DYNAMICS OF A ONE-DIMENSIONAL ELECTRON BUNCH WITH INITIALLY RECTANGULAR DENSITY PROFILE INJECTED INTO HOMOGENEOUS PLASMA

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Dynamics of an electron bunch with initially rectangular density profile injected into plasma is studied in a 1D model using the PIC method. The dependences of the maximal beam density and the maximal amplitude of a wake wave field on the model parameters (plasma density and temperature, beam density, velocity, and duration) are obtained and interpreted. The deformation index as a qualitative characteristic of a deformation of the bunch initial density profile is proposed, and its dependence on the model parameters is studied.

1. Introduction

Dynamics of electron bunches in plasma is a topic of increasing importance due to its broad range of applications. Acceleration of electrons by wake fields excited in plasma by relativistic electron bunches [1] takes the prominent place among them.

In [2,3], it was proposed to use the transition radiation of an electron bunch for the diagnostics of a non-uniform plasma. Excitation of wake waves in homogeneous and inhomogeneous plasmas by an electron bunch was studied in [4] in order to estimate the electron bunch energy losses in such a plasma during its motion. But the calculations in that work were carried out in the given current approximation, so no perturbation of the electron bunch density during its motion was considered. However, it is obvious that, due to the wake wave field, the bunch density can vary substantially. This results in a change of the transition radiation spectrum excited by the bunch. Thus, to realize the proposed diagnostics of an inhomogeneous plasma, it is necessary to find out the shape of an electron bunch that will ensure its minimal deformation during the motion in plasma. The first step to solve this problem is the study of the dynamics of an electron bunch with initially rectangular profile.

Dynamics of relativistic electronic bunches with initially rectangular profile moving in plasma was studied in [5] by means of the one-dimensional computer simulation. But no detailed description and explanation of the observed effects was proposed. The dependence of the dynamics of electron bunches on the system parameters was not also studied.

In the present article, we study the dependence of the dynamics of a non-relativistic electron bunch with initially rectangular density profile during its motion through plasma on the model parameters by using a computer simulation within the PIC method.

2. Model Description, Simulation Method, and Parameters

Let us consider a homogeneous isotropic plasma with hot electrons ($T_e \gg T_i$), with the initial charge densities of electrons and ions being equal. The electron bunch ($n_b \ll n_p$) with initially rectangular density profile is injected into the plasma along the x-axis from the conductive wall ($x = 0$) and later absorbed by the wall ($x = L$) on the opposite side of the plasma. The evolution of the bunch was caused by a self-consistent wake wave field excited by this bunch.

Modified package PDP1 [6,7] was used in the simulation. Table 1 presents the range of key parameters. In this package, all the charged particles (plasma electrons and ions, as well as beam electrons) are modeled by a set of charge planes perpendicular to the x-axis moving under the action of a self-consistent electrostatic field. Plasma particles were absorbed by the walls.

Now let us discuss the dependences of the maximal bunch density and maximal wake field on parameters of the analyzed model.

3. Dynamics of an Electron Bunch in a Wake Wave Field

Simulation results of the wake wave excitation are presented in Fig. 1. One can see that there is no field in front of the bunch. A wake wave is excited by its forward front. The field behind the front decelerates bunch electrons. At the distance $\lambda/2$ ($\lambda = 2\pi v_b/\omega_p$, where v_b is the initial speed of a bunch, and ω_p is the electron plasma frequency) from the front, the field sign changes, and electrons are accelerated there. As a result, the longitudinal focusing of electrons in a microbunch takes place. After the formation of a sharp density maximum, the overtaking of electrons occurs, which corresponds to the wave-front braking in the phase space. This results in the bifurcation of the initial density maximum. Far from the forward front, the bunch density distribution looks to be quasiperiodic with spatial period λ (Fig. 1,*b*).

The longitudinal focusing of electrons in microbunches leads to the bunch density decrease on the forward front (Fig. 1,*b*). As a result, the wake wave amplitude decreases far from an injector. In this region, the wake wave is actually excited by the sharp maximum of the first microbunch (Fig. 1,*a*). Consequently, the sharp maxima of the wake field are observed simultaneously with the microbunches focusing. One can see that the wake field excited by the next microbunch is cophased with the field excited by its predecessor (Fig. 1,*a*).

On the bunch forward front, where the wake field vanishes, the velocity of electrons remains almost constant during their motion in the plasma. At the same time, electrons on the bunch back front are substantially decelerated by the wake field (Fig. 1,*b*).

