SYNERGETIC APPROACH TO THE DESCRIPTION OF A STEP-LIKE TRANSITION BETWEEN THE MOTION REGIMES OF ACTIVE PARTICLES

O.V. YUSHCHENKO, A.I. OLEMSKOI[1]

Sumy State University
(2, Rymskyi-Korsakov Str., Sumy 40007, Ukraine),

[1]**Institute of Applied Physics, Nat. Acad. of Sci. of Ukraine**
(58, Petropavlivska Str., Sumy 40030, Ukraine; e-mail: alex@ufn.ru)

UDC 539.2

A synergetic model has been developed to describe transitions between the motion modes in an ensemble of active particles of the type of biological systems. We show that states of the system are reduced to either the random motion of particles or a translational displacement of the ensemble as a whole, which depends on the excitation level of the system determined by an internal parameter. The phase portraits of the system were used to study a step-like transition between the motion regimes of active particles for various ratios between the characteristic times of hydrodynamic mode variations. The model parameters, which promote the transition of the system into the ordered state, have been analyzed.

1. Introduction

Since the 19-th century, a lot of scientists have been interested in the idea of describing the behavior of biological systems by means of methods of theoretical physics (see works [1, 2] and references therein). This problem is associated with a complicated structure of biological systems, which possess plenty of components that interact with one another in a nonlinear way, so that the behavior of the entire system does not imitate the behavior of its components. On the other hand, natural systems are dissipative, and their self-organization demands an inflow of additional energy from outside.

Systems, where self-organization takes place, have been widely studied recently [3]. Investigations are not confined to objects of inorganic nature, when individual features of system components are practically not considered. The synergetic approach was used to describe a lot of social and financial systems, such as the crashes of financial markets, the dynamics of exchange rates, the distribution of social layers in a society, social revolutions, traffic, and others. In this connection, the following question arises: Can the synergetic approach be also used for the description of the behavior of living organisms?

The application of models based on the methods of nonlinear dynamics and the theory of self-organization allowed such processes as the formation of biological relief, regeneration, biological rhythms, muscle contraction, associative memory, and so on to be explained [2, 4]. At the same time, there remain a good many biological processes which have not been understood appropriately. Their investigations are reduced to discovering the mechanisms of structurization. Such researches occupy one of the central places in natural and engineering sciences and consider systems that are characterized by an infinite number of stable states. For instance, the study of the dynamics of the predator–prey system reveals the presence of space-time structures in the form of propagating fronts, regular and stochastic oscillations, and concentric and spiral waves [4]. The latter are observed in the ocean as the rotational motion of plankton on a kilometer scale, or they can arise in various bacterial colonies [5, 6].

A particular place among the problems mentioned above is occupied by the description of possible modes for the collective motion of an ensemble of living organisms–a flock of birds, a swarm of insects, a bacterial

ISSN 2071-0194. Ukr. J. Phys. 2009. V. 54, N 7

colony, and so on. The result of researches [4–6] testify that such a motion does not depend on the biological accumulation level, ranging from cells and the simplest microorganisms to highly organized fishes and birds. This allows such accumulations to be presented as ensembles of active Brownian particles. Every such particle, possessing a reserve of internal energy, can change its internal state described by the parameter θ. Taking the interaction between particles into account, the dynamics of such an ensemble subjected to the action of chemical reagents–e.g., pheromones–can be predicted. The release of such a reagent leads to a space-time distribution $h(\mathbf{r}, t)$ for the concentration of chemical products, which is equivalent to an emergence of a field $\mathbf{f} = \nabla h(\mathbf{r}, t)$ that is a gradient of the concentration of those products. It is remarkable that the response of a particle to a chemical signal is not only its shift in space, but it is also a formation of chemical products, the composition of which is defined by the internal state parameter θ. As a result, it turns out that a group of active particles can move as follows:

– the rotational motion around the empty center of masses, so that the ensemble acquires a toroidal shape (the center of masses remains at rest in this case);

– the translational motion, when the ensemble moves as a whole, forming a dense group;

– the step-like motion (a consecutive alternation of the modes described above).

In work [2], an attempt was made to classify those motion modes. However, the scheme developed there was not self-consistent. This is related to the fact that one of the parameters that define the system behavior is introduced in an artificial way (for example, the internal parameter can accept only two values, $\theta = \pm 1$). In this work, the phenomenological scheme is studied, in the framework of which the self-organization of an ensemble of active particles is considered self-consistently. Our approach is based on the Lorentz three-parameter system which corresponds to the simplest field representation of a self-organizing system. Section 2 is devoted to the development of a model and the study of the monotonous self-organization mode. In Section 3, a step-like transition between motion modes of active particles is researched, and the estimation for the model parameters that promote the transition of the system to the ordered state is given. In Section 4, the analysis of the kinetic scenario of the transition is made on the basis of phase portraits of the system.

2. Synergetics of the Collective Behavior of Active Particles

By analogy with a condensed medium [7], a self-organizing system can be presented making use of a self-consistent description of the time dependences of the order parameter, conjugate field, and control parameter. For the sake of definiteness, let the group of active particles be represented by a flock of birds, the center of masses of which can either remain at rest or move translationally. Then, the order parameter, which distinguishes between those states, is reduced to the average velocity $\mathbf{v}$ of birds' motions. Respectively, the conjugate field is given by the long-range force $\mathbf{f} \equiv \nabla h(\mathbf{r}, t)$ of the chemical type, whereas the control parameter θ characterizes the internal state, which is determined by the reaction of particles to this force. As a result, the problem is reduced to the expression of the change rates $\dot{v}$, $\dot{f}$, and $\dot{\theta}$ of the indicated quantities in terms of their magnitudes v, f, and θ (below, for simplicity, we consider a one-dimensional case).

