DEPENDENCE OF NUCLEAR BINDING ENERGY ON THE SPECIFIC CHARGE IN THE RANGE OF SMALL ISOTOPIC NUMBERS

M.M. POPUSHOY

Odessa State Academy of Building and Architecture
(4, Didrikhson Str., Odessa 65029, Ukraine)

UDC 539.17

On the basis of the S-matrix formalism, approximated linear dependences of the nuclear binding energy on the specific charge ZA^{-1} have been found for isotopic (isotonic) nuclei ${}^{A}_{Z}Q$ with $J^{\pi} = \text{const}$ in the range of isotopic numbers $Y = A - 2Z = 0, \pm1, \pm2, \ldots$.

On the basis of the Schrödinger equation, the analytical structure of the S-matrix in the complex plane of the Coulomb coupling constant $a = \mu Zz$ $(2e^2 = h = c = 1)$, which is caused by the complex values of charge numbers Z and z at fixed values of the reduced mass μ and the orbital moment l, is known to describe the Coulomb effects in the system

$$ {}^{A_1-A}_{\quad z}X + {}^{A}_{Z}Y \leftrightarrow {}^{A_1}_{z+Z}Q, \tag{1}$$

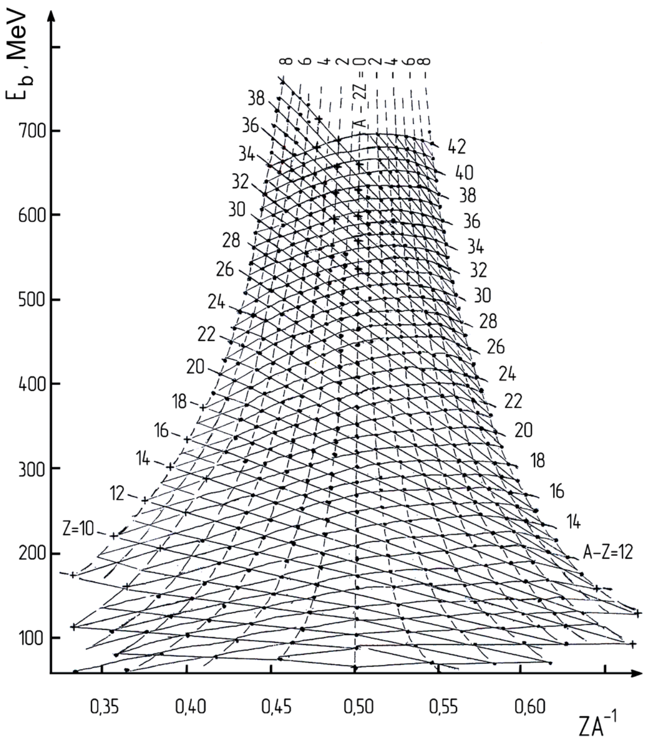
where X, Y, and Q are isobaric nuclear multiplets ($A = \text{const}$, $A_1 = \text{const}$, and $J^{\pi} = \text{const}$) [1]. For instance, the bound states and resonances of compound systems ${}^{A_1}_{z+Z}Q$, which arise when a fixed particle (cluster) ${}^{A_1-A}_{\quad z}X$ with $z = \text{const}$ and $A_1 - A = \text{const}$ is scattered by isobaric nuclei ${}^{A}_{Z}Y$ ($A = \text{const}$) with $J^{\pi} = \text{const}$, can be described by a trajectory of a pole of the S-matrix in the complex a-plane, which is caused by complex Z-values. In a specific case of α-decay of the compound system, the relation between the resonance energy E_r and the width Γ_r, obtained from the behavior of the trajectory of the pole $\alpha(E) = \mu Z(E)z$ when the energy $E \to 0_+$, is the Geiger–Nuttall law for isobaric nuclei with $J^{\pi} = \text{const}$; i.e. the quantities $\lg \Gamma_r$ and $E_r^{-1/2}$ are linearly interdependent [2].

The analytic properties of the S-matrix in the complex plane, which originates due to the complex values of the reduced mass μ at fixed physical values of Z and z, correspond to the description of baryon effects in system (1), where X, Y, and Q are isotopic nuclei with $J^{\pi} = \text{const}$ [3]. For instance, in the case of α-decay of compound systems ${}^{A_1}_{z+Z}Q$ with $z + Z = \text{const}$ and $A_1 = \text{var}$, the relation between the resonance parameters E_r and Γ_r, found from the specific trajectory of the pole $a(E) = \mu(E)Zz$ at $E \geq 0$, is the Geiger–Nuttall law for isotopic nuclei with fixed J^{π}-characteristics [4]. It should be noted that the state of a quantum system is invariant with respect to its description by a pole of the S-matrix in the complex plane of each of the spectral parameters that are included into the Schrödinger equation irrespective of a specific model, provided that the values of other parameters are physically reasonable [5]. Therefore, in terms of the S-matrix formalism of a two-fragment problem, the universality of the law for isobars and isotopes means that resonance states of compound systems do not depend on the way of their description by poles of the S-matrix in the complex a-plane, provided that $\mu = \text{const}$ and $Z = \text{var}$, or $Z = \text{const}$ and $\mu = \text{var}$.