Table 1

Parameter	Value
Length of the system, L	200 cm
Number of cells in the system	1000
Number of particles in one large particle	$10^6 - 10^7$
Plasma density, n_p	$10^6 - 10^9$ (cm^{-3})
Plasma electrons temperature, T_e	2×10^5 K
Plasma ions temperature, T_i	4×10^4 K
Initial bunch velocity, v_b	$1 \times 10^9 - 4 \times 10^9$ cm/s
Bunch duration, t_b	$10^{-7} - 10^{-8}$ s
Initial bunch density, n_b	$10^4 - 10^6$ (cm^{-3})

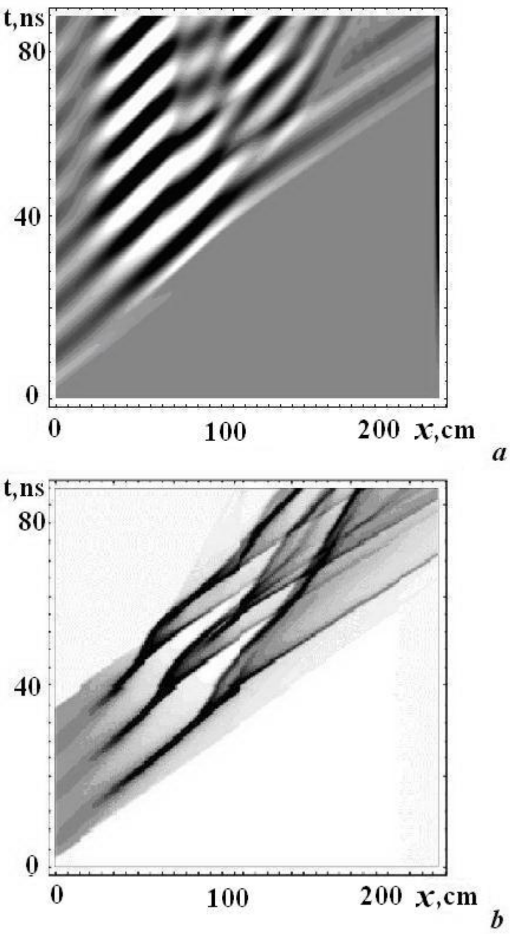

Fig. 1. Space-time distributions of wake fields (*a*) and bunch density (*b*): $v_b = 3 \times 10^9$ cm/s, $t_b = 3\,T_{\text{Langm}} = 3.3 \times 10^{-8}$ s, $n_b = 1.1 \times 10^6$ cm^{-3}, $n_p = 10^8$ cm^{-3} (dark areas correspond to higher density)

The distribution of wake fields changes essentially after the particles overtaking. The number of microbunches grows due to their bifurcation, so the distance between bunches does not correspond to the length of wake waves any more. As a result, the field of this wave noticeably decreases (Fig. 1,*a*).

4. Dependence of the Wake Wave Maximal Amplitude on Model Parameters

We now discuss the dependences of the maximal bunch density and the maximal wake field on parameters of the analyzed model.

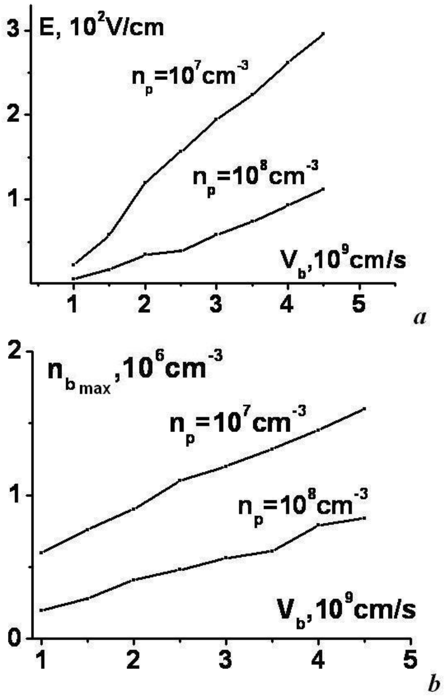

Fig. 2. Dependences of the maximal wake field (a) and the maximal bunch density (b) on the initial bunch velocity: $n_b = 1 \times 10^5$ cm^{-3}

