FOCUSING PROPERTIES OF POLYMER-STABILIZED LIQUID CRYSTAL LENSES

S.P. PAVLIUCHENKO, S.L. SUBOTA, V.YU. RESHETNYAK

Taras Shevchenko Kyiv National University, Faculty of Physics
(2, Academician Glushkov Ave., Kyiv 03680, Ukraine; e-mail: sveta_pavl@ukr.net)

UDC 532.738; 535

Focusing properties of gradient polymer-stabilized liquid crystal (G-PSLC) lenses are studied. We propose a theoretical model that describes the dependence of the G-PSLC lens focal length on the applied voltage. The model involves the strong light absorption during the polymerization process resulting in a modification of the boundary conditions. Our results can be used to develop G-PSLC lenses without moving parts which will allow the electro-optical zooming.

1. Introduction

Liquid-crystal-based gradient-index lenses are a new class of active-optical elements which have been intensively studied [1–13]. All these elements form a subclass of the LC-based switchable devices that are used as displays, modulators, and detectors [2, 3].

In recent years, there arises a significant interest in lenses with variable focal length. These lenses have many potential applications [3], for example, in medicine. Eyes lose their flexibility with age, which sometimes makes it difficult to shift the focus from near to far or vice versa. To combat the problem, LC lenses have been proposed. Today, a lot of possible designs for liquid crystal lenses are available. Among them, there are the lenses that are created by using electrodes with holes [4,5] or non-planar electrodes [6–9]; lenses employing Fresnel zones [10]; G-PSLC lenses [2, 11], *etc.* The operation of all these spectacle lenses is based on the electrical control over the refractive index of a thin layer of a nematic liquid crystal.

In this paper, we focus on adaptive gradient polymer-stabilized lenses. These lenses are fabricated using the photopolymerization technique with a Gaussian beam [2]. The mixture of a planar oriented nematic liquid crystal (NLC) and a few percent of a photopolymerizable monomer is illuminated by a laser beam with a Gaussian spatial intensity distribution, which induces a spatially inhomogeneous polymer network [12]. In the cell, there occurs the light absorption of the incident beam during polymerization. As for the absorption, one may consider two models of this problem. The first model is when there is a weak light absorption in the cell. In this case, a polymer network is created throughout the cell bulk [13]. Otherwise, when there is the strong light absorption in the cell, a network can be formed close to one cell wall only. In this case, the effect of the polymer network is only to modify the boundary conditions at that wall, that is the new feature in the present work.

The electro-optical response of the system to a uniform electric field is inhomogeneous but centrally symmetric. The refractive index profile is qualitatively similar to the director spatial distribution. The cell acts as a positive lens for the extraordinary polarized light component passing through it. By varying the applied voltage, one can change the refractive index profile and hence control the focal length of the lens.

The present work is organized as follows. In the first section, we describe the cell geometry and find the director reorientation under the action of an externally applied voltage in the cell with a polymer network. In the second section, we study the dependence of the focal length on the applied voltage for a G-PSLC lens. Finally, we draw some conclusions.

2. Director Reorientation under Action of Externally Applied Voltage in the Cell with a Polymer Network

Let us consider a nematic LC cell of thickness L with spatially modulated boundary conditions. The geometry of the problem is shown in Fig. 1. At the one wall of the LC cell surfaces, there is the strong anchoring and the director is parallel to the OX axis. For another wall, we assume that the anchoring energy varies by the law

$$W(\rho) = C\exp(-\gamma\rho^2), \tag{1}$$

as NLC is illuminated by a Gaussian beam during the polymerizaion process. Here, $C = wN_p$, where w is a parameter that describes anchoring the LC director with the polymer network, and N_p is the density of the polymer network. Varying the concentration of polymer in the liquid cell, we can vary the constant C. In the

 ISSN 2071-0194. Ukr. J. Phys. 2009. V. 54, N 3

sequel, the center of the beam with a Gaussian spatial intensity distribution will be named the center of the cell. Let a dc electric field be applied along the OZ axis. The field reorients the director, and it plausible to assume that the director remains in the XZ plane.