In the case where complex values of the coupling Coulomb constant are resulted from complex values of μ and Z, the real part of a trajectory of the S-matrix pole describes the scattering of cluster X by isotonic nuclei ${}^{A}_{Z}Y$ ($A - Z = \text{const}$) with $J^{\pi} = \text{const}$, and it is a linear function of the energy E of resonance states of system (1), i.e.

$$\operatorname{Re} a(E) \approx \gamma_1 + \gamma_2 E, \tag{2}$$

where γ_1 and γ_2 are constants [6]. Equation (2) is determined by the behavior of $\operatorname{Re} a(E)$ in the direct product of complex regions $\operatorname{Re}\mu(E)>0$ and $\operatorname{Re} Z(E)>0$,



Dependence of the binding energy of nuclei ${}^{A}_{Z}Q$ on the specific charge ZA^{-1}. Solid lines pass through isotopes and isotones; dashed lines are drawn through nuclei with a fixed value of neutron (proton) excess

where the coefficients γ_1 and γ_2 depend on the physical values of the orbital moment l and the electric charge of cluster X.

An important role of formula (2) with respect to its terms should be emphasized. Namely, the real part of the coupling Coulomb constant a depends linearly on the resonance energies at $\mu = \text{const}$ and $Z = \text{var}$, or $\mu = \text{var}$ and $Z = \text{const}$, or $\mu = \text{var}$ and $Z = \text{var}$. This conclusion is confirmed by the data on α-decay [4, 6], pick-up of nucleons and nucleon pairs [7, 8], and decay of the ground nuclear states into arbitrary isotopic non-magic daughter fragments with $J^\pi = \text{const}$ [9]. Moreover, on the basis of the equidistance of decay energies, the effective mass formulas for heavy even-even ($J^\pi = 0^+$) isotopes [10] and isotones [11] were derived.

Works [12, 13] generalize results of the S-matrix formalism to the case where the complex values of the Coulomb coupling constant $a = \mu Z z$ are associated with the complex masses and the charge numbers of fragments ${}^{A_1-A}_{z}X$ and ${}^{A}_{Z}Y$ with $J^\pi = \text{const}$. In this case, the real part of a trajectory of the S-matrix pole $a(E)$ satisfies an equation similar to Eq. (2) in the region that is a direct product of complex regions $\mathrm{Re}\,\mu(E) > 0$, $\mathrm{Re}\,Z(E) > 0$, and $\mathrm{Re}\,z(E) > 0$. This means that if the ground state of the system Q ($A_1 = \text{const}$ and $z + Z = \text{const}$) decays into variable daughter fragments with $J^\pi = \text{const}$, the decay energy

$$E = M(z + Z; A_1) - M(z; A_1 - A) - M(Z; A),$$

where $z \neq \text{const}$ and $Z \neq \text{const}$, and the real values of the coupling Coulomb constant

$$a = \frac{M(z; A_1 - A)M(Z; A)}{M(z; A_1 - A) + M(Z; A)} zZ$$

are connected with each other by an approximately linear relation, which is confirmed by the results of calculations dealing with a two-fragment decay of the ground states of ^{212}Po, ^{214}Rn, ^{216}Ra, ^{218}Th, ^{238}U, and ^{240}Cm nuclei, where the variable fragment X is α-cluster structures in the ground state: ^{4}He, ^{8}Be, ... , ^{28}Si [12, 13].

In recent work [14], the results obtained within the S-matrix formalism at variable electric charges and a reduced mass were generalized to the case of the separation of a compound system into free nucleons. In that work, the approximate linear dependences of the binding energy of non-magic nuclei ${}^{A}_{Z}Q$ on the values of Coulomb parameter ZA^{-1} (specific charge) were found, by taking into account that those nuclei belong to isotopes, isotones, isobars, and nuclei with a fixed neutron excess. The given linear dependences of E_b on the physical values of the parameter ZA^{-1} are satisfactory for nuclei ${}^{A}_{Z}Q$ with $60 \leq Z \leq 100$ and $14 \leq A - 2Z \leq 58$ [14].

