# NONLINEAR SWITCHING OF MICROWAVE PULSES IN LAYERED MEDIA WITH PARAELECTRIC

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Interaction of microwave electromagnetic (EM) waves with a layered structure that includes a paraelectric layer is investigated theoretically. An oblique incidence of an EM wave is considered, when the total internal reflection and the resonant transmission at several frequencies occur. It is demonstrated that this structure modulates effectively EM waves. The modulation mechanism is a change of the dielectric permittivity of a paraelectric layer due to a moderate bias electric field. The interaction of strong incident EM pulses with this structure is also considered in the case of the absence of a bias electric field. The nonlinear switching of short pulses takes place there, when both the maximum amplitude and the shape of a pulse change essentially under a small change of the amplitude of the incident pulse. This can be considered as the bistability of nonlinear pulses.

## 1. Introduction

Last time, much attention has been given to the electromagnetic wave propagation in layered structures both periodic and aperiodic [1–5]. Various nonlinear phenomena, both resonant and nonresonant, have been observed there. The most papers are devoted to the optical wave range, where it is possible to realize the structures with low losses. But a fabrication of multilayer structures is complicated, so it is simpler to realize the wave phenomena in structures with several layers, where a resonant tunneling of EM waves can occur. In the simplest case, these structures include several dielectric layers 1,2,3 with dielectric permittivities $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3 = \varepsilon_1$; the surrounding medium has a dielectric permittivity $\varepsilon_4$, see Fig. 1. The following inequalities are satisfied: $\varepsilon_1 < \varepsilon_4 < \varepsilon_2$. When the EM wave is incident at an angle $\theta \neq 0$, its tunneling can occur through layers 1 and 3, and such a structure possesses the resonant transmission at a collection of frequencies [6,7]. The EM tunneling itself now is the subject of intensive investigations, where the problems of delay time, superluminous propagation, and causality are under wide discussions.

If layer 2 is chosen as a nonlinear dielectric, then one can observe nonlinear wave phenomena in this structure. We are interested in the microwave range, where the paraelectric layers possess high values of dielectric nonlinearity and also relatively low losses [8,9]. The following materials are suitable: $BaTiO_3$, $SrTiO_3$, and $KTaO_3$ (the last one at low temperatures). All materials possess high values of dielectric permittivities which depend on the temperature and the electric field [9]. Therefore, it is possible to realize various dynamic nonlinear phenomena in these structures, like the auto-switching of powerful EM pulses and, probably, the modulation instability of long incident pulses. Note that the interaction of EM pulses with layered nonlinear structures in the microwave range has been investigated insufficiently, as compared with optics. For the practical

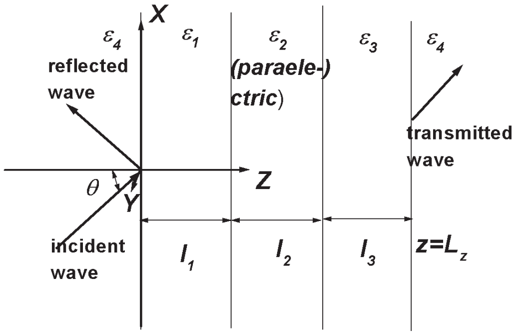


Fig. 1. Geometry of the problem

purposes, these structures can be effective modulators, nonlinear limiters, and switches.

## 2. Geometry and Model

The paper is devoted to the investigations of the switching of linear and nonlinear microwave EM waves in the layered structure with a paraelectric layer under the resonant tunneling. The effective amplitude modulation of linear EM waves takes place when a bias electric field is applied to the paraelectric layer. When the short powerful pulses are incident obliquely on the structure, the auto-switching occurs without a bias. The geometry of the structure is shown in Fig. 1.

The layered structure with three layers is considered. An EM wave is incident at $z = 0$. The thicknesses of layers are comparable with the wavelength of the EM wave within the media. For investigations of the interaction of strong pulses with this structure, it is necessary to derive the equation for the slowly varying wave amplitude. The system is uniform along the $OY$ axis. For sake of simplicity, the transverse polarization of EM waves is considered. The basic equation for the electric field $E \equiv E_y$ of an EM wave in the layered structure is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) E = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left((\varepsilon(z) + \Delta\varepsilon)E\right), \qquad (1)$$

where $\varepsilon(z)$ is the linear part of dielectric permittivity, and $\Delta\varepsilon$ is its nonlinear part in the paraelectric. The nonlinear part can be either by a bias electric field or the electric field of a powerful microwave pulse.