Bearing in mind that the order parameter $v(t)$ is the main one, and its behavior is inherited by both the force $f(t)$ and the internal parameter $\theta(t)$, let us take their relation for the average acceleration in the linear form

$$\tau_v \dot{v} = -v + A_v f. \tag{1}$$

Here, the first term on the right-hand side is responsible for the relaxation of the velocity to the zero value within the time interval τ_v, and the second term describes the linear reaction of the acceleration $\dot{v}$ to the field f growth ($A_f > 0$ is the coupling constant).

For the conjugate field, the equation is accepted in the form

$$\tau_f \dot{f} = -f + A_f v\theta, \tag{2}$$

where the first term has again a relaxation origin with the characteristic time τ_f, and the second one represents the positive feedback between the average velocity of motion and the internal state parameter, on the one hand, and the rate of conjugate field variation, on the other hand ($A_f > 0$ is the coupling constant). Just this coupling is responsible for the increase of the conjugate field which is responsible for self-organization.

The last equation of the system evolution describes the relaxation of the internal state parameter θ which plays the role of control parameter:

$$\tau_\theta \dot{\theta} = (\theta_e - \theta) - A_\theta v f. \tag{3}$$

In contrast to Eqs. (1) and (2), the first term in Eq. (3) describes the relaxation of the parameter θ to a final value θ_e, which is driven by an external influence, rather than to zero (τ_θ is the corresponding relaxation time, and $A_\theta > 0$ is the coupling constant). According to expression (3), the *negative* feedback between the long-range force and the motion velocity, on the one hand, and the change rate of the internal state parameter, on the other hand, give rise, in accordance with the Le Chatelier principle, to a reduction of this parameter.

According to work [8], the system of synergetic equations (1)–(3) is the simplest field scheme that demonstrates the self-organization effect. To analyze this system, it is convenient to take advantage of dimensionless variables, i.e. to reckon the time t, motion velocity v, conjugate field f, and internal state parameter θ on the following respective scales:

$$t_v, \quad v_c \equiv (A_f A_\theta)^{-1/2},$$

$$f_c \equiv (A_v^2 A_f A_\theta)^{-1/2}, \quad \theta_c \equiv (A_v A_f)^{-1}. \tag{4}$$

Then, the behavior of a group of active particles is presented by the dimensionless system of equations

$$\dot{v} = -v + f, \tag{5}$$

$$\sigma \dot{f} = -f + v\theta, \tag{6}$$

$$\delta\,\dot{\theta} = (\theta_e - \theta) - vf, \tag{7}$$

where the ratios between the characteristic relaxation times are designated as follows:

$$\sigma \equiv \frac{\tau_f}{\tau_v}, \quad \delta \equiv \frac{\tau_\theta}{\tau_v}. \tag{8}$$

The monotonous regime of self-organization is realized, when the relaxation time τ_v for the average velocity is much larger than the corresponding parameters for the conjugate field τ_f and the control parameter τ_θ [3]:

$$\sigma, \delta \ll 1. \tag{9}$$

Since the dimensionless rates $\dot{v}$, $\dot{f}$, and $\dot{\theta}$ are of the same order of magnitude, conditions (9) allow the left-hand sides of Eqs. (6) and (7) to be neglected, which brings about the relations

$$f = \theta_e \frac{v}{1+v^2}, \quad \theta = \frac{\theta_e}{1+v^2}. \tag{10}$$

Therefore, a spontaneous growth of the average velocity in the interval confined by a maximal value v_c results in an increase of the long-range force f and the decrease of the internal state parameter θ to a value of $\theta_e/2$, which is driven by the external influence.

The substitution of the first of equalities (10) into formula (5) gives rise to the Landau–Khalatnikov equation

$$\dot{v} = -\frac{\partial E}{\partial v}, \tag{11}$$

the form of which is defined by the kinetic energy of motion

$$E = \frac{v^2}{2} - \frac{\theta_e}{2} \ln\left(1 + v^2\right), \tag{12}$$

which is measured in v_c^2 units. For small values of the internal state parameter θ_e, the dependence $E(v)$ is monotonously growing, with a minimum at $v = 0$, which corresponds to the quiescent center of masses of the group (a disordered state). As θ_e grows to values that exceed the critical level θ_c, there appears a minimum at

$$v_0 = \sqrt{\theta_e - 1} \tag{13}$$

which corresponds to a translational motion (an ordered state). In this case, the long-range force attains the finite value $f_0 = v_0$, and the internal state parameter diminishes to the critical value $\theta_0 = 1$.

This analysis demonstrates that the system of equations (5)–(7) allows one to describe the self-consistent scenario of the spontaneous transition of a group of active particles into the translational motion regime.

3. Step-like Transition

The behavior of the system of active particles described above supposes that the characteristic time of an average velocity variation τ_v remains constant for the transition from the rotational to the translational motion mode. Actually, the system reaction to an increase of the average velocity v can induce the growth of the time τ_v. Such a slowing down brings about a transformation of a continuous transition to a step-like one. Let us use the simplest approximation

$$\tau_v = \tau_0 \left(1 + \frac{\kappa}{1 + v^2/v_\tau^2}\right)^{-1} \tag{14}$$

which is defined by positive constants τ_0, κ, and v_τ. Then, in the framework of adiabatic approximation (9), Lorentz system (5)–(7) is associated with the kinetic energy of motion

$$E = \frac{v^2}{2} - \theta_e \ln\left(1+v^2\right) + \frac{\kappa v_1^2}{2} \ln\left[1+\left(\frac{v}{v_1}\right)^2\right],$$
$$v_1 \equiv \frac{v_\tau}{v_c} \tag{15}$$

which differs from expression (12) by the last term. According to formula (15), if the internal state parameter θ_e is small, the dependence $E(v)$ monotonously grows with a minimum at the point $v = 0$ which corresponds to the rotational mode of motion. At the value