This work aims at finding the character of the dependence of the binding energy of isotopic (isotonic) nuclei on the specific charge ZA^{-1} in the range of low neutron (proton) excess, i.e. on the isotopic number $Y = A - 2Z = 0, \pm1, \pm2, \ldots$. It has to be noted that, in the range $Y \leq 0$, the nuclear binding energies are known for a restricted number of nuclei. For example, in work [15], the binding energies for $Y = 0, -1, -2$, and -3 were given for Z up to $Z = 30, 25, 22$, and 20, respectively, while the data were absent for $Y = -4$ and -5. However, in works [16, 17], on the basis of the rule of the equidistant energies of separation (decay) of isobaric multiplet members, the relations between the ground state masses of mirror nuclei were established. These relations were used to calculate the mass excess ΔM for a large group of proton-rich nuclei and to carry out the analysis of the results obtained. On the basis of known values for ΔM and the relation

$$E_b(Z;A) = A\Delta_n - Z\Delta_{nH} - \Delta M, \tag{3}$$

where $\Delta_n = 8.071$ MeV is the neutron mass excess with respect to the atomic mass unit, and $\Delta_{nH} = 0.782$ MeV is the difference between the masses of a neutron and a hydrogen atom, it is easy to find the binding energy $E_b(Z;A)$ for neutron-deficient nuclei with $A - 2Z = -1, -2, \ldots$.

The dependence of E_b on the physical values of specific charge ZA^{-1}, which was calculated with the use of the known binding energies for nuclei ${}^A_Z Q$ with $A-2Z = 0 \div 8$ [15] and the values of $E_b(Z;A)$ determined with the help of formula (3) for the known mass excesses ΔM at $A - 2Z = -1 \div -8$ [16, 17], is depicted in the Figure. The Figure demonstrates that isotopic nuclei with neutron (proton) excess–i.e. if $|A - 2Z| > 0$ for them–are located along straight lines, i.e. their binding energy linearly depends on ZA^{-1}. The variation of straight line slope at the transition to heavier isotopes evidences for a growing role of the Coulomb component in the nuclear-Coulomb potential. The preservation of the linear character of the $E_b(ZA^{-1})$-dependence at $Z =$ const indicates that the change of even *p–p* interactions at the increase of the neutron number does not result in a substantial variation of the Coulomb component of the nuclear-Coulomb potential. In this case, for nuclei with $A - 2Z =$ const, the transition from even isotopes to odd ones corresponds to a lower increment of the binding energy than it is at the transition from odd to even ones, which testifies to a presence of an even energy in the Bethe–Weizsäcker formula [15, p. 198], with the sign of this energy depending on the nucleon number parity.

The dependence of the parameter E_b for nuclei with $A-Z =$ const on ZA^{-1} demonstrates that the increment of binding energy in isotonic neutron-deficient ($A-2Z < 0$) nuclei decreases as the specific charge grows. The

Z	A	E_b, MeV	Z	A	E_b, MeV
6	18	116.2	18	44	374.9
8	22	162.9	32	64	548.1
8	24	174.4	34	68	580.8
10	26	205.9	34	70	601.5
10	28	220.0	36	72	612.2
12	18	99.6	36	74	633.3
12	30	246.6	38	76	644.3
12	32	261.6	38	78	665.5
14	34	287.2	38	80	686.6
14	36	303.0	40	80	675.4
16	24	130.5	40	82	697.2
16	40	33.4	40	84	719.0
18	28	165.9			

deviations from linearity in the range $ZA^{-1} > 0.5$ are more pronounced for heavier nuclei, which testifies to a growing role of the Coulomb component of the nuclear binding energy in the well-known Bethe–Weizsäcker formula. From the character of the $E_b(ZA^{-1})$-dependence, it also follows that the behavior of isotopic and isotonic lines, when changing over to lighter nuclei, reflects the fact that the binding energy becomes a symmetric function of the isotopic number $Y = A - 2Z$, i.e. the proton (neutron) excess, as the Coulomb repulsion gets weaker. This symmetry is also confirmed by the presence of the isotopic (isobaric) binding energy term in the Bethe–Weizsäcker formula.