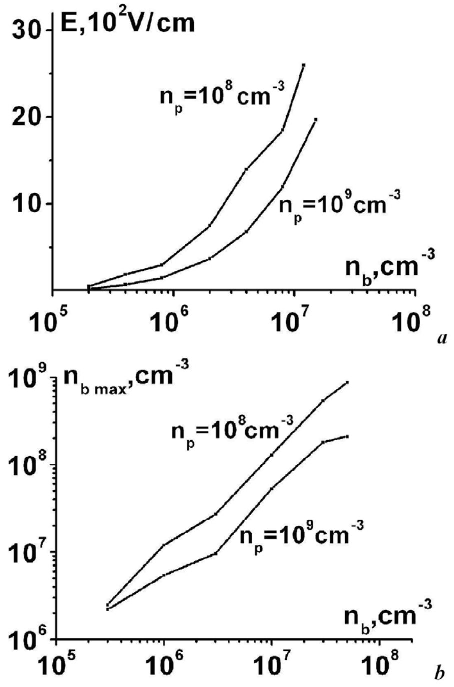

Fig. 3. Dependences of the maximal wake field (a) and the maximal bunch density (b) on the initial bunch density: $v_b = 3 \times 10^9$ cm/s, $t_b = 1 \times 10^{-8}$ s

4.1. Dependence on the bunch velocity and density

Figure 2 shows the dependences of the maximal wake field (a) and the maximal electron bunch density (b) on the initial bunch velocity for two different densities of a background plasma. The bunch length is three times more than the wake wave length. The maximal wake field amplitude grows almost directly proportionally to the bunch velocity. A maximum of the wake field is excited at the time moments of microbunch focusing, as has already been noticed in Section 3.

The length of excited wake waves grows with a bunch velocity, $\lambda = 2\pi v/\omega_p$, where ω_p is the Langmuir frequency of the background plasma. As a result, the number of electrons in a microbunch increases. Thus, with increase of the bunch velocity, the maximal electron density in a microbunch increases, and the amplitude of an electric field excited by this microbunch increases as well. The point, where the field reaches its maximum, moves gradually from an injector with increase of the bunch velocity.

Figure 3,*a,b* presents the maximal wake field and the maximal electron density in a bunch depending on the initial density of this bunch. The excited field grows with the bunch density; moreover, the dependence is close to a linear one. This result is expected, because the growth of the bunch density results in the increase of the maximal density of microbunches and the corresponding wake field.

4.2. Dependence on the bunch duration

The dependences of the maximal wake field and the maximal bunch density on the duration of a bunch are plotted in Fig. 4.

While the length of a bunch is less or of the order of the length of wake waves, the increase of a bunch length leads to the growth of its maximal density (Fig. 4,*b*). At the further growth of the bunch length, the number of microbunches grows. The sequence of microbunches coherently excites the wake wave, so its amplitude grows with increase of the number of microbunches (Fig. 7,*b,c*).

4.3. Dependence on the background plasma density

Figure 5 presents the dependences of the maximal excited electric field (a) and the electron density in a bunch (b) on the background plasma density. As the

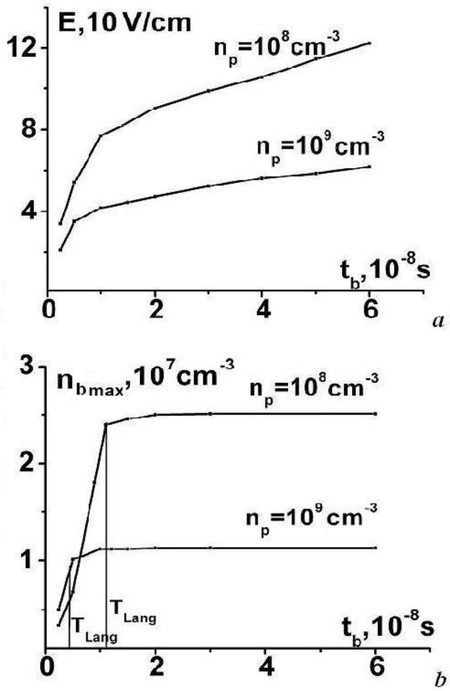

Fig. 4. Dependences of the maximal wake field (a) and the maximal bunch density (b) on the initial bunch duration: $v_b = 3\times10^9$ cm/s, $n_b = 2 \times 10^6$ cm^{-3}