We consider a solution of Eq. (1) as

$$E = A(z,t)\exp(i(\omega t - k_x x)) + \text{c.c.} \qquad (2)$$

Here, $A(z,t)$ is the wave amplitude, $\omega$ is the carrier frequency; $k_x = (\omega/c)(\varepsilon_4)^{1/2}\sin\theta$ is the $x$-component of the wave vector of the wave; $\theta \neq 0$ is the incidence angle. The following condition should be satisfied: $|\partial A \partial t| \ll \omega|A|$. Thus, a slow variation of $A(z,t)$ in the course of time $t$ only is assumed [10]. A dependence of the amplitude $A$ on the coordinate $z$ can be arbitrary. After the substitution of (2) into Eq. (1), we obtain the following equation for $A(z,t)$:

$$\frac{\partial A}{\partial t} + \frac{i\omega}{2}\left(1 + \frac{\Delta\varepsilon}{\varepsilon(z)}\right)A -$$

$$-\frac{\omega\varepsilon''}{2\varepsilon(z)}A - \frac{ik_x c^2}{2\omega\varepsilon(z)}A + \frac{ic^2}{2\omega\varepsilon(z)}\frac{\partial^2 A}{\partial z^2} = 0. \qquad (3)$$

Here, $\Delta\varepsilon = -a(|A|^2 + E_0^2)$ is the dielectric nonlinearity of the paraelectric layer, where $E_0$ is the bias electric field [8,9]. Other layers are assumed linear. The imaginary part $\varepsilon''$ of the linear dielectric permittivity leads to some dissipation of waves. Below, we investigate two cases separately, where either the modulation of a linear EM wave due to the bias electric field occurs or there are the nonlinear phenomena without bias.

In Eq. (3), the boundary conditions at the internal interfaces are satisfied according to [10]. But this equation should be added additionally by boundary conditions at $z =0$ and $z = L_z$. The amplitude $F(t)$ of the incident wave is given at $z = 0$: $E_{\text{inc}}(z = 0, t) = (1/2)F(t) \times \exp(i\omega t - ik_x x) + \text{c.c.}$ At $z = L_z$, only the outgoing wave is present. From the conditions of continuity of the tangential components of the electric ($E_y$) and magnetic ($H_x$) fields, the following boundary conditions have been derived:

$$\frac{i}{k_{z4}}\frac{\partial A}{\partial z} + A = F(t) \quad \text{at} \quad z = 0;$$

$$\frac{i}{k_{z4}}\frac{\partial A}{\partial z} - A = 0 \quad \text{at} \quad z = L_z. \qquad (4)$$

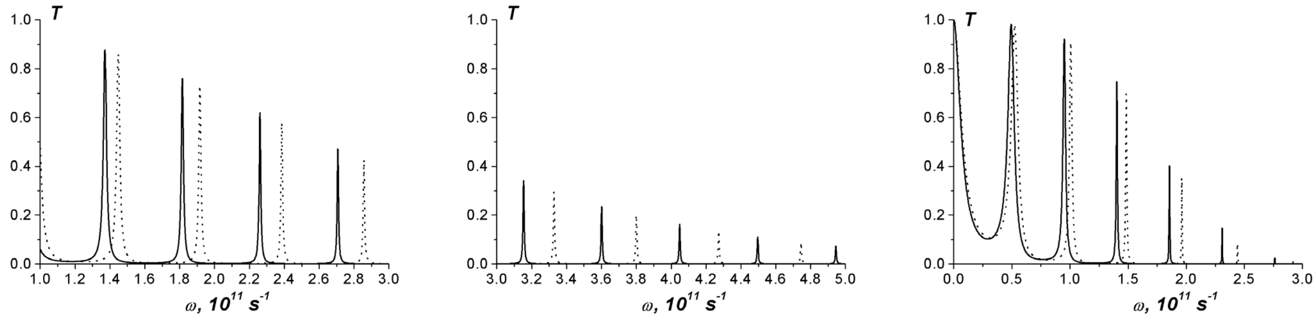
Here, $k_{z4} = (\omega/c)(\varepsilon_4)^{1/2}\cos\theta$.

Equation (3) with boundary conditions (4) has been solved by difference methods with the use of stable implicit schemes [11]. The validity of the obtained results has been checked by a variation of the temporal and spatial steps of discreteness.