$$\theta_c^0 = 1 + v_1^2(\kappa - 1) + 2v_1\sqrt{\kappa\left(1 - v_1^2\right)}, \tag{16}$$

there appears a plateau in the dependence $E(v)$. At $\theta_e > \theta_c^0$, this plateau transforms into a minimum that corresponds to a non-zero value of the average velocity ($v_0 \neq 0$, the translational motion) and a maximum (the unstable state) that separates the minima at $v_0 = 0$ and $v_0 \neq 0$. As the internal state parameter grows further, the minimum corresponding to the translational motion mode becomes deeper, whereas the height of the barrier that separates stationary states falls down to achieve zero at the critical value of the internal state parameter

$$\theta_{c0} = 1 + \kappa. \tag{17}$$

The stationary values of the average velocity are determined by the relations

$$v_0^m = v_{00}\left\{1 \mp \left[1 + \left(\frac{v_1}{v_{00}^2}\right)^2 (\theta_e - \theta_{c0})\right]^{1/2}\right\}^{1/2}, \tag{18}$$

$$v_{00}^2 \equiv \frac{1}{2}\left[(\theta_e - 1) - (1+\kappa)v_1^2\right], \tag{19}$$

where the upper sign corresponds to the unstable state v^m, when the kinetic energy of motion is maximal, and the lower one to the stable state v_0 (the translational motion). If the internal state parameter increases slowly, the stationary average velocity v_0 has a jump from zero to $\sqrt{2}v_{00}$ at the point $\theta_e = \theta_{c0}$. Afterwards, its magnitude smoothly grows. If θ_e decreases backward, the stationary value of average velocity v_0 first smoothly diminishes until the point $\theta_e{=}\theta_c^0$, $v_0{=}v_{00}$ is reached; then, it vanishes in a jump-like manner. Hence, a hysteresis takes place at $v_1 \equiv v_\tau/v_c < 1$, which stems from the presence of a barrier in the dependence of the kinetic energy of motion (15). The growth of the internal state parameter variation rate is accompanied by the narrowing of the hysteresis loop.

Reckoning the average motion velocity v, long-range force f, and internal state parameter θ on the v_c, f_c, and θ_c scales, respectively–they are defined by equalities (4)–one can study possible evolution modes of the system which are determined by different ratios among the characteristic times τ_0, τ_f, and τ_θ. In so doing, it is necessary to remember that the effective time of the average velocity variation τ_v in initial Eq. (5) is given by formula (14).

3.1. Case $\tau_f \ll \tau_0, \tau_\theta$

Let us proceed from studying the nonmonotonous behavior and consider the case where the relaxation time of the long-range force is the shortest, so that the long-range force can trace the variations of the average velocity and the internal state parameter. As a result, the oscillations of the long-range force can be neglected, and the value $\dot{f} = 0$ can be adopted in Eq. (6). This yields the relation

$$f = v\theta. \tag{20}$$

Making allowance for relation (20) in Eqs. (5) and (7), we arrive at the system of equations (the time is measured in units of τ_0)

$$\dot{v} = -v\left[(1-\theta) + \kappa\left(1 + v^2/v_1^2\right)^{-1}\right], \tag{21}$$

$$\dot{\theta} = \delta^{-1}\left[\theta_e - \theta\left(1+v^2\right)\right]. \tag{22}$$

Its behavior is determined by the value of the internal state parameter θ_e, which describes the degree of system nonequilibrium, and by the ratio between the characteristic variation times of the internal state parameter and the average velocity

$$\delta = \frac{\tau_\theta}{\tau_0}. \tag{23}$$

In the general case, the standard analysis [9] of system (21), (22) testifies that its phase portrait is characterized by the presence of three singular points $D(\theta_e, 0)$, $O(\theta_-, v_-)$, and $S(\theta_+, v_+)$, the coordinates $\theta_\pm$ and $v_\pm$ of which are defined by the equalities

$$\theta_\pm = \frac{\left(1+v_{00}^2\right) \pm \sqrt{\left(1+v_{00}^2\right)^2 - \theta_e\left(1-v_1^2\right)}}{1 - v_1^2}, \tag{24}$$

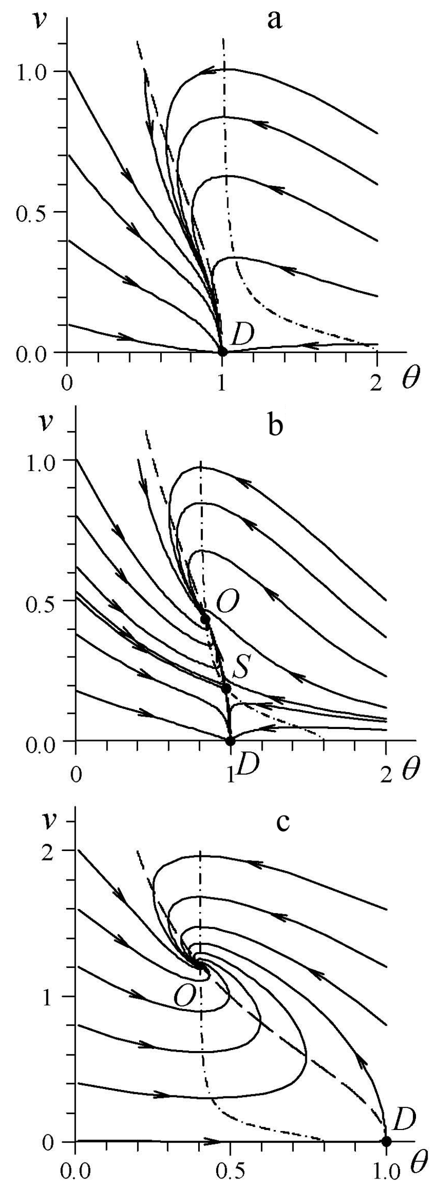

Fig. 1. Phase portraits of the step-like transition between the motion modes of active particles for various values of the internal state parameter $\theta_e = \theta_c$ (a), $1.25\theta_c$ (b), and $2.5\theta_c$ (c). $\kappa = 1$, $v_1 = 0.1$, and $\tau_f \ll \tau_0 = \tau_\theta$