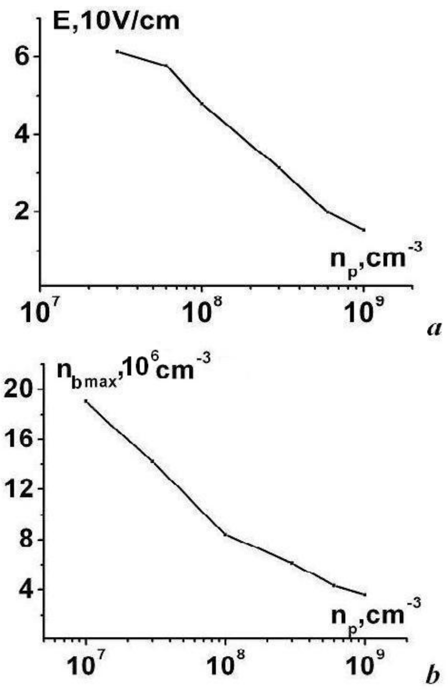

Fig. 5. Dependences of the maximal wake field (a) and the maximal bunch density (b) on the background plasma density: $v_b = 2 \times 10^9$ cm/s, $t_b = 3 \times 10^{-8}$ s, $n_b = 1 \times 10^6$ cm^{-3}

background plasma density increases, the wake wave amplitude decreases. Probably, the explanation of the dependences obtained follows from the fact that the plasma frequency is proportional to $n_p^{1/2}$. Thus, an increase of the background plasma density results in a decrease of both the period of wake waves and the length of microbunches. This effect leads to a reduction of the density maxima of microbunches.

5. Deformation Index and Its Dependence on Model Parameters

It is convenient to introduce some integral parameter for the description of the bunch deformation during the motion in plasma. Let us define the deformation index in the form

$$\sigma = \frac{\int\limits_{-\infty}^{+\infty} [n_{0b}(x) - n_b(x)]^2\, dx}{\int\limits_{-\infty}^{+\infty} n_{0b}(x)^2\, dx}. \qquad (1)$$

Here $n_{0b}(x)$ and $n_b(x)$ is the bunch density profile at the initial time moment and at the given time moment, respectively.

The deformation index is a dimensionless parameter similar to the square of a relative standard deviation. If the density distribution of an electron bunch remains constant, then the deformation index is equal to zero. With increase of the bunch density perturbation, this parameter grows.

5.1. Deformation index behavior along the bunch trajectory

The spatial dependence of the deformation index for the same bunch parameters as in Fig. 1 is presented in Fig. 6.

The deformation index is calculated for the bunch at the time moment, when its forward front is situated at a distance x from an injector. One can see that the deformation index increases monotonously with this distance not far from an injector. This part of the graph corresponds to a gradual focusing of microbunches (Fig. 1,b). The maximum of the deformation index is reached at the time moment of the maximal focusing of the first microbunch (distributions of the bunch density and the field at this time moment are shown in Fig. 7,a).

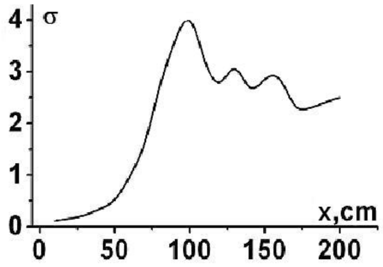

Fig. 6. Spatial dependence of the deformation index: $n_b = 2 \times 10^5$ cm^{-3}, $v_b = 2\times10^9$ cm/s, and $t_b = 3.3\times10^{-8}$ s, $n_p = 10^8$ cm^{-3}

