}$, $M_\odot$ pc^{-3} (km/s)$^{-3}$ (5)	m_{DEG}, keV (6)	m_{FD}, keV (7)
Leo IV	[51, 55]	116^{+26}_{-34}	3.3±2.8	2.82 $\times 10^{-5}$	0.406	0.483
Coma Berenices	[51, 55]	77±10	4.6±2.3	4.59 $\times 10^{-5}$	0.459	0.546
Canes Venatici II	[51, 55]	74^{+14}_{-10}	4.6±2.4	4.97 $\times 10^{-5}$	0.468	0.557

[5]Note that this velocity is somewhat lower than the total rotational velocity of stars around the galaxy center. This is due to the presence of two additional components - disk and buldge – formed by the luminous matter.

Because of the presence of baryonic matter in galaxies, there are some uncertainties of the choice of $r_{\rm gal}$. To be conservative, we use $r_{\rm gal}$, where the Dark Matter contribution is equal to the contribution from baryonic matter (the sum of bulge and disk components).

For the subsequent analysis, we used two spiral galaxies with the best studied haloes – our Galaxy and the Andromeda galaxy. The velocity profiles for these galaxies are shown, e.g., in [53, 54]. The results for $r_{\rm gal}$, $v_h(r_{\rm gal})$, as well as the obtained bounds for $\bar{F}$, are shown in Table 2.

The average value of the lower mass bound from Table 2 implies $m_{\rm DEG} > 34$ eV, $m_{\rm FD} > 40$ eV. Therefore, if the dSph dynamics (in contrast to the dynamics of spiral galaxies) is *not* due to Dark Matter, the lower mass bound in the DM particles is ∼40 eV, depending on the DM species.

2.3. Galaxy groups

The Dark Matter profiles in galaxy groups are obtained from the analysis of the X-ray thermal emission distribution from the hot gas halo. In this paper, we used the data from [56], where the distributions of Dark Matter, the baryonic matter in galaxies, and the hot intergalactic gas are derived. Similar to the previous subsection, to take into account the uncertainties of the DM and hot gas distributions, we use the phase-space density values calculated for the distance $r_{\rm gr}$, where the DM mass $M_{\rm gr} = M(r_{\rm gr})$ starts to dominate over the baryonic mass. The maximal velocity of DM particles is estimated as

$$v_\infty = \sqrt{\frac{2G_N M_{\rm gr}}{r_{\rm gr}}}. \qquad (16)$$

Whence (using (9)) we obtain the conservative estimate of the maximal phase-space density,

$$\bar{F}_{\rm gr} = \frac{9}{32\sqrt{2}\pi^2 G_N^3/2 M_{\rm gr}^1/2 r_{\rm gr}^{3/2}}. \qquad (17)$$

The results are presented in Table 3. The averaged values of the mass bound derived from Table 3 are $m_{\rm DEG} > 24$eV, $m_{\rm FD} > 29$eV. Therefore, if the dSph dynamics, in contrast to the dynamics of spiral galaxies and galaxy groups, is not due to Dark Matter, the lower bounds on the DM particle mass is ∼ 25–30 eV, depending on the DM model.

3. Results and Their Discussion

Our main results are presented in Tables 1–3. Summarizing the results, we obtain the following lower bounds for DM particle masses:

$$m_{\rm DEG} > 0.41\,\text{keV}\ , \quad m_{\rm FD} > 0.48\,\text{keV}, \qquad (18)$$

from dSphs,

$$m_{\rm DEG} > 0.034\,\text{keV}\ , \quad m_{\rm FD} > 0.040\,\text{keV}, \qquad (19)$$

T a b l e 2. Parameters of selected spiral galaxies from [53, 54] (columns 1–5) and obtained lower mass bounds for different DM types (columns 6 and 7). $m_{\rm DEG}$ denotes the model-independent bound coming from the Pauli principle (2), $m_{\rm FD}$ – model-dependent bound for DM particles with the momentum distribution (5)