$$v_\pm = \sqrt{(\theta_e - \theta_\pm)/\theta_\pm}. \tag{25}$$

Point D is associated with the Lyapunov parameters

$$\lambda_D = \frac{\delta(\theta_e - \theta_{c0}) - 1}{2\delta}\left(1 \pm \sqrt{1 + \frac{4\delta(\theta_e - \theta_{c0})}{[\delta(\theta_e - \theta_{c0}) - 1]^2}}\right), \tag{26}$$

where, according to formula (17), the quantity $\theta_{c0} = 1 + \kappa$ defines the stability loss point. Therefore, for $\theta_e < \theta_{c0}$, point D is a stable node, whereas, for $\theta_e > \theta_{c0}$, it is a saddle point. The Lyapunov parameters of points $O(\theta_-, v_-)$ and $S(\theta_+, v_+)$ are determined by their coordinates (24) and (25), making use of the relations

$$\lambda_\pm = \lambda_0\left(1 \pm \sqrt{1+\Delta}\right),$$

$$\lambda_0 = \frac{\theta_e - \theta_\pm}{\kappa v_1^2 \theta_\pm}(\theta_\pm - 1)^2 - \frac{\theta_e}{2\delta\theta_\pm},$$

$$\lambda_0^2 \Delta = \frac{2}{\delta}(\theta_e - \theta_\pm)\left(\frac{\theta_e(1-\theta_\pm)^2}{v_1^2 \kappa \theta_\pm^2} - 1\right). \tag{27}$$

In the interval $\theta_c^0 < \theta < \theta_{c0}$, point S is a saddle, and point O a stable node or focus.

Those results testify that the phase portrait of the system changes in the following way, as the internal state parameter grows (see Fig. 1). At $\theta_e < \theta_c^0$, when dependence (15) grows monotonously, points S and O are absent, and point D is a stable node; this configuration corresponds to the rotational motion mode of active particles. If characteristic value (16) is exceeded, there occurs a bifurcation in the system. It consists in the appearance of saddle S and stable node/focus O, which are defined by coordinates (24) and (25). As the parameter θ_e grows further, the saddle that corresponds to the energy barrier in the dependence $E(v)$ approaches point D and, at the point θ_{c0}, absorbs it. A further growth of θ_e gives a picture that corresponds to the translational mode of motion.

Figure 2 demonstrates how the phase portrait, which corresponds to the translational motion of the ensemble of active particles ($\theta_c^0 < \theta_e < \theta_{c0}$), changes with increase in the ratio $\delta = \tau_\theta/\tau_0$ between the relaxation times. In the vicinity of point O, the trajectories quickly converge to a universal section MOS in the adiabatic approximation $\tau_\theta \ll \tau_0$ (Fig. 2,a), whereas, in the inverse case $\tau_\theta \gg \tau_0$, there appears a mode of damped oscillations (Fig. 2,c). In addition, in the range of low average velocities, there emerges a separatrix which evidences for the presence of a barrier in the dependence $E(v)$. The researches of time dependences of the path passed by a point along the phase trajectory [10] testifies to a deceleration near the *big river channel* MOS, which corresponds to the vicinity of the ordered state minimum (the translational mode).

The features found in the system evolution can be described on the basis of the dependence $E(v,\theta)$ of the kinetic energy of motion E on the average velocity v and the internal state parameter θ. In so doing, it is worth proceeding from the fact that, in the course of evolution, the system is mainly located in the vicinity of extrema of the dependence $E(v,\theta)$. Since the relaxation time along each of the axes v and θ is reciprocal to the dependence $E(v,\theta)$ curvature along the corresponding axis[1], the condition $\tau_\theta \ll \tau_0$ means that the dependence $E(v,\theta)$ changes much more quickly along the θ-axis than along the v-one. As a result, it turns out that the surface of the function $E(v,\theta)$ has a narrow groove along the universal trajectory caused by the dependence $\theta(v)$ in form (10). As is seen from Fig. 2,a, the system quickly rolls down into the groove along the axis θ, which corresponds to a large curvature. It is just this groove that provides a universal character of the system evolution, because the dependence $E(v,\theta)$ always looks like a parabola near its extrema:

$$E \approx E(v_0,\theta_0) + \frac{\chi_v^{-1}}{2}(v-v_0)^2 + \frac{\chi_\theta^{-1}}{2}(\theta-\theta_0)^2, \qquad (28)$$

where the values of v_0 and θ_0 determine the extremum position, and the susceptibilities χ_v and χ_θ do the curvatures.

In view of the aforesaid, it may seem that, in the approximation $\tau_0 \ll \tau_\theta$, which is opposite to the adiabatic one, the dependence $E(v,\theta)$ should also have a groove, so that the origin of damped oscillations presented in Fig. 2,c becomes incomprehensible. However, it is worth remembering that the susceptibilities χ_v and χ_θ in equality (28) are connected with the relaxation times τ_0 and τ_θ by different relations $\chi_\theta \propto \tau_\theta$ and $\chi_v \propto \tau_0|\theta_e-\theta_{c0}|^{-1}$ near the minimum point. Since $|\theta_e-\theta_{c0}| \ll 1$, the curvature $\chi_v^{-1} \propto \tau_0^{-1}|\theta_e-\theta_{c0}|$ of parabola (28) along the v-axis turns out comparable with the curvature $\chi_\theta^{-1} \propto \tau_\theta^{-1}$ of the dependence $E(v,\theta)$ along the θ-axis, despite that the value of τ_0 is small. In other words, if $\tau_0 \ll \tau_\theta$, the dependence $E(v,\theta)$ is paraboloid-like near the minimum that corresponds to the translational motion mode, with the curvatures along the v- and θ-axes being small and comparable with each other. As a result, when the configuration point rolls down to the minimum, it rotates, lying on the surface of this paraboloid. Such a rotation obviously corresponds to damped oscillations illustrated in Fig. 2,c.