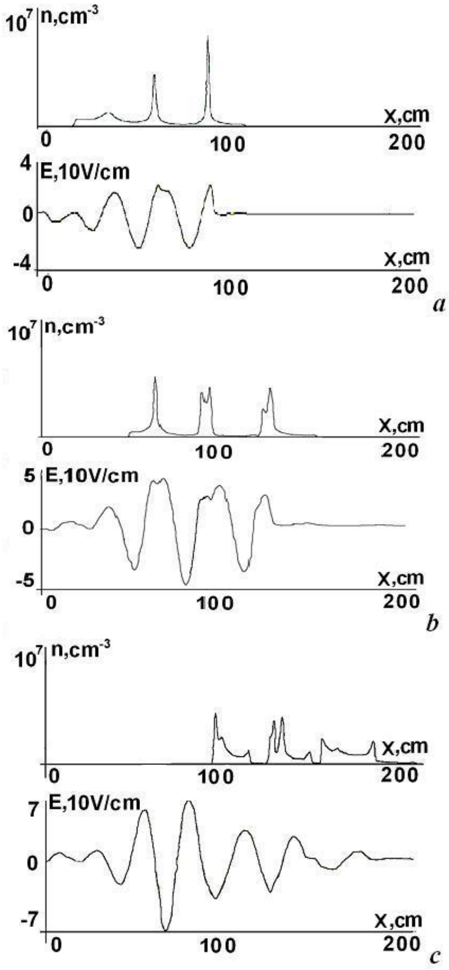

Fig. 7. Spatial distributions of the bunch density and the electric field at the time moments $t = 3.5 \times 10^{-8}$ s (a), $t = 5.2 \times 10^{-8}$ s (b), $t = 6.9 \times 10^{-8}$ s (c): $v_b = 3 \times 10^9$ cm/s, $t_b = 3T_{\mathrm{Langm}} = 3.3 \times 10^{-8}$ s, $n_b = 1.1 \times 10^6$ cm^{-3}, $n_p = 10^8$ cm^{-3}

Further, the first bunch starts to break up, because of the electrons overtaking, and the deformation index decreases. The subsequent local maxima are reached at the time moments of the maximal focusing of the second and third microbunches and the formation of the secondary density maxima, respectively (Fig. 1,b and Fig. 7,c).

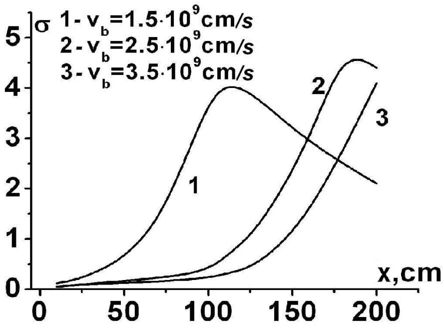

Fig. 8. Spatial dependence of the deformation index for different bunch velocities: $t_b = 1.1 \times 10^{-8}$ s, $n_b = 2 \times 10^5$ cm^{-3}, $n_p = 1 \times 10^8$ cm^{-3}

5.2. Dependence on the parameters of a bunch and the background plasma

Figure 8 presents the spatial dependences of the deformation index for different bunch velocities. One can see that the growth rate of this parameter at small distances from an injector decreases with increase of the initial bunch velocity. Indeed, with increase of the bunch velocity, the length of wake waves grows, as well as the microbunches length. As a result, the microbunch focusing takes place later on. One can see that the maximal value of the deformation index is almost independent of the bunch velocity. This fact can be concerned with the last stage of the microbunch focusing, when the sharp density maximum is formed. At this stage, the deformation index remains almost constant.

From the dependences presented in Fig. 9, it is clear that the growth rate of the deformation index at small distances from an injector increases with the bunch length. Indeed, for little bunch lengths (on the scale of wake waves' length), all the bunch electrons are located in the same phase of the wake field, so they are decelerated or accelerated simultaneously. For long bunches, the grouping takes place. It is caused by the spatial variation of the initial velocity perturbation due to the wake field.

Figure 10 demonstrates that, with increase of the bunch density, the wake field amplitude grows (compare

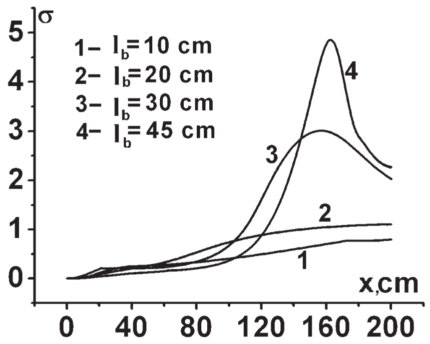

Fig. 9. Spatial dependence of the deformation index for different bunch lengths: $n_b = 2 \times 10^5$ cm^{-3}, $v_b = 3 \times 10^9$ cm/s, $n_p = 1 \times 10^8$ cm^{-3}, the wake wave length is 32 cm

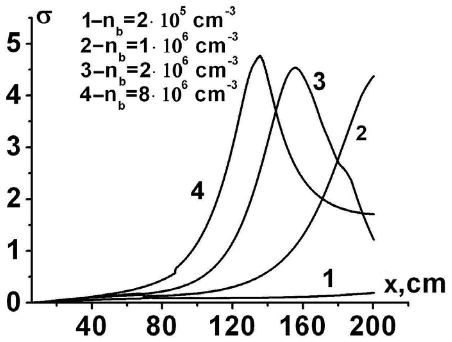

Fig. 10. Spatial dependence of the deformation index for different bunch densities: $v_b = 3 \times 10^9$ cm/s, $t_b = 1.1 \times 10^{-8}$ s, $n_p = 1 \times 10^8$ cm^{-3}.

with Fig. 3,*a* in Section 4.1). In this case, the perturbation of the bunch density increases with the field.