From the symmetry reason, the director field is set by the relation

$$\mathbf{n} = (\cos\theta(\rho, z), 0, \sin\theta(\rho, z)). \tag{2}$$

The free energy functional includes terms concerning the elastic and electric forces (F_{el} and F_E, respectively) and the interaction with the polymer network F_{pol}:

$$F = F_{\text{el}} + F_E + F_{\text{pol}}. \tag{3}$$

Here,

$$F_{\text{el}} = \frac{1}{2}\int K_1(\triangledown \mathbf{n})^2\,dV + \frac{1}{2}\int K_2(\triangledown \times \mathbf{n})^2\,dV +$$

$$+\frac{1}{2}\int K_3(\mathbf{n}\times\triangledown\times\mathbf{n})^2\,dV,$$

$$F_E = -\frac{1}{2}\int \mathbf{DE}\,dV, \quad F_{\text{pol}} = -\frac{1}{2}\int W(\rho)(\mathbf{ne})^2\,dS.$$

The term F_{pol} describes the interaction of LC with the polymer network formed by UV illumination; K_1, K_2, and K_3 are the elastic constants of pure LC; $\mathbf{E}$ is the direction of the easy axis at the bottom substrate; $\mathbf{D}$ is the electric displacement vector.

Supposing the NLC to be an ideal dielectric, we write $\triangledown\mathbf{D} = 0$ and consequently $\frac{\partial D_z}{\partial z} = 0$. In this case, we have neglected the derivative $\frac{\partial}{\partial \rho}$ in comparison with the derivative $\frac{\partial}{\partial z}$, because the characteristic length of the director inhomogeneity in the z-direction (cell thickness) is much less than the characteristic size of the director inhomogeneity in ρ, which is given by the size of the beam spot, $\backsim 1$ mm. Solving this equation, we obtain $D_z = \text{const}$. Likewise, combining the equation $\triangledown\times\mathbf{E} = 0$ with the boundary conditions $E_x = E_y = 0$ at the cell walls, we obtain $\mathbf{E} = (0, 0, E(z))$. As a result, we get

$$D_z = \varepsilon_{zz}E_z = (\varepsilon_\perp + \varepsilon_\alpha \sin^2\theta(z))E_z = \text{const}. \tag{4}$$

Consequently, the voltage U across the nematic cell is given by the relation

$$U = \int_0^L E\,dz = D_z\int_0^L (\varepsilon_\perp + \varepsilon_\alpha\sin^2\theta(z))^{-1}\,dz. \tag{5}$$

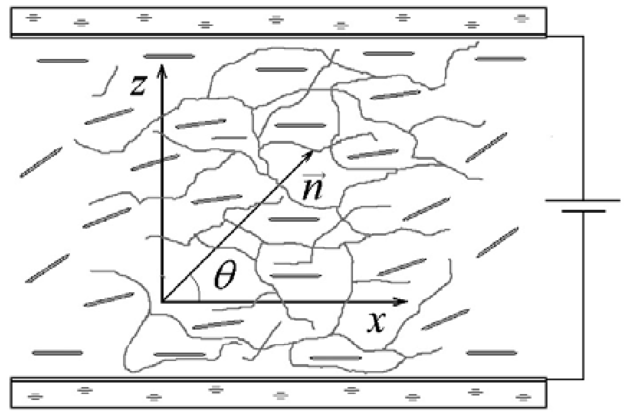

Fig. 1. LC cell geometry

The thermodynamic functional takes the form

$$F = \pi K\int_0^\infty \rho\,d\rho\int_0^L (\theta_z)^2 + (\theta_\rho)^2)\,dz -$$

$$-U^2\pi\int_0^\infty \rho\,d\rho\left(\int_0^L(\varepsilon_\perp + \varepsilon_\alpha\sin^2\theta(z))^{-1}\,dz\right)^{-1} -$$

$$-\pi\int_0^\infty W(\rho)(\cos\theta(\rho,z))^2\rho\,d\rho. \tag{6}$$

Here, we use the one-elastic-constant approximation $K_1 = K_2 = K_3 = K$ and neglect the derivative $\frac{\partial}{\partial\rho}$ in comparison with the derivative $\frac{\partial}{\partial z}$.

