A MODEL OF GENERATION OF PHOTON-NUMBER ENTANGLED STATES WITH STIMULATED PARAMETRIC DOWN CONVERSION

O.O. GURIN, V.C. USENKO[1], C.V. USENKO

Taras Shevchenko Kyiv National University
(Bld. 1, 2, Academician Glushkov Ave., Kyiv 03680, Ukraine),

[1]**Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine**
(14-b, Metrolohichna Str., Kyiv 03680, Ukraine)

UDC 530.145

We consider the process of generation of photon-number entangled states of light in the stimulated nonlinear parametric down conversion process and build a simple model describing the generation not involving the traditional parametric approximation. The equations of motion for the system of a pumping and a two-mode outgoing field are solved for the case of a strong correlation between two modes, and the evolution of the state parameters of the generated modes is obtained. The solution is briefly analyzed for particular types of photon-number entangled states.

1. Introduction

Nonclassical features of the states of light are of utmost interest within the past decades, as they became the basis for the quantum information theory and its applications, in particular, quantum computation, teleportation, and cryptography [1]. All of these directions are based on the fundamental principles of quantum mechanics and involve mainly the essential resource of quantum entanglement. The recent developments in the field are based on the continuous variables information coding [2] into the parameters of multilevel quantum systems as distinct from the discrete variables coding with two-level system states. In the optical implementations, it corresponds mainly to the consideration of multiphoton quantum states as the information carriers. At that, the special attention is given to entangled multiphoton states, in particular, to photon-number entangled states of light (PNES), which provide with the strong correlation between the intensity measurements of the two spatial modes. Several communication protocols were proposed and partially implemented on the basis of the photon-number entangled states with the use of the intensity difference [3, 4] or intensity fluctuations coding [5]. Such states can be generally generated in the nonlinear process of parametric down conversion (PDC) [6–9] being the well-known typical example of the quantum-mechanical event when the pumping photons excite the atoms of the nonlinear media, which emit photon pairs upon relaxation. While such pairs were observed independently in the first experiments, the generated multiphoton beams started to manifest the nonclassical features with increase in the process gain in optical parametric amplifiers, in particular, the strong correlation between modes (showed in the sub-shotnoise signal-idler intensity difference fluctuations), and became referred to as the twin beams [10–12]. At that, the modes marginal statistical distributions were thermal. The recent advances in the experimental technics (in particular, the use of powerful short laser pulses for the pumping and effective multiport photon-counters and CCD cameras), as well as the usage of cavity-based parametric oscillators, enabled several groups to obtain strongly correlated states of light which possess the Poisson statistics [13, 14]. Meanwhile, the traditional approach to the description of the PDC process is based on the parametric approximation, when the pumping mode is considered as classical [7]. In the present paper, we develop a simple model of PNES generation without the parametric approximation, thus keeping a nonlinearity in the operator equations. We make the statistical assumptions relevant to the considered states and obtain the solutions describing the evolution of state parameters. At that, we assume that the pumping is so intense that the atoms of the nonlinear media are excited coherently, and the PDC process is stimulated. This work is a step toward the exact model describing the PDC-based generation of PNES states.

2. Photon-Number Entangled States

PNES can be written in the Fock number basis as

$$|\Psi\rangle\rangle = \mathcal{N}^{1/2} \sum_n c_n \,|\mathrm{n},\mathrm{n}\rangle\rangle, \tag{1}$$

where $|\mathrm{n,n}\rangle\rangle = |n\rangle_1 \otimes |n\rangle_2$, indices $1, 2$ standing for the respective modes, and $\mathcal{N}$ is a normalization factor.

Their features are evident thereby: the amplitude mean values for each of the modes are zero, while the amplitudes are strongly correlated, i.e. in terms of the modes quantum operators $\hat{a}_1 = \hat{a}_2 = 0$ and $\hat{a}_1\hat{a}_2 \equiv \lambda \neq 0$. These statistical properties of the states are used within the construction of the PNES generation model.

3. Generation

Let us describe the possible generation of PNES states by means of a PDC process. As was already mentioned, we assume that the pumping pulses are intense enough to coherently excite the whole electron subsystem of the nonlinear media and so the pumping energy directly turns to the energy of a generated two-mode electromagnetic field excitation. The system can be described by the Hamiltonian

$$\hat{H} = \hbar\omega_1\hat{a}_1^+\hat{a}_1 + \hbar\omega_2\hat{a}_2^+\hat{a}_2 + \hbar\omega_0\hat{a}_0^+\hat{a}_0 + \\ + i\hbar\chi\left(\hat{a}_1^+\hat{a}_2^+\hat{a}_0 - \hat{a}_1\hat{a}_2\hat{a}_0^+\right) \tag{2}$$

which includes the interaction term, but doesn't include quantum operators standing for the electron subsystem excitation and relaxation, and covers only the energy swap between the pumping and emitted fields. We assume that the excitation and the relaxation take place immediately, i.e. the model is adiabatic.