## 3. Simulations and Discussion of Results

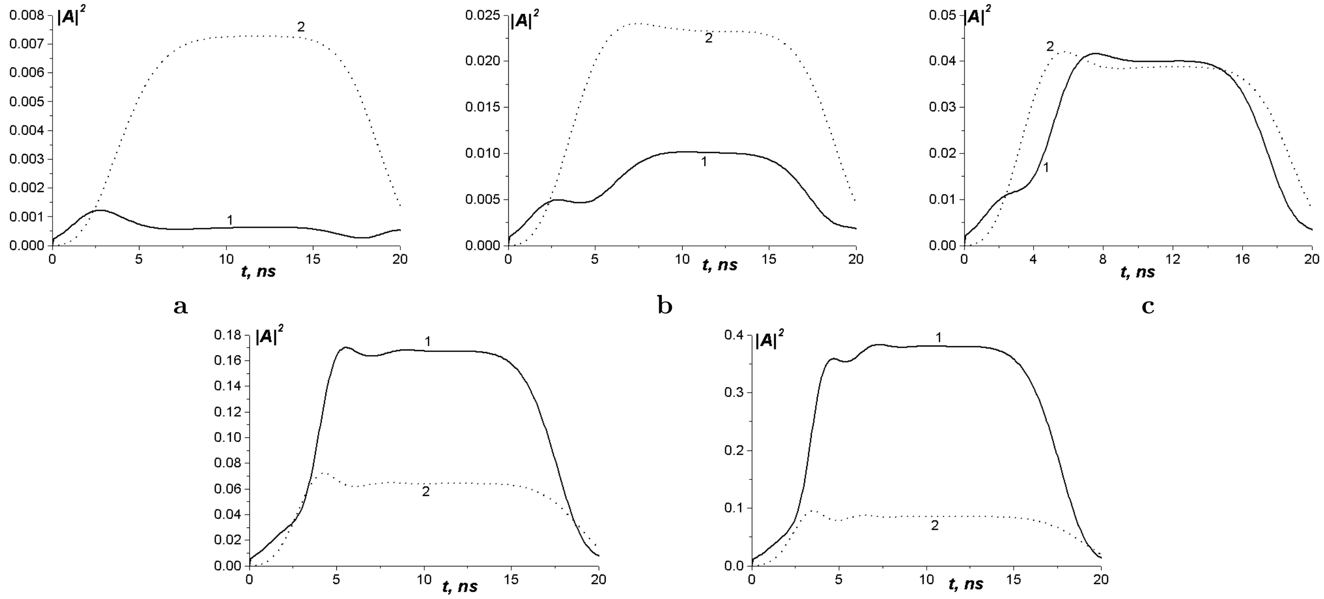
The parameters of the structure used in simulations are: $\varepsilon_1 = \varepsilon_3 = 36$, $\varepsilon_2 = 1800$ (paraelectric layer), $\varepsilon_4 = 144$ (surrounding medium). The values of dielectric permittivity of the paraelectric can reach the values of 5000. . .10000, so the used value is not maximally possible [9]. We took the dielectric losses within the structure into account: $\varepsilon_1''/\varepsilon_1 = -10^{-4}$, $\varepsilon_2''/\varepsilon_2 = -10^{-3}$. The results of the simulations are of qualitative character and tolerant to some changes of the parameters.

Initially, we have investigated the resonant transmission of an obliquely incident continuous EM wave. The amplitude of the wave is small here, so the change of the dielectric permittivity of the paraelectric is due to the bias field only: $\Delta\varepsilon = -aE_0^2$. It is possible to observe the sharp peaks of the resonant transmission. This phenomenon can be used for the effective



**a** **b** **c**

Fig. 2. Transmission coefficients of a continuous wave with small amplitude for the thicknesses of the layers $l_1 = l_2 = l_3 = 0.05$ cm. Parts it a), it b) are for the incidence angle $\theta = 30°$ ($b$ is the same as $a$, but in details), part it c) is for $\theta = 45°$



**a** **b** **c**

**d** **e**

Fig. 3. Interaction of powerful microwave pulses with a layered structure. The thicknesses of the layers are $l_1 = l_2 = l_3 = 0.05$ cm. The carrier frequency is $\omega = 1.80{\times}10^{11}$ $\mathrm{s}^{-1}$, the incidence angle is $\theta = 30°$; $t_1 = 10$ ns, $t_0 = 9$ ns. The solid line (1) is the reflected wave, the dotted line (2) is the transmitted wave. Part it a) is for $A_0 = 0.1$, it b) is for $A_0 = 0.2$, it c) $A_0 = 0.3$, it d) $A_0 = 0.5$, it e) $A_0 = 0.7$

modulation of EM waves by changing the dielectric permittivity of the paraelectric layer, when the constant bias electric field $E_0$ is applied, see Fig. 2. The regions of resonant transmission can be used also for observing the nonlinear phenomena.