By minimizing functional (6), we obtain the Euler–Lagrange equation with boundary conditions:

$$\begin{cases} \theta_{zz} + \dfrac{D^2\varepsilon_\alpha}{2K}\dfrac{\sin 2\theta}{\left[\varepsilon_\perp + \varepsilon_\alpha\sin^2\theta(z)\right]^2} = 0, \\ \theta(z=0,\rho)_z - \dfrac{W(\rho)}{2K}\sin 2\theta(z=0,\rho) = 0, \\ \theta(z=L,\rho) = 0. \end{cases} \tag{7}$$

First, we consider the case of a small director deviation from the initial state and determine the dependence of the threshold voltage on the position in

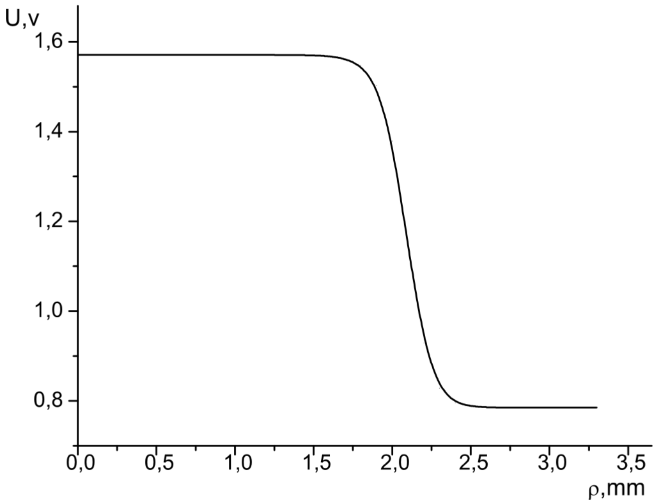

Fig. 2. Dependence U on the distance ρ from the cell center

the cell. So, we rewrite (7) as

$$\begin{cases} \theta_{zz}+\dfrac{D^2\varepsilon_\alpha}{K\varepsilon_\perp^2}\theta=0, \\ \theta_z(z=0,\rho)-\dfrac{C\exp(-\gamma\rho^2)}{K}=0, \\ \theta(z=L,\rho)=0. \end{cases} \tag{8}$$

The general solution to system (8) is

$$\theta(z,\rho)=A\sin\frac{D\sqrt{\varepsilon_\alpha}}{\varepsilon_\perp\sqrt{K}}z+B\cos\frac{D\sqrt{\varepsilon_\alpha}}{\varepsilon_\perp\sqrt{K}}z. \tag{9}$$

Using the boundary conditions of system (8), we get the equation for V:

$$\tan V=-\frac{L}{S}\exp(-\gamma\rho^2), \tag{10}$$

where $V=\dfrac{U}{U_0}$, U is the voltage which is applied to the nematic cell; $U_0=\pi\sqrt{\dfrac{K}{\varepsilon_\alpha}}$; and $S=\dfrac{CL^2}{K}$. The equation has been solved numerically using the following parameters for LC E7 [14]: $\varepsilon_\alpha=13.8$; $\varepsilon_\perp^2=5.2$; $L=10^{-5}$ m; $K=1.14\times10^{-11}$ H; $C=10^{-4}$ J/m^2, $\gamma=3\times10^6$ m^{-2}. In Fig. 2, we present the dependence of the minimal voltage when the LC reorientation starts on the distance from the cell center.