Profile (1)	Reference (2)	$r_{\rm gal}$, kpc (3)	$v(r_{\rm gal})$, km/s (4)	$\bar{F}_{\rm gal}$, $M_\odot$ pc^{-3} (km/s)$^{-3}$ (5)	$m_{\rm DEG}$, keV (6)	$m_{\rm FD}$, keV (7)
MW, A1	[53]	3.0±0.6	150±10	3.45×10^{-9}	0.043	0.051
M31. C1	[53]	3.8±0.8	180±10	1.79×10^{-9}	0.036	0.043
M31a	[54]	6.0±1.2	140±10	0.93×10^{-9}	0.031	0.037
M31b	[54]	6.5±1.3	140±10	0.79×10^{-9}	0.030	0.035
M31c	[54]	6.0±1.2	150±10	0.86×10^{-9}	0.030	0.036

T a b l e 3. Parameters of chosen galaxy groups from [56] (columns 1–5) and obtained lower mass bounds for different DM types (columns 6 and 7). $m_{\rm DEG}$ denotes the model-independent bound coming from the Pauli principle (2), $m_{\rm FD}$ – model-dependent bound for DM particles with the momentum distribution (5)

Object (1)	Reference (2)	$M_{\rm gr}$, $10^{11} M_\odot$ (3)	$r_{\rm gr}$, kpc (4)	$\bar{F}_{\rm gr}$, $M_\odot$ pc^{-3} (km/s)$^{-3}$ (5)	$m_{\rm DEG}$, keV (6)	$m_{\rm FD}$, keV (7)
Abell 262	[56]	0.65±0.07	6.5±1.7	5.34×10^{-10}	0.027	0.032
NGC 533	[56]	0.50±0.05	5.0±1.5	9.02×10^{-10}	0.031	0.037
MKW 4	[56]	2.5±0.3	12.0±3.6	1.08×10^{-10}	0.018	0.022
IC 1860	[56]	1.3±0.1	9.5±2.9	2.14×10^{-10}	0.021	0.026

from spiral galaxies, and

$$m_{\rm DEG} > 0.024\,\text{keV}\ , \quad m_{\rm FD} > 0.029\,\text{keV}, \tag{20}$$

from galaxy groups.

We should also note that our bounds on $m_{\rm FD}$ are robust, by assuming that the influence of baryons is not responsible for the *increase* of the phase-space density during the process of structure formation. If this assumption is not true, only $m_{\rm DEG}$ bounds are robust.

Because spiral galaxies and galaxy groups are much more disperse than dSphs and have much larger velocity dispersions, the bounds obtained from the analysis of the phase-space density of spiral galaxies and galaxy groups are significantly weaker than those obtained from dSphs. Nevertheless, such an analysis is reasonable, because the possibility for a multicomponent Dark Matter to exist is not yet excluded (see, e.g., [57]; we should also note that the presence of a nonzero neutrino mass implies that a small part of Dark matter is made of relic neutrinos) or the possible co-existence of Dark Matter and modified gravity (see, e.g., [58–60]). Therefore, producing the restrictions on the phase-space density of DM-dominated objects of various types is important.

3.1. Influence of aspherical shapes of DM halos

We analyze a change of bound (2) due to a deviation of a DM halo from the spherical shape. A similar asphericity affects both the phase-space volume of the halo proportional to $\left(V v_{\max}^3\right)^{-1}$, where V is the volume of the inner part of a dSph (which is equal to $\frac{4}{3}\pi R^3$ for a spherically symmetric dSph, see the text above Eq.(2)) and $v_{\max}$, the escape velocity from the system[6]. To obtain the lower bound on $m_{\rm DEG}$, we consider the dSph as a homogeneous ellipsoid with semiaxes a, b, and c and assume the ellipticity of its 2D projection[7] $\epsilon \lesssim 0.5$. Because we observe only a 2D projection of such an ellipsoid, there are two possibilities:

Prolate dSph: $c > b \simeq a$. We see the axes b and c related to the "averaged" radius R via $b = R(1-\epsilon)^{1/2}, c = R(1-\epsilon)^{-1/2}$. Therefore, the spatial volume

$$V = \frac{4}{3}\pi abc \approx \frac{4}{3}\pi R^3(1-\epsilon)^{1/2} \approx \frac{4}{3}\pi R^3(1-0.5\epsilon). \tag{21}$$

The gravitational potential for $\epsilon \lesssim 0.5$ is dominated by monopole and quadrupole components,

$$\phi \approx \phi^{(0)} + \phi^{(2)}. \tag{22}$$

The maximal value of the potential occurs near the end of the minor semiaxis:

$$|\phi_{\max}| \equiv \frac{v_\infty^2}{2} = \frac{G_N M}{a} - \frac{G_N D_{zz}}{4a^3}, \tag{23}$$

where $D_{zz} = \frac{2M(c^2-a^2)}{5}$ – the quadrupole moment of the system [61]. For $\epsilon \ll 1$, we obtain

$$\frac{V v_\infty^3|_{\rm prolate}}{V v_\infty^3|_{\rm spherical}} \approx 1 + 0.05\epsilon, \tag{24}$$

which gives us the correction for $m_{\rm DEG}$ to be smaller than 1% (for $\epsilon = 0.5$).

Oblate dSph: $c \simeq b > a$. We observe the axes a and c; therefore, the spatial volume V changes by $(1-\epsilon)^{-1/2} \approx 1 + 0.5\epsilon$. The maximum of the gravitational potential is then given by

$$|\phi_{\max}| \approx \frac{G_N M}{a} + \frac{G_N D_{xx}}{2a^3} \approx \frac{G_N M}{R}(1+0.1\epsilon), \tag{25}$$

where D_{xx} is given by the same expression, as D_{zz} above. The maximal phase-space volume changes in the oblate case by $\approx 1 + 0.65\epsilon$, so the correction for $m_{\rm DEG}$ will constitute about 8% for $\epsilon \simeq 0.5$.

Thus, the departure from the spherical symmetry for DM halos of dSphs changes the limit on $m_{\rm DEG}$ by less than $\lesssim 10\%$ for the case of the axes ratio of 1:2. This uncertainty is below several others; therefore, we will consider dSphs to be spherical in what follows.

3.2. Influence of baryons

In this paper, we suggested that a conservative way to put the bound on a DM particle mass may be based on the requirements that the maximum of the observed coarse-grained phase space density should not exceed the maximum of the initial distribution function of DM particles. The maximum of the coarse-grained distribution function in the final state may be conservatively estimated from the observed quantities. This bound relies on the assumption that the maximum of the distribution function is not significantly increased by the interaction with baryons.

Although DM consists of the non-interacting particles, the remaining part of the galaxy – the baryons – interact with one another and dissipate their energy, finally concentrating toward the center. The baryons,

[6]Here, we neglect the influence of the mass inside the sphere with radius R.

[7]Throughout this paper, we define the *ellipticity* ϵ in a way similar to that in [34] (see also [55]), i.e. $\epsilon \equiv 1 - b/a$, where a and b are the *semimajor* and *semiminor* axis, respectively. Thus, the case of $\epsilon = 0.5$ corresponds to the ratio of axes to be equal to 1:2.

which are condensed at the center, influence a shape of the DM halo gravitationally, increasing the central DM density [62, 63]. The opposite effect is the energy feedback from SNae, galactic winds, and reionization, which creates the strong outflow, significantly decreasing the mass of the gas and thereby affecting the DM halo shape. Such a feedback is thought to be responsible to the formation of dwarf spheroidals from gas-rich dwarf spiral/irregular galaxies [64–68]. Clearly both gas condensation and feedback strongly influence the central PSD of DM [69] and can lead, in principle, to the violation of inequality (4). Numerical studies of galaxy mergers show that baryons can lead to an increase of the phase-space density during a merger (see, e.g., [70]). However, the method used in this work – the coarse-graining of the PSD over a large phase-space region – reduces the influence of baryons. Indeed, we take the spatial averaging over the radius $R \sim r_h$ which includes the external part of the system, where the amount of baryons is small. Additional studies are necessary to estimate effects of baryons and make our bounds more robust.