It is possible to realize the 99-% modulation degree of transmission in a vicinity of the frequency of the incident EM wave near the resonant peaks of transmission. The value of modulation of the dielectric permittivity due to the bias field is moderate: $\Delta\varepsilon/\varepsilon_2 = -0.1$. Such a modulation is reached in the bias fields of $\sim$1 kV/cm in typical paraelectrics [8, 9].

A more interesting phenomenon is the interaction of strong short pulses with the structure without a bias field ($E_0 = 0$). The incident pulse is rectangular-like and determined by the function $F(t) = A_0 \exp(-((t - t_1)/t_0)^6)$. The condition $|dF/dt| \ll \omega F$ has been satisfied, because the durations of the fronts of incident pulses is about 1 … 3 ns, and the period of oscillations is $2\pi/\omega \approx 30$ ps. The results of simulations are given in Figs. 3 and 4.

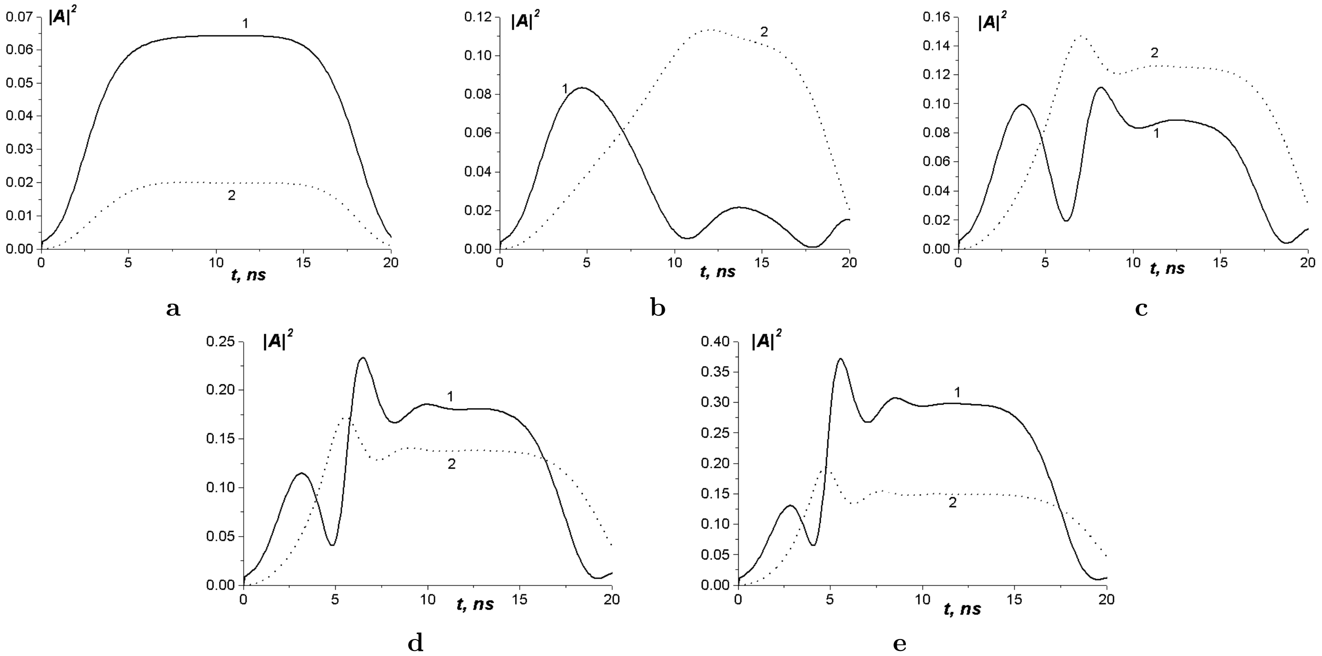


Fig. 4. Interaction of powerful microwave pulses with a layered structure. The thicknesses of the layers are $l_1 = l_2 = l_3 = 0.05$ cm. The carrier frequency is $\omega = 1.83\times10^{11}$ s$^{-1}$, the incidence angle is $\theta = 30°$; $t_1 = 10$ ns, $t_0 = 9$ ns. Part it a) is for $A_0 = 0.3$, it b) $A_0 = 0.4$, it c) $A_0 = 0.5$, d) $A_0 = 0.6$, it e) $A_0 = 0.7$