We now find the solution of system (7) in the general case. First, we rewrite this system in the form

$$\begin{cases} \theta_{zz}+a\dfrac{\sin2\theta}{[1+b\sin^2\theta]^2}=0, \\ \theta_{0z}-\dfrac{W(\rho)\sin2\theta_0}{2K}=0, \\ \theta(z=L,\rho)=0, \end{cases} \tag{11}$$

where

$$a=\frac{D^2\varepsilon_\alpha}{2K\varepsilon_\perp^2};b=\frac{\varepsilon_\alpha}{\varepsilon_\perp};\theta(z=0,\rho)=\theta_0. \tag{12}$$

Integrating the first equation of system (11) two times and using the second and third equations (boundary conditions), we obtain

$$\begin{cases} z=\frac{1-\gamma}{2\sqrt{(\alpha-c)(1+\gamma)}(1+p)}\Big[P(\arcsin\sqrt{\frac{(1+\gamma)(1-\cos2\theta)}{2(1-\gamma\cos2\theta)}}, \\ \frac{2p}{(1+\gamma)(1+p)},\sqrt{\frac{2\gamma}{1+\gamma}})-P(\arcsin\sqrt{\frac{(1+\gamma)(1-\cos2\theta_0)}{2(1-\gamma\cos2\theta_0)}}, \\ \frac{2p}{(1+\gamma)(1+p)},\sqrt{\frac{2\gamma}{1+\gamma}})\Big]; \\ 4L^2(\alpha-c)(1+\gamma)(1+p)^2=(1-\gamma)^2\times \\ \times P^2\left(\arcsin\sqrt{\frac{(1+\gamma)(1-\cos2\theta_0)}{2(1-\gamma\cos2\theta_0)}},\frac{2p}{(1+\gamma)(1+p)},\sqrt{\frac{2\gamma}{1+\gamma}}\right), \end{cases} \tag{13}$$

where we defined

$$c=\frac{\alpha}{(1-\gamma\cos2\theta_0)}-\frac{W^2\sin^22\theta_0}{4K^2},$$

$$p=\frac{c\gamma}{\alpha-c},\quad \gamma=\frac{b}{2+b},\quad \alpha=\frac{4a}{b(2+b)},$$

$$P\left(\arcsin\sqrt{\frac{(1+\gamma)(1-\cos2\theta)}{2(1-\gamma\cos2\theta)}},\frac{2p}{(1+\gamma)(1+p)},\frac{2\gamma}{1+\gamma}\right)$$

is an elliptic integral of the third kind.

Though system (11) has been solved analytically, the practical use of its solution (13) is difficult, so some approximations have been made for finding a numerical solution to system (7).

As the minimum deviation of the angle reorientation direction is $\theta=0$, and the maximum one is $\theta=\frac{\pi}{2}$, the values of the electric displacement vary negligibly. That's why, we can simplify formula (5) as

$$D_z=D=\frac{U\varepsilon_\perp}{L}. \tag{14}$$

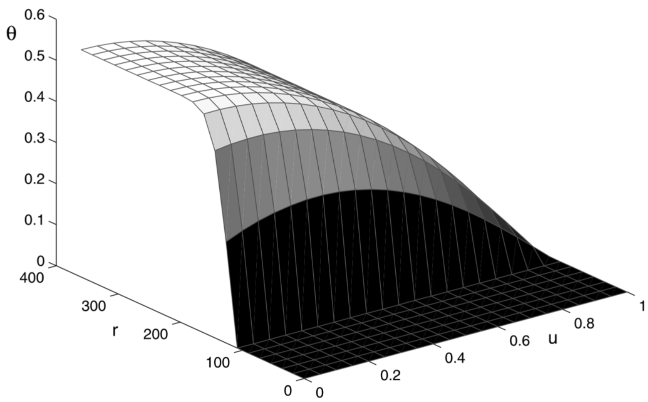

Fig. 3. Director reorientation angle versus the coordinates r and u of the nematic cell at the voltage U=0.8 V

Then system (7) takes the form

$$\begin{cases} \theta_{zz} + \dfrac{U^2\varepsilon_\alpha\varepsilon_0}{2KL^2}\dfrac{\sin 2\theta}{\left[1+\dfrac{\varepsilon_\alpha}{\varepsilon_\perp}\sin^2\theta\right]^2} = 0, \\ \theta(z=0,\rho)_z - \dfrac{W(\rho)}{2K}\sin 2\theta(z=0,\rho) = 0, \\ \theta(z=L,\rho) = 0. \end{cases} \tag{15}$$