3.3. Influence of Zeldovich velocities

We would also like to stress that the initial velocities of DM particles in our approach are *thermal* velocities, and they should not be confused with the so-called *Zeldovich* velocities [1]. Numerical simulations of the galaxy formation do not start at the time, when the DM phase-space distribution is spatially uniform (redshifts $z \gtrsim 10^3$). Instead, the initial (linear) stage of the structure formation is computed analytically in the framework of the so-called *Zeldovich approximation* [1]. This approximation is commonly used to set up initial conditions for the numerical simulations of the non-linear stage of the structure formation [71–73] which starts at redshifts $z \sim 10$. The peculiar (*Zeldovich*) velocities acquired by DM particles at this stage due to the structure formation and included into the initial conditions are normally $\sigma \sim 10$ km/s. Apart from Zeldovich velocities, DM particles also possess thermal velocities which are discussed in this paper. For cold enough Dark Matter, these thermal velocities are much smaller than Zeldovich ones and, thus, are often neglected and not included into initial conditions. Therefore, the numerical studies of PSD evolution[8] (see, e.g., [74–79]) essentially investigate the change of PSD from the Zeldovich to final stage. It was found in some of these works that the PSD changes by $10^2 - 10^3$ in the process of collapse [75]. This change of PSD can be understood as being simply an evolution from initial Zeldovich velocities $\sigma_i \sim 10$ km/s to the final (virial) ones $\sigma_f \sim 10^2$ km/s (with $Q_i/Q_f \sim (\sigma_f/\sigma_i)^3 \sim 10^3$).

Because initial thermal velocities may be much smaller than Zeldovich ones, the initial PSD may differ from the final (observed) PSD not by a factor of 2–3, but by many orders of magnitude. This fact does not contradict the results of simulations described in, e.g., [75] and, therefore, cannot be used to obtain an *upper* bound on the mass of DM particles (c.f [32, 80]).

[8] Most of these studies use the quantity $Q(r) = \rho(r)/\sigma^3(r)$ as a PSD estimator

D.I. is grateful to to the Scientific and Educational Center of the Bogolyubov Institute for Theoretical Physics in Kyiv, Ukraine, and especially to V. Shadura, for the creating wonderful atmosphere for young Ukrainian scientists. The work of D.I. is supported from the “Cosmomicrophysics” programme, the Program of Fundamental Research of the Physics and Astronomy Division of the National Academy of Sciences of Ukraine, and from the grant No. F/16-417-2007 of the Governmental Fund of Fundamental Research of the National Academy of Sciences of Ukraine.

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Received 13.05.09

ОБМЕЖЕННЯ ЗНИЗУ НА МАСУ ЧАСТИНОК ФЕРМІОННОЇ ТЕМНОЇ МАТЕРІЇ

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Р е з ю м е

В даній роботі побудовано нові обмеження знизу на масу частинок ферміонної темної матерії (ТМ), використовуючи аналіз фазової густини ТМ для широкого класу астрономічних об'єктів (сферичних карликових галактик, спіральних галактик та груп галактик). Для кожного типу об'єктів, отримано два обмеження на масу частинок ТМ. Перше, модельно-незалежне обмеження не залежить від деталей еволюції системи та розподілу темної матерії. Сильніше, модельно-залежне обмеження є артефактом вибраної моделі початкового розподілу ТМ (для прикладу розглянуто теплові релікти з релятивістським спектром). Після цього обговорено області можливого застосування отриманих обмежень.