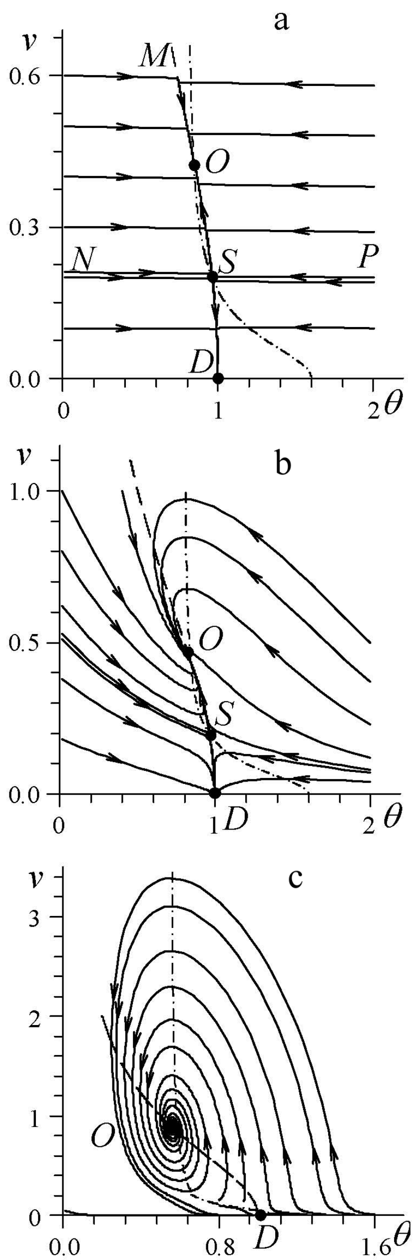

Fig. 2. Phase portraits of the step-like transition for various values of the internal state parameter and various ratios between the times of hydrodynamic mode change ($\kappa = 1$ and $v_1 = 0.1$): (a) $\theta_e = 1.25\theta_c$, $\tau_f \ll \tau_0 = 100\tau_\theta$; ($b$) $\theta_e = 1.25\theta_c$, $\tau_f \ll \tau_0 = \tau_\theta$; and ($c$) $\theta_e = 1.8\theta_c$, $\tau_f \ll \tau_\theta = 10\tau_0$

It is worth bearing in mind that the critical growth of the susceptibility χ_v in formula (28), which was

[1]It follows from the fact that the indicated curvature is reciprocal to the corresponding susceptibility, which is proportional, in its turn, to the corresponding relaxation time [11].

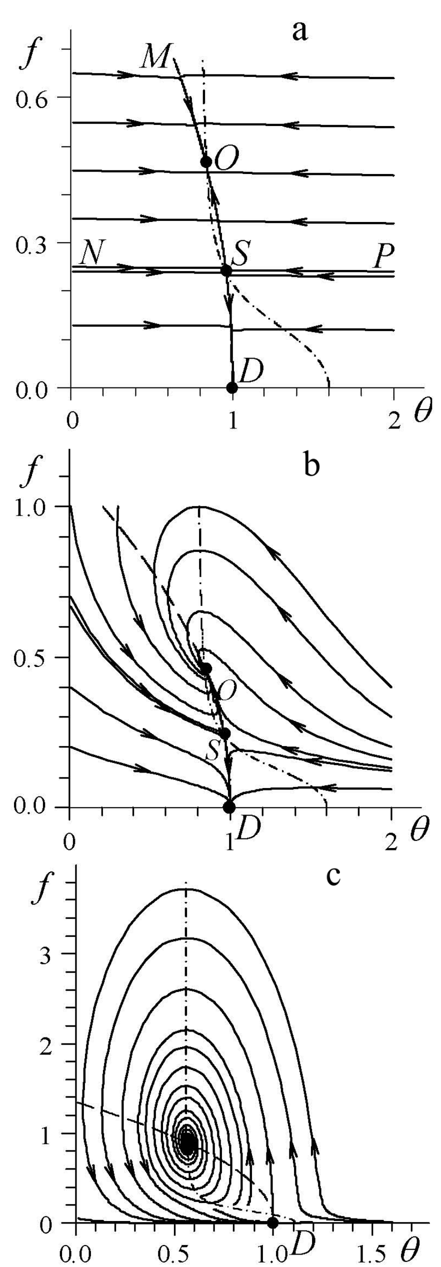

Fig. 3. Phase portraits of the step-like transition ($\kappa = 1$ and $v_1 = 0.1$): (*a*) $\theta_e = 1.25\theta_c$, $\tau_0 \ll \tau_f = 100\tau_\theta$; (*b*) $\theta_e = 1.25\theta_c$, $\tau_0 \ll \tau_f = \tau_\theta$; and (*c*) $\theta_e = 1.8\theta_c$, $\tau_0 \ll \tau_\theta = 10\tau_f$

described above, has a collective character typical of a self-organizing system [7]. It does not manifest itself in the vicinity of the maximum in the dependence $E(v, \theta)$. Just for this reason, the separatrix is not twisted in the approximation $\tau_0 \ll \tau_\theta$ (see Fig. 2,*c*).