The quantum operators for each of the two modes of the outgoing field satisfy the Heisenberg equations

$$i\hbar\frac{d}{dt}\hat{a}_{1,2} = \left[\hat{a}_{1,2}\hat{H}\right] = \hbar\omega_{1,2}\hat{a}_{1,2} + i\hbar\chi\hat{a}_{2,1}^+\hat{a}_0, \tag{3}$$

and the similar equations hold for the Hermitian conjugate quantities. The equations lead to ones for the operators $\hat{n}_{1,2} = \hat{a}_{1,2}^+\hat{a}_{1,2}$ which correspond to the photon numbers in each of the two generated modes:

$$i\hbar\frac{d}{dt}\hat{n}_1 = i\hbar\frac{d}{dt}\hat{n}_2 = i\hbar\chi\hat{a}_2\hat{a}_1\hat{a}_0^+ + i\hbar\chi\hat{a}_1^+\hat{a}_2^+\hat{a}_0. \tag{4}$$

In this case, the difference between photon numbers in the two modes vanishes,

$$\frac{d}{dt}\left(\hat{n}_1 - \hat{n}_2\right) = 0, \tag{5}$$

which means that

$$\hat{n}_1 - \hat{n}_2 = 0, \tag{6}$$

if both modes are not excited initially.

Introducing the operators $\hat{A} = \hat{a}_1\hat{a}_2$, $\hat{A}^+ = \hat{a}_1^+\hat{a}_2^+$ and $\hat{N} = \hat{n}_2 + \hat{n}_1$, we obtain the system of equations

$$\frac{d}{dt}\hat{A} = -i\omega\hat{A} + \chi\left(\hat{N}+1\right)\hat{a}_0, \tag{7}$$

$$\frac{d}{dt}\hat{A}^+ = i\omega\hat{A}^+ + \chi\left(\hat{N}+1\right)\hat{a}_0^+, \tag{8}$$

$$\frac{d}{dt}\hat{N} = 2\chi\left(\hat{A}\hat{a}_0^+ + \hat{A}^+\hat{a}_0\right). \tag{9}$$

We suppose that the pumping is coherent, and the generated state is PNES, i.e.

$\langle\hat{A}\rangle = \langle\langle\Psi|\,\hat{A}\,|\Psi\rangle\rangle = \lambda\,(t)\,;$

$\langle\hat{a}_0\rangle = \langle\alpha\,|\hat{a}_0|\,\alpha\rangle = \alpha\,(t)\,;$

$\langle\hat{A}^+\rangle = \langle\langle\Psi|\,\hat{A}^+\,|\Psi\rangle\rangle = \overline{\lambda}\,(t)\,;$

$\langle\hat{a}_0^+\rangle = \left\langle\alpha\,\left|\hat{a}_0^+\right|\,\alpha\right\rangle = \overline{\alpha}\,(t)\,;$

and that $(\hat{N} = N\,(\lambda\,(t)) \equiv N\,(t)$.

Since we describe the generated state as photon-number entangled, but not as a pair of coherent states, we may assume its statistical independence from the pumping state, rather than the independence between the two modes. The overall system state can be presented as $|\alpha\rangle\otimes|\Psi\rangle\rangle$, while $|\Psi\rangle\rangle \neq |\alpha_1\rangle\otimes|\alpha_2\rangle$, which is the manifestation of modes' entanglement, i.e. $\hat{a}_1\hat{a}_2\hat{a}_0^+ = \hat{a}_1\hat{a}_2\hat{a}_0^+$, $\hat{A}\hat{a}_0^+ = \lambda\,(t)\,\overline{\alpha}\,(t)$ and $\hat{N}\hat{a}_0 = N\,(t)\,\alpha\,(t)$. Thus, the system of equations for the operators turns to the same for the state parameters:

$$\frac{d}{dt}\lambda\,(t) = -i\omega\lambda\,(t) + \chi\,(N\,(t)+1)\,\alpha\,(t)\,, \tag{10}$$

$$\frac{d}{dt}\overline{\lambda}\,(t) = i\omega\overline{\lambda}\,(t) + \chi\,(N\,(t)+1)\,\overline{\alpha}\,(t)\,, \tag{11}$$

$$\frac{d}{dt}N\,(t) = 2\chi\left(\lambda\,(t)\,\overline{\alpha}\,(t) + \overline{\lambda}\,(t)\,\alpha\,(t)\right). \tag{12}$$