For all figures, the value of $|A| = 1$ corresponds to the value of modulation of the dielectric permittivity of the paraelectric $\Delta\varepsilon/\varepsilon_2 = -0.02$. Therefore, the dielectric nonlinearity is moderate. Two cases are considered below: the carrier frequency of the incident pulse coincides with the maximum of the linear transmission (resonant frequency), and the carrier frequency is somewhat higher than one for the maximum transmission.

In the first case, it is possible to observe the switching from the transmission into the total reflection regime under a gradual increase of the maximum value of the incident pulse amplitude, see Fig. 3. This can be explained by the nonlinear mismatch between the carrier frequency of the pulse and the linear resonant frequency of the structure.

The second case is more interesting. If the carrier frequency of the pulse is chosen in the vicinity of the transmission peaks, then the maximum amplitude and also the shape of transmitted and reflected pulses depend on the amplitude and the duration of the incident pulse. To get the switching into the transmission regime (nonlinear transparency), it is necessary to choose properly the maximum amplitude of the incident pulse, see Fig. 4. If the amplitude of the incident pulse is higher than the optimum one, then the transmission regime does not occur. The explanation of this fact is clear. The linear transmission peaks are narrow, see Fig. 2. Therefore, for shifting the carrier frequency into the transmission region, the modulation of the dielectric permittivity should be equal to the corresponding value. At different durations of the incident pulse, the shapes of transmitted and reflected pulses are changed essentially. It is possible to consider this switching as a bistability of short powerful pulses.

In the case of long powerful pulses, the modulation instability can be observed in these structures. But, for this purpose, it is better to use the simpler structures that include only two layers: $l_1$ (with the dielectric permittivity $\varepsilon_1$) and $l_2$ (with $\varepsilon_2$) and a normal incidence of a microwave. It is also necessary to suppress the generation of higher harmonics (third and fifth ones), because the generation of higher harmonics leads to nonlinear losses.

## 4. Conclusions

The layered structure with a few layers that includes a paraelectric layer can be used for observation of the resonant tunneling of EM waves in the microwave range. The resonant tunneling can be obtained under the oblique incidence of the EM wave onto a three-layer structure. Such a structure can be used as an effective modulator and a switch in the microwave range. For small input wave amplitudes, the modulation of the transmitted or reflected wave is due to a change of the dielectric permittivity of the paraelectric layer in the bias electric field. In the case of strong input pulses, the auto-switching can occur, where both the maximum amplitudes of transmitted and reflected pulses and their shapes change essentially under a gradual change of the maximum amplitude of the incident pulse. To obtain the switching from the reflection to the transmission regime, it is necessary to choose properly the incident pulse amplitude to compensate exactly the mismatch between the carrier frequency of the pulse and the frequency of the linear resonant transmission.

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## НЕЛІНІЙНЕ ПЕРЕКЛЮЧЕННЯ НВЧ-ІМПУЛЬСІВ У ШАРУВАТИХ СЕРЕДОВИЩАХ З ПАРАЕЛЕКТРИКОМ

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Резюме

Досліджено теоретично взаємодія НВЧ електромагнітних (ЕМ) хвиль з шаруватою структурою, яка містить параелектричний шар. Розглянуто похиле падіння ЕМ-хвилі, коли має місце повне внутрішнє відбиття і резонансне проходження на декількох частотах. Показано, що ця структура ефективно модулює ЕМ-хвилю. Механізм модуляції є в зміні діелектричної проникності параелектричного шару при впливі помірного постійного електричного поля. Разглянуто також взаємодію з даною структурою потужних падаючих ЕМ-імпульсів у відсутності постійного електричного поля. Нелінійне переключення коротких імпульсів спостерігається, коли максимальна амплітуда і форма імпульсу суттєво змінюються при невеликій зміні амплітуди падаючого імпульсу. Дане явище можна розглядати як бістабільність нелінійних імпульсів.