3.2. Case $\tau_0 \ll \tau_f, \tau_\theta$

From the analytical point of view, this case is the most laborious one, because the substitution of the effective time for the average velocity variation (formula (14)) into initial equation (5), where $\dot{v}$ has to be put equal zero, gives rise to a cubic equation (see work [12]). It is convenient to write down its solution in the form

$$3v = f + v_+(f) + v_-(f), \tag{29}$$

where the functions

$$v_\pm(f) = \left\{ f\left(f^2 + f_1^2\right) \pm 3v_1 \sqrt{3\left[\left(f^2 + f_2^2\right)^2 + f_3^4\right]} \right\}^{\frac{1}{3}} \tag{30}$$

and the constants f_1, f_2, and f_3 defined by the equalities

$$f_1^2 \equiv \frac{9}{2} v_1^2 (2 - \kappa),$$

$$f_2^2 \equiv \frac{1}{8} v_1^2 \left[36(2 - \kappa) - (8 - \kappa)^2\right],$$

$$f_3^2 \equiv \frac{1}{8} v_1^2 \sqrt{\kappa(8 - \kappa)^3} \tag{31}$$

are introduced. The substitution of relation (29) into Eqs. (6) and (7)) transforms them to

$$\dot{f} = -f + \frac{1}{3}\theta \left[f + v_+(f) + v_-(f)\right], \tag{32}$$

$$\tau\dot{\theta} = (\theta_e - \theta) - \frac{1}{3} f \left[f + v_+(f) + v_-(f)\right], \tag{33}$$

where the time is measured in τ_f-units, and the ratio $\tau \equiv \tau_\theta / \tau_f$ between the relaxation times is introduced.

Though neither the singular points nor the corresponding Lyapunov parameters can be find analytically in this case, the numerical study of the phase portrait (see Fig. 3) testifies that the behavior of the system coincides with that analyzed in Section 3.1. Here, similarly to the previous case, there appears a separatrix for those values of the internal state parameter θ and the long-range force f, which correspond to the energy barrier that separates the translational and rotational motion modes. Then, by analogy with the previous section, it is easy to see that, for $\tau_\theta \ll \tau_f$, when the universality of the system evolution manifests itself as much as possible (see Fig. 3,*a*), the dependence $E(f, \theta)$ has a narrow groove along the universal section of trajectories. The presence of the damped oscillation mode in the opposite case $\tau_\theta \gg \tau_f$ (Fig. 3,*c*) evidences for a critical growth of the long-range force susceptibility, $\chi_f \propto \tau_f |\theta_e - \theta_{c0}|^{-1}$. The main reason for this growth is obviously the critical growth of the average velocity susceptibility, $\chi_v \propto \tau_0 |\theta_e - \theta_{c0}|^{-1}$. The presence of the pronounced functional relation (29) between the average velocity and the long-range force provides the growth of the long-range force susceptibility.

3.3. Case $\tau_\theta \ll \tau_0, \tau_f$

If we put $\dot{\theta} = 0$ in formula (7), we obtain the relation

$$\theta = \theta_e - vf. \tag{34}$$

Its substitution into Eq. (6) gives the equation

$$\dot{f} = \frac{1}{\sigma}\left[\theta_e v - f(1+v^2)\right], \tag{35}$$

where $\sigma \equiv \tau_f/\tau_0$. In this case, Eq. (5) looks like

$$\dot{v} = -v\left(1 + \frac{\kappa}{1+v^2/v_1^2}\right) + f. \tag{36}$$

The phase portrait of system (35), (36) is characterized by the presence of three singular points $D(0,0)$, $O(f_+, v_+)$, and $S(f_-, v_-)$, the coordinates $v_\pm$ and $f_\pm$ of which are given by the equalities

$$v_\pm = \left\{v_{00}^2 \pm \sqrt{v_{00}^4 + \left[\theta_e - (1+\kappa)\right] v_1^2}\right\}^{1/2}, \tag{37}$$

$$f_\pm = \theta_e \frac{v_\pm}{1+v_\pm^2}. \tag{38}$$

The Lyapunov parameters of singular point D are determined by the expression

$$\lambda_D = -\frac{1}{2}\left(\theta_{c0} + \sigma^{-1}\right)\left[1 \pm \sqrt{1 + \frac{4(\theta_e - \theta_{c0})}{\sigma(\theta_{c0}+\sigma^{-1})^2}}\right]. \tag{39}$$

If $\theta_e < \theta_{c0}$, this point is a stable node, whereas if $\theta_e \geq \theta_{c0}$, it is a saddle. The Lyapunov parameters of points $O(f_+, v_+)$ and $S(f_-, v_-)$ depend on their coordinates (37) and (38) in the following manner:

$$\lambda_\pm = \lambda_0\left(1 \pm \sqrt{1+\Delta}\right),$$

$$\lambda_0 = \frac{(f_\pm - v_\pm)^2}{\kappa v_1^2} - \frac{1}{2}\left(\frac{1+v_\pm^2}{\sigma} + \frac{f_\pm}{v_\pm}\right),$$