Let us separate the slow amplitudes: $\lambda\,(t) = e^{-i\omega t}\Lambda\,(t)$; $\alpha\,(t) = e^{-i\omega t}a\,(t)$. By supposing that a is real, i.e. Λ is real, we obtain the system

$$\frac{d}{dt}\Lambda\,(t) = \chi\,(N\,(t)+1)\,a\,(t)\,, \tag{13}$$

$$\frac{d}{dt}N\,(t) = 4\chi\Lambda\,(t)\,a\,(t)\,. \tag{14}$$

The solutions of the equations of motion (13) and (14) have the form

$$\Lambda(t) = \sinh\tau(t)\cosh\tau(t);\;\; N = 2\sinh^2\tau(t), \tag{15}$$

where

$$\tau(t) = \chi \int\limits_{-\infty}^{t} a(t')dt', \tag{16}$$

assuming that the initial conditions are $t \to -\infty : \tau \to 0, N \to 0, \Lambda \to 0$. The solutions describe the connection between the time profiles of the pumping and generated modes. The effect of the pumping profile is specified by the integral characteristic (16). In particular, in the case of a pulse pumping, we have $a(t) = \{t < 0 : 0;\ t > T : 0;\ 0 < t < T : a\}$: $\tau = \{t < 0 : 0;\ t > T : \chi aT;\ 0 < t < T : \chi at\}$. Result (15) is in good agreement with the available description of the PDC process [7].

4. Particular Cases

In order to establish a link between the model and the physical reality, we briefly analyze it for the two relevant particular types of PNES states, namely the twin beam (TWB) and two-mode coherently correlated (TMC) states.

The twin beam states which are also referred to as squeezed vacuum states can be presented in the form (1) as

$$|x\rangle\rangle = \sqrt{(1-x^2)} \sum_n x^n \, |n, n\rangle\rangle, \tag{17}$$

where, generally, $x \in \mathbb{C}$ and $0 \le |x| \le 1$, but we assume x as real without loss of generality. TWB states are Gaussian and are widely used for CV teleportation and dense coding. The marginal statistical distributions of the TWB modes are equal to thermal states.

The TMC states being the degenerate pair-coherent states [15] can be given as

$$|\lambda\rangle\rangle = \frac{1}{\sqrt{I_0(2|\lambda|)}} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \, |n, n\rangle\rangle, \tag{18}$$

where $I_0(x)$ is the modified Bessel function. These non-Gaussian states are eigenstates for the product of the modes' annihilation operators $\hat{a_1}\hat{a_2}\,|\lambda\rangle\rangle = \lambda\,|\lambda\rangle\rangle$ and possess the sub-Poisson photon-number statistics for each of the modes.

In order to estimate which of the states is closely described by the model, while the mode amplitudes have zero mean values, we introduce the observables which we call pair-quadratures similarly to the mode quadratures and construct them of the operators $\hat{A} = \hat{a}_1\hat{a}_2$, $\hat{A}^+ = \hat{a}_1^+\hat{a}_2^+$ as $\hat{C_+} = \hat{A} + \hat{A}^+$ and $\hat{C_-} = \frac{\hat{A}-\hat{A}^+}{i}$. We estimate the evolution of a pair-quadrature dispersion $D_C = \hat{C}^2 - \hat{C}^2$, where $C \equiv C_\pm$ within the model, by directly differentiating the dispersion obtained with averaging by the time-dependent solutions,

$$\left.\frac{dD_C}{dt}\right|_{\text{model}} =$$

$$= \frac{d}{dt}\Big(\langle\langle\Psi(\mathrm{t})|\, \hat{C}^2 \,|\Psi(\mathrm{t})\rangle\rangle - \langle\langle\Psi(\mathrm{t})|\, \hat{C} \,|\Psi(\mathrm{t})\rangle\rangle^2\Big), \tag{19}$$

and assuming that the state evolution is described by the time dependence of state parameters (15). We compare this with the exact evolution of the pair-quadrature dispersion obtained directly by averaging the pair-quadrature derivatives by the particular states either TWB (17) or TMC (18):

$$\left.\frac{dD_C}{dt}\right|_{\text{exact}} = \frac{d\hat{C}}{dt}\hat{C} + \hat{C}\frac{d\hat{C}}{dt} - 2\hat{C}\frac{d\hat{C}}{dt}. \tag{20}$$

Here, $\frac{d\hat{C}}{dt}$ is obtained from the Heisenberg equation $\frac{d\hat{C}}{dt} = \frac{1}{i\hbar}[\hat{C}, \hat{H}]$ for the system Hamiltonian (2).