It is convenient to rewrite system (15) in terms of the dimensionless parameters $u = \dfrac{z}{L}$ and $r = \dfrac{\rho}{L}$ as

$$\begin{cases} \theta_{uu} + \dfrac{U^2\varepsilon_\alpha\varepsilon_0}{2K}\dfrac{\sin 2\theta}{\left[1+\dfrac{\varepsilon_\alpha}{\varepsilon_\perp}\sin^2\theta\right]^2} = 0, \\ \theta(u=0,rL)_u - \dfrac{\exp(-\gamma(rL)^2)L}{2K}\sin 2\theta = 0, \\ \theta(u=L,rL) = 0. \end{cases} \tag{16}$$

System (16) has been solved numerically. Figures 3–5 show the dependence of the director reorientation angle θ on the coordinates of the nematic cell r and u at various applied voltages. We see that the director reorientation profile behaves itself at the cell center as the director profile in the case with strong anchoring. Far from of the center, it resembles the profile in the case with weak anchoring at one wall and with strong anchoring at another one. The intermediate region with finite anchoring is narrow, because the Gaussian function in formula (1) changes very quickly with increase in the distance in the radial direction. Since only this region

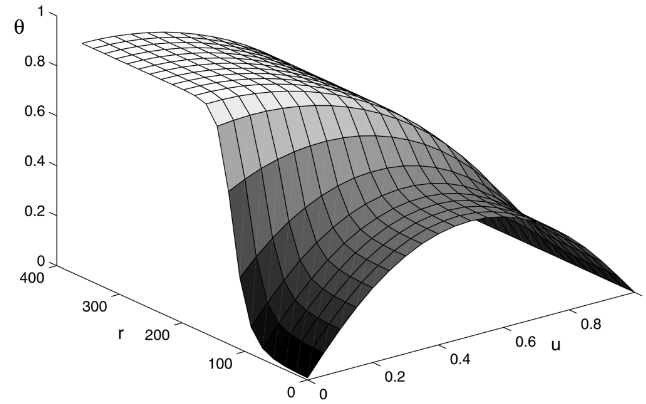

Fig. 5. Director reorientation angle versus the coordinates r and u of the nematic cell at the voltage U=1.4 V

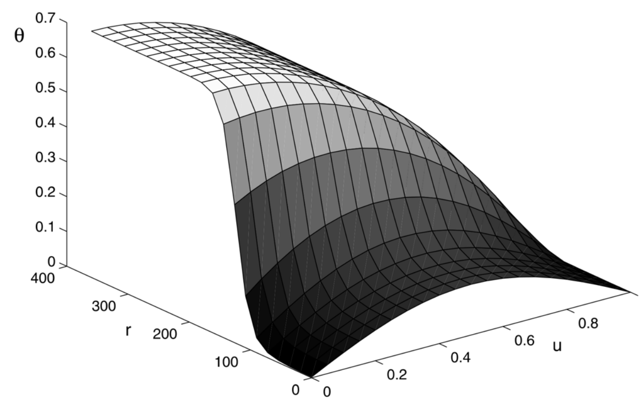

Fig. 4. Director reorientation angle versus the coordinates r and u of the nematic cell at the voltage U=1 V

makes the main contribution to the creation of the lens, the above-mentioned fact will hamper the modeling of adaptive lenses.