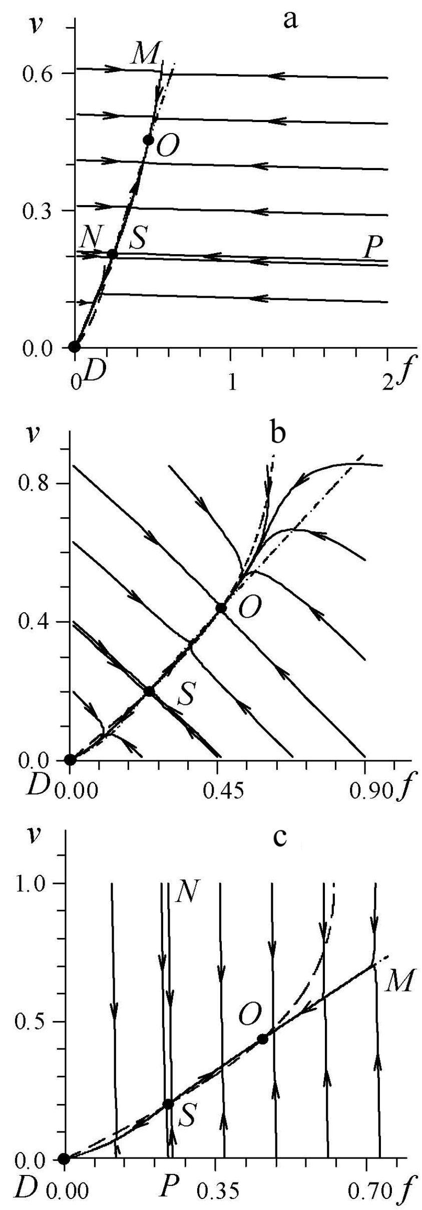

Fig. 4. Phase portraits of the step-like transition ($\kappa = 1$, $v_1 = 0.1$, and $\theta_e = 1.25\theta_c$): (*a*) $\tau_\theta \ll \tau_0 = 100\tau_f$, (*b*) $\tau_\theta \ll \tau_0 = \tau_f$, and (*c*) $\tau_\theta \ll \tau_f = 100\tau_0$

$$\lambda_0^2\Delta = \frac{\left(1-v_\pm^2\right) f_\pm}{\tau v_\pm} - \frac{1+v_\pm^2}{\sigma}\left[\frac{f_\pm}{v_\pm} - \frac{2\left(f_\pm - v_\pm\right)^2}{\kappa v_1^2}\right]. \tag{40}$$

In the interval $\theta_c^0 < \theta < \theta_{c0}$, where the step-like transition is realized, point S is a saddle, and point O a stable node or focus.

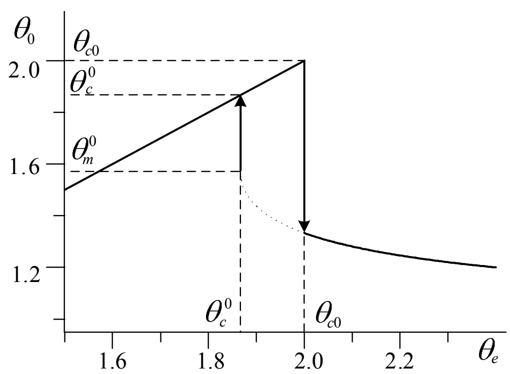

Fig. 5. Dependence of the stationary value of the internal state parameter θ_0 on the parameter θ_e driven by an external influence ($\kappa = 1$ and $v_1 = 0.5$)

Numerical calculations of the phase portrait give a pattern shown in Fig. 4. A distinctive feature in this case is the circumstance that the universality of the system evolution is observed not only if $\tau_f \ll \tau_0$ (Fig. 4,*a*), but also for the opposite relation $\tau_f \gg \tau_0$ (Fig. 4,*c*), when the emergence of oscillations might be expected. This phenomenon is related to the fact that the curvatures of parabolas along the axes f and v are determined by the magnitudes of inverse susceptibilities $\chi_f^{-1} \propto \tau_f^{-1}|\theta_e - \theta_{c0}|$ and $\chi_v^{-1} \propto \tau_0^{-1}|\theta_e - \theta_{c0}|$. Therefore, if $\tau_f \gg \tau_0$, the relation $\chi_f^{-1} \ll \chi_v^{-1}$ is fulfilled, which means that the curvature of the dependence $E(v, f)$ along the axis of the long-range force f is much smaller than that along the axis of the average velocity v. Therefore, in Fig. 4,*c*, the trajectories, along which the configuration point quickly rolls down to the *big river channel*, are oriented practically along the v-axis.

This research demonstrates that Lorentz equations (5)–(7) allow one to describe the main features of the transition between the motion modes of active particles, namely, between the rotational and translational ones. The phenomenological description is reached, if the dependence $E(v)$ of the kinetic energy on the average velocity is used. The step-like transition is realized, if the time of the average velocity relaxation depends on v, according to Eq. (14). Then, the function $E(v)$ is given by Eq. (15) that depends on the parameters κ and v_τ. The dependence $E(v)$ has an energy barrier, which separates the minima corresponding to the translational and rotational motions, provided that $v_1 \equiv \frac{v_\tau}{v_c} < 1$. In this case, if the parameter θ_e falls within the interval $(\theta_c^0, \theta_{c0})$, the transition occurs in a step-like manner, whereas, at $\theta_e > \theta_{c0}$, it is continuous. The equilibrium value of the average velocity is determined by equalities (18) and (19).

An essential feature of a self-organizing system consists in that the stationary value of the internal parameter θ_0 does not coincide with that of the parameter θ_e driven by an external influence. For the step-like transition, the value of θ_- (see Eq. (24), which corresponds to the minimum in the dependence $E(v)$, is realized. Since θ_c and θ_- are the minimal values of the internal parameter, which mark the start of the transition into the translational motion mode, the indicated fact means that the negative feedback between the average velocity and the long-range force (the last term in formula (7)) stimulates so a strong reduction of the internal state parameter that the latter ensures self-organization, only if it has a limit value.

According to Fig. 5, the stationary value of the internal parameter θ_0 coincides with the θ_e-value in the interval $0 < \theta_e < \theta_c^0$. If $\theta_e > \theta_c^0$, the stationary value of the internal parameter θ_0 becomes two-valued and slowly falls down from

$$\theta_m^0 = \theta_{c0}\left(1 + v_1\sqrt{\frac{\kappa}{1 - v_1^2}}\right) \tag{41}$$

at $\theta_e = \theta_c^0$ to θ_c as $\theta_e \to \infty$. Provided that θ_e slowly grows from 0 to θ_{c0}, the stationary value of the internal parameter linearly grows in the same interval. After a jump downward at the point $\theta_e = \theta_{c0}$, the quantity θ_0 slowly decreases in accordance with dependence (24) for θ_-. If θ_e decreases backward, the stationary value of the internal parameter θ_0 slowly grows and, at the point θ_c^0, changes in a step-like manner from θ_m^0 to θ_c^0. In the interesting range of the parameters v_1 and κ—it is confined by the value

$$\kappa_{\min} = \frac{v_1^2}{1 - v_1^2}, \tag{42}$$

–θ_m^0 is less than θ_c^0. Therefore, the minimal value of the internal parameter θ_0 is less than θ_e in the interval $(\theta_c^0, \theta_{c0})$.