Crosses on the lines that display the dependence of E_b on ZA^{-1} for nuclei with $Z =$ const correspond to even-even nuclei ($J^\pi = 0^+$), the binding energies of which were found on the basis of the approximate relation

$$2E_b(Z;A) \approx E_b(Z;A-2) + E_b(Z;A+2)$$

and using the known values of binding energy for the nearest nuclei-isotopes with the same parity. The corresponding calculated values of E_b are quoted in the Table. Possible deviations (of about 1 MeV) of the binding energies calculated for those nuclei are stimulated by neglecting the difference between the neutron and proton masses in the expression for the reduced mass of the A-nucleon system [14]. One can see that the binding energy difference amounts to 54.1 MeV for mirror nuclei ^{28}Ne and ^{28}Ar with $|A - 2Z| = 8$. For heavier mirror even-even nuclei with $Y = A - 2Z = \pm 8$ (see the Figure)–for instance, ^{68}Zn and ^{68}Sr–

$$\Delta E_b = E_b(30;58) - E_b(38;68) = 86.8 \text{ MeV},$$

which confirms the character of the E_b deviation from the isotopic symmetry at the transition to heavier isobars with $J^\pi =$ const due to the growing role of the Coulomb component of E_b in the Bethe–Weizsäcker formula.

To summarize, it should be noted that, for the A-nucleon problem, the fundamental parameter of the Schrödinger equation–the Coulomb coupling constant $a = \mu Zz$ – characterizes the magnitude of the specific charge of the compound system. It is an argument that reflects the linear character of the nuclear binding energy dependence in a good approximation with regard for the classification of the nuclei as isotopes, isotones, isobars, and nuclei with fixed neutron excess in the range of high isotopic numbers. We also note that the linear behavior of the real part of the S-matrix pole trajectory in the complex plane of the parameter $a(E) = Z(E)A^{-1}(E)$ [14] does not depend on the specific shape of the interaction potential. Hence, the main role in the separation of the compound system ${}^A_ZQ$ is played by properties of the parent nucleus, rather than by those of free nucleons. Therefore, in the range of magic values for Z or $A - Z$, the modification in the behavior of the short-range potential gives rise to a deviation from the linear dependence mentioned above.

1. I.V. Poplavskii, Fiz. Elem. Chast. At. Yadra **27**, 1099 (1996).
2. I.V. Poplavskii and M.N. Popushoy, Izv. Akad. Nauk SSSR Ser. Fiz. **50**, 47 (1986); **50**, 52 (1986).
3. M.N. Popushoy, Fiz. Elem. Chast. At. Yadra **34**, 1485 (2003).
4. M.N. Popushoy, Yad. Fiz. **54**, 1262 (1991).
5. M.N. Popushoy, Teor. Mat. Fiz. **69**, 466 (1986).
6. M.N. Popushoy, Izv. Ross. Akad. Nauk Ser. Fiz. **57**, 166 (1993); **61**, 775 (1997).
7. M.N. Popushoy, Yad. Fiz. **55**, 645 (1992).
8. I.V. Poplavskii and M.N. Popushoy, Ukr. Fiz. Zh. **38**, 1289 (1993).
9. M.N. Popushoy, Yad. Fiz. **57**, 398 (1994).
10. I.V. Poplavskii and M.N. Popushoy, Yad. Fiz. **55**, 2915 (1992).
11. I.V. Poplavskii and M.N. Popushoy, Ukr. Fiz. Zh. **38**, 815 (1993).
12. M.N. Popushoy, Yad. Fiz. **58**, 632 (1995).
13. I.V. Poplavskii and M.N. Popushoy, Izv. Ross. Akad. Nauk Ser. Fiz. **59**, 150 (1995).
14. M.N. Popushoy, Ukr. Fiz. Zh. **53**, 5 (2008).
15. V.A. Kravtsov, *Atomic Masses and Nuclear Binding Energies* (Atomizdat, Moscow, 1974) (in Russian).
16. I.V. Poplavskii and M.N. Popushoy, Yad. Fiz. **59**, 627 (1996).
17. I.V. Poplavskii, Izv. Ross. Akad. Nauk Ser. Fiz. **62**, 1004 (1998).

Received 17.08.08.
Translated from Ukrainian by O.I. Voitenko

ЗАЛЕЖНІСТЬ ЕНЕРГІЇ ЗВ'ЯЗКУ ЯДЕР ВІД ПИТОМОГО ЗАРЯДУ В ОБЛАСТІ МАЛИХ ЗНАЧЕНЬ ІЗОТОПІЧНОГО ЧИСЛА

М.М. Попушой

Р е з ю м е

На основі S-матричного формалізму встановлено наближені лінійні залежності енергії зв'язку ізотопних (ізотонних) ядер ${}^A_ZQ$ з $J^\pi = \mathrm{const}$ в інтервалі значень ізотопічного числа $Y = A - 2Z = 0, \pm 1, \pm 2, ...$ від питомого заряду ZA^{-1}.