3. Light Propagation through Polymer-Stabilized LC Subject to Externally Applied Voltage

In the previous section, we have found the spatial profile of the LC director subject to an externally applied voltage. The refractive index for the extraordinary beam propagating in the LC cell is given by the relation

$$n(\psi) = \frac{\sqrt{\tilde{\varepsilon}_\parallel\tilde{\varepsilon}_\perp}}{\left[\tilde{\varepsilon}_\parallel\cos^2\psi + \tilde{\varepsilon}_\perp\sin^2\psi\right]^{1/2}}, \tag{17}$$

where ψ is the angle between the director (optical axis) and the light beam wave vector, and $\hat{\tilde{\varepsilon}}$ is the dielectric

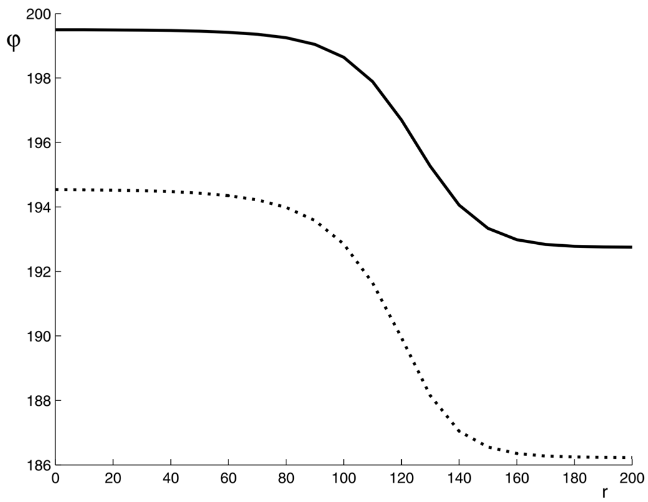

Fig. 6. Dependence of the phase φ on the distance r from the cell center at voltages of 1 V (solid curve) and 1.7 V (dashed curve)

tensor at the optical frequency. In our case, $\psi = \frac{\pi}{2} - \theta$, where θ is the director deviation calculated in the first section.

Formula (17) can be rewritten in terms of θ as

$$n(\theta) = \frac{\sqrt{\varepsilon_\parallel \varepsilon_\perp}}{\left[\varepsilon_\perp + \varepsilon_\alpha \sin^2\theta\right]^{1/2}}. \tag{18}$$

The electric field applied to a nematic liquid crystal cell creates a lens-like director distribution. We treat the LC cell illuminated with a testing beam like a thin phase plate. Then the phase retardation for a testing light beam passing through the cell will depend on the distance from the center of the UV Gaussian beam that created the polymer network:

$$\varphi(\rho) = \int\limits_0^L kn(\rho, z)\, dz, \tag{19}$$

where k is the wave vector of the testing beam.

Figure 6 shows the dependence of the phase lag φ on the distance r from the cell center at various applied voltages.

The focusing properties of the resulting lens are determined by the phase retardation dependence on the distance from the beam center (the cell center). To determine the focal length of the lens, we fit (19) to a parabola (Fresnel approximation) [15]:

$$\varphi(\rho) = a - \frac{k\rho^2}{2f}, \tag{20}$$

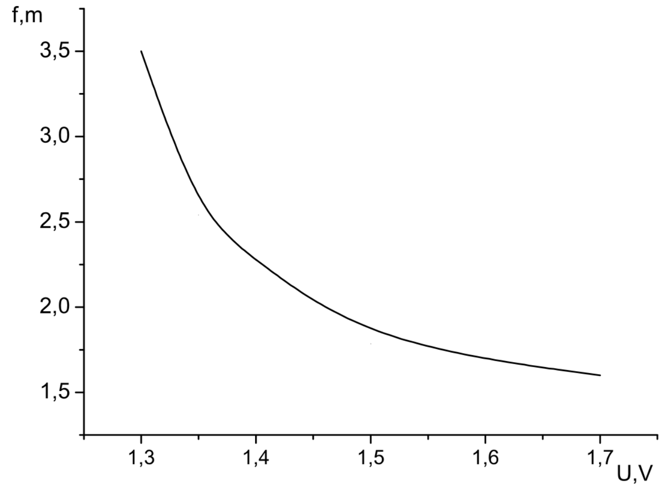

Fig. 7. Focal length versus the applied voltage U

where $a = \varphi(0)$ is the phase shift at the center of the initial UV beam, and f is the focal length of the G-PSLC lens.

In Fig. 7, we show the focal length versus the applied voltage (LC E7, $\sqrt{\varepsilon_\parallel} = n_e = 1.738$, $\sqrt{\varepsilon_\perp} = n_0 = 1.518$).