The kinetic scenario of the transition is represented by the phase portraits illustrated in Figs. 1 to 4. In the case of the step-like transition, there occurs a bifurcation at $\theta_e = \theta_c^0$. As a result, there appear node D which characterizes the rotational motion, saddle S which corresponds to the energy barrier in the dependence $E(v)$, and node/focus O which characterizes the translational mode of motion. As the internal state parameter grows within the interval $(\theta_c^0, \theta_{c0})$, saddle S approaches node D and absorbs it at the point θ_{c0}, whereas node/focus O shifts toward increasing both the average velocity and the long-range force.

4. Conclusions

According to the dimensionless system of Lorentz equations (21), (6), and (7), the scenario of the transition from the rotational to the translational motion mode is governed by a set of three synergetic and three kinetic parameters. The leading role in the first group is played by the internal parameter θ_e, the ratio of which to the critical value θ_c determines the motion mode (see Fig. 1). The other two synergetic parameters, κ and $v_1 \equiv v_\tau/v_c$, the values of which define dispersion law (14) for the time of the average velocity change, determine the step-like transition region. It is confined by the minimal value (42). If moving away from the latter, the region extends. A characteristic feature of the step-like transition is that a separatrix appears in the phase portrait, which gives rise to a critical dependence of the system evolution on the choice of the initial system state. Such a behavior corresponds to the division of the space of states into motion mode regions, and the corresponding boundary between them is given by the separatrix.

The kinetic parameters, which govern the system behavior, are the characteristic times τ_0, τ_f, and τ_θ of the average velocity, long-range force, and internal parameter changes, respectively, in independent regimes. Our research shows that the universal scenario of the system evolution is realized, if the latter time is minimal. In this case, the system reaches the universal section MOS (see Figs. 2 to 4) in a short time interval τ_θ. The position of this section depends only on the synergetic parameters rather than on the ratios among the time scales τ_0, τ_f, and τ_θ. Such a situation is evidently realized in reality. If the internal parameter change time grows abnormally, the phase portraits become twisted near the stable state point.

To summarize, we note that, in order to obtain actual values for the average velocity, long-range force, and internal parameter, their dimensionless counterparts v, f, and θ must be multiplied by the corresponding scales v_c, f_c, and θ_c, given by equalities (4). The results of our consideration testify that the most disposed to the evolution are systems which have a low critical threshold θ_c, the overcoming of which provides the transition to the translational motion mode. According to the last of equalities (4), such a transition is achieved in the simplest way, if the values of the coupling parameters A_v and A_f in Eqs. (1) and (2) are large.

1. H. Markl, A.v. Humboldt–Magazin **65**, 13 (1995).
2. W. Ebeling and F. Schweitzer, cond–mat/0211606 v1.
3. H. Haken, *Synergetics*(Springer, Berlin, 1978).
4. A.B. Medvidinskii, S.V. Petrovskii, I.A. Tikhonova *et al.*, Usp. Fiz. Nauk **172**, 1, 31, (2002).
5. W. Alt and G. Hoffmann, *Biological Motion* (Springer, Berlin, 1990).
6. G.R. Ivanitskii, A.B. Medvidinskii, and M.A. Tsyganov, Usp. Fiz. Nauk **164**, 1041 (1994).
7. A.I. Olemskoi and A.A. Katsnelson, *Synergetics of Condensed Matter* (URSS, Moscow, 2003) (in Russian).
8. A.I. Olemskoi, Physica A **310**, 223 (2002).
9. A. Andronov, A. Vitt, and S. Khaykin, *Theory of Oscillations* (Pergamon Press, Oxford, 1966).
10. A.I. Olemskoi, A.V. Khomenko, Zh. Èksp. Teor. Fiz. **110**, 2144 (1996).
11. A.A. Katsnelson and A.I. Olemskoi, *Microscopic Theory of Nonhomogeneous Structures* (Mir, Moscow, 1990).
12. L.D. Landau and E.M. Lifshits, *Statistical Physics, Part 1* (Pergamon Ptress, Oxford, 1980).
13. H. Haken, *Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices* (Springer, New York, 1993).
14. A.I. Olemskoi, *Fractals in Condensed Matter Physics* (Gordon and Breach, London, 1996); A.I. Olemskoi and A.Ya. Flat, Usp. Fiz. Nauk **163**, 5 (1993).

Received 12.11.08.
Translated from Ukrainian by O.I. Voitenko

СИНЕРГЕТИЧНИЙ ПІДХІД
ДО ОПИСУ ПЕРЕРИВЧАСТОГО ПЕРЕХОДУ
МІЖ РЕЖИМАМИ РУХУ АКТИВНИХ ЧАСТИНОК

О.В. Ющенко, О.І. Олемской

Р е з ю м е

Розвинуто синергетичну модель, яка дозволяє представити перехід між режимами руху ансамблю активних частинок типу біологічних систем. Показано, що стани системи зводяться до безладного руху частинок або до поступального зміщення їх ансамблю як цілого, залежно від ступеня збудженності системи, що визначається внутрішнім параметром. На основі фазових портретів системи досліджено переривчастий перехід між режимами руху активних частинок для різних співвідношень між часами зміни гідродинамічних мод. Проаналізовано параметри моделі, що сприяють переходу системи до впорядкованого стану.