For the $\hat{C_+}$ pair-quadrature, the resulting derivative within the "exact" approach is

$$\left.\frac{dD_{C+}}{dt}\right|_{\text{exact}} = \frac{2\chi}{\hbar}\hat{Q}\hat{C_+}, \tag{21}$$

where $\hat{Q} = \hat{a_0} + \hat{a_0}^+$ is the pumping quadrature. Calculating this derivative for the particular type of states (TWB or TMC) and comparing it with the expression obtained by the calculation through averaging by time-dependent solutions within the model, we get for TWB:

$$\left.\frac{dD_{C+}}{dt}\right|_{\text{exact}}^{\text{TWB}} = \left.\frac{dD_{C+}}{dt}\right|_{\text{model}}^{\text{TWB}} = 8\chi\alpha\frac{x(x^2+1)}{(1-x^2)^2}. \tag{22}$$

While, for TMC, we have

$$\left.\frac{dD_{C+}}{dt}\right|_{\text{exact}}^{\text{TMC}} = 8\chi\lambda\alpha \ne \left.\frac{dD_{C+}}{dt}\right|_{\text{model}}^{\text{TMC}} = 4\chi\lambda\alpha. \tag{23}$$

Thus, the developed model describes more likely the generation of thermally distributed TWB states than that of sub-Poisson TMC states, since the pair-quadrature uncertainty grows faster if the state is initially modeled as TMC; whereas this evolution for TWB is exactly defined by the generation model.

5. Conclusions

In this work, we built a simple model describing the generation of photon-number entangled states in the stimulated parametric down conversion process. At that, we didn't make the traditional parametric approximation and considered a pumping mode as quantum. The special statistical properties of the states enabled obtaining the solution for the equations of motion describing the process of generation in terms of the evolution of state parameters. The solution represents the relation between time-profiles of the pumping and generated modes. The model is briefly analyzed with respect to the particular types of photon-number entangled states and describes more likely the generation of a twin-beam state.

1. N. Gisin, G. Ribordy, W. Tittel, H. Zbinden, Rev. Mod. Phys. **74**, 145 (2002).
2. S.L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2004).
3. A.C. Funk and M.G. Raymer, Phys. Rev. A **65**, 042307 (2002).
4. K. Hayasaka, Y. Zhang, and K. Kasai, Opt. Lett. **29**, 1665 (2004).
5. V.C. Usenko and B.I. Lev, Phys. Lett. A **348**, 17 (2005); V. Usenko and C. Usenko, J. Russ. Laser Res. **25**, 361 (2004).
6. D.N. Klyshko, JETP Letters **6**, 490 (1967).
7. D.F. Walls and G.J. Milburn, *Quantum Optics* (Springer, Heidelberg, 1995), Chap. 5.
8. L. Mandel and E. Wolf, *Optical Coherence and Quantum optics* (Cambridge University Press, Cambridge, 1995), Chap. 22.4.
9. J. Peřina, Z. Hradil, and B. Jurčo, *Quantum Optics and Fundamentals of Physics* (Kluwer, Dordrecht, 1994), chap. 8.
10. O. Aytur and P. Kumar, Phys. Rev. Lett. **65**, 1551 (1990).
11. A. Heidmann *et al.*, Phys. Rev. Lett. **59**, 2555 (1987).
12. M. Vasilyev, S.-K. Choi, P. Kumar, and G.M. D'Ariano, Phys. Rev. Lett. **84**, 2354 (2000).
13. J. Laurat, T. Coudreau, N. Treps, A. Maitre, and C. Fabre, Phys. Rev. Lett. **91**, 213601 (2003).
14. O. Haderka, J. Perina jr., M. Hamar, and J. Perina, Phys. Rev. A **71**, 033815 (2005); J. Perina jr., O. Haderka, and M. Hamar, arXiv:quant-ph/0310065.
15. G.S. Agarwal, Phys. Rev. Lett. **57**, 827 (1986); G.S. Agarwal and A. Biswas, J. Opt. B **7**, 350 (2005).

Received 15.07.08

МОДЕЛЬ ГЕНЕРАЦІЇ СТАНІВ,
ПЕРЕПЛУТАНИХ ЗА ЧИСЛАМИ ФОТОНІВ, ШЛЯХОМ
СТИМУЛЬОВАНОГО ПАРАМЕТРИЧНОГО РОЗСІЯННЯ

О.О. Гурін, В.К. Усенко, К.В. Усенко

Р е з ю м е

Розглянуто процес генерації станів світла, переплутаних за числами фотонів, за допомогою стимульованого параметричного розсіяння та побудовано просту модель, яка описує таку генерацію та не включає в себе традиційне параметричне наближення. Рівняння руху системи, що складається з накачки та двомодового вихідного поля, розв'язані для випадку сильної кореляції між модами, отримано еволюцію параметрів стану генерованих мод. Розв'язок коротко проаналізований для конкретних типів станів, переплутаних за числами фотонів.