4. Conclusions

In this paper, we have proposed a theoretical model which describes the focusing properties of the nematic liquid crystal lens, as well as the dependence of the G-PSLC lens focal length on the applied voltage. We have determined the dependence of the minimum voltage required to start the director reorientation on the distance to the cell center. We have calculated the director profile of NLC in the cell with spatially modulated boundary conditions subject to an externally applied electric field and have estimated the focal length of the G-PSLC lens. The results show that the focal length decreases with increase in the applied voltage. These results can be used to develop G-PSLC lenses that have no moving parts and which allow the electro-optical zooming.

We acknowledge NATO grant CBP.NUKR.CLG. 981968 which supported a visit of V.Yu. Reshetnyak to Southampton (UK), where some part of this work was carried out. We are also grateful to the Royal Society for the support of the visits of SLS and VYR to Southampton at the early stage of this project. We are also grateful to T. Galstian (Quebec) and Timothy J. Sluckin (UK) for the fruitful discussions.

1. G. Li, D.L. Mathine, P. Valley *et al.*, Proc. Natl. Acad. Sci. (USA) **103**, 6100 (2006).

2. V. Presnyakov and T. Galstian, J. of Appl. Phys. **97**, 103101-1 (2005).

3. T.C. Kraan, T. van Bommel, and R.A.M. Hikmet, J. Opt. Soc. Am. A. **24**, 3467 (2007).

4. F. Naumov, G.D. Love, M.Yu. Loktev, and F.L. Vladimirov, Optics Express **4**, 344 (1999).

5. M. Ye and S. Sato, Jpn. J. Appl. Phys. **41**, L 571 (2002).

6. B. Wang, M. Ye, M. Honma, T. Nose, and S. Sato, Jpn. J. Appl. Phys. **41**, L 1232 (2002).

7. H. Ren, Y.-H. Fan, S. Gauza, and S.-T. Wu, Appl. Phys. Lett. **84**, 4789 (2004).

8. H. Ren and S.-T. Wu, Optics Express **14**, 11292 (2006).

9. H. Ren, D.Fox, B.Wu and S.-T. Wu, Optics Express **15**, 11328 (2007).

10. Y.-H. Fan, H. Ren and S.-T. Wu, Optics Express **11**, 3080 (2003).

11. V. Presnyakov, K. Asatryan, and T. Galstian, Optics Express **10**, 865 (2002).

12. V.Yu. Reshetnyak, S.L. Subota, and T.V. Galstian, Mol. Cryst. Liq. Cryst. **454**, 187/[589]-200/[602] (2006).

13. V.Y. Reshetnyak, S.M. Shelestiuk, S.L. Subota, S.P. Pavliuchenko, and T.J. Sluckin, Proceedings of SPIE, **6587**, 658709 (2007).

14. P.S. Drzaic and A. Muller, Liq. Cryst. **5**, 1467 (1989).

15. J.W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 2002).

Received 09.04.08

ФОКУСУЮЧІ ВЛАСТИВОСТІ
РІДКОКРИСТАЛІЧНИХ НЕМАТИЧНИХ ЛІНЗ

С.П. Павлюченко, С.Л. Субота, В.Ю. Решетняк

Р е з ю м е

Лінза, фокусна відстань якої керується електричним полем, утворюється в нематичному рідкому кристалі в процесі фотополімерізації в неоднорідному світловому полі гаусового пучка світла. В роботі припускається, що внаслідок сильного поглинання світла, полімеризація відбувається в тонкому шарі поблизу поверхні комірки з боку опромінення. Неоднорідний розподіл полімеру приводить до просторово неоднорідних межових умов на директор рідкого кристала. Знайдено аналітично та чисельно кут переорієнтації директора нематичного рідкого кристала, показник заломлення утвореної лінзи та набіг фази, який набуває електромагнітна хвиля при проходженні крізь лінзу. Отримано фокусну відстань лінзи залежно від величини прикладеної напруги та параметрів комірки.