Figure 11 shows the dependence of the spatial behavior of the deformation index on the background plasma density. It was already noted (see Section 4.3) that, with increase of the background plasma density, the maximal bunch density decreases, as well as the wake wave field.

As a result, the deformation index decreases at large distances from an injector. At the same time for higher plasma densities, the focusing of the first microbunch occurs at smaller distances from an injector, which results in the corresponding spatial dependence of the deformation index.

6. Conditions of the Bunch Minimal Deformation

From the simulation results, it is possible to obtain the following conditions for the minimal deformation of an electron bunch with initially rectangular density profile during its motion in a homogeneous plasma.

1. Duration of a bunch should be much less than the period of Langmuir oscillations of a background plasma.
2. Bunch velocity should be as large as possible. One can expect that the relativistic increase of the electrons' mass will lead to an additional decrease of the bunch deformation (our calculations were carried out for the non-relativistic case).
3. The density of a bunch should be as small as possible.
4. If the length of the bunch trajectory is less than the length of the first density maximum formation, then a smaller deformation of the bunch density is reached in a less dense plasma.

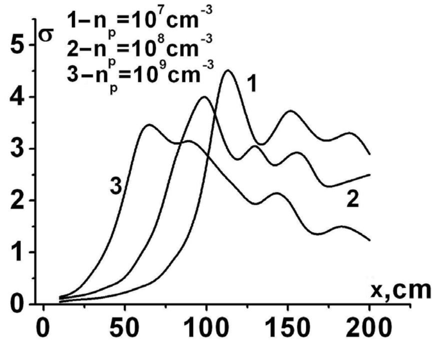

Fig. 11. Spatial dependence of the deformation index for different background plasma densities: $n_b = 2{\times}10^5$ cm^{-3}, $v_b = 2{\times}10^9$ cm/s

7. Conclusion

1. Dynamics of the electron bunch with initially rectangular density profile in a wake wave field is similar to the phase focusing of an electron beam with initial velocity modulation. At small distances from an injector, the wake wave is excited by the bunch forward front. At large distances, this wave is excited mainly by microbunches formed due to the focusing of the initial bunch by the wake field.
2. Dependences of the maximal bunch density and the maximal wake field on the model parameters are defined, primarily, by a variation of the microbunches' length at a variation of the parameters. Sometimes, it can be caused by the excitation of a wake wave by a coherent sequence of microbunches.
3. The proposed deformation index (an analog of the standard deviation for random processes) can be the integral characteristic of a deformation of the

initial bunch density profile. During the motion of a bunch with initially rectangular density profile through a plasma, the deformation index first grows, then reaches a maximum value at the time moment of the first microbunch focusing and further decreases nonmonotonously. The dependence of the deformation index on the model parameters is defined mainly by the factors that determine values of the maximal bunch density and the maximal wake field.
4. The use of short weak bunches for the inhomogeneous plasma diagnostics via transition radiation [2,3] results in the small amplitude of this radiation. This amplitude can be increased if a plasma inhomogeneity is situated at the distance of the first microbunch focusing. In this case, the bunch with duration equal to the wake wave period can be used.

In this work, we have considered only bunches with primarily rectangular density profile. But the wake wave amplitude excited by a bunch, as well as the bunch deformation rate, essentially depends on the shape of its initial density profile [8]. The study of this dependence will be a subject of the further researches.

Preliminary results of this work were reported in [9,10].

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ДОСЛІДЖЕННЯ ДИНАМІКИ ЕЛЕКТРОННОГО ЗГУСТКУ З ПРЯМОКУТНИМ ПРОФІЛЕМ КОНЦЕНТРАЦІЇ, ІНЖЕКТОВАНОГО В ОДНОРІДНУ ПЛАЗМУ

І.О. Анісімов, Ю.М. Толочкевич

Резюме

Досліджена динаміка одновимірного електронного згустку з прямокутним початковим профілем концентрації, інжектованого в однорідну плазму, за допомогою комп'ютерного моделювання методом макрочастинок в комірках. Пояснено залежності максимальної концентрації електронного згустку і максимальної амплітуди кільватерної хвилі від параметрів моделі (концентрація плазми, концентрація згустку, початкова швидкість і тривалість згустку). Введено показник деформації як кількісна характеристика деформації початкового профілю концентрації згустку і досліджено його залежність від параметрів моделі.