TO THE MOLECULAR THEORY OF THE PROPAGATION OF HIGH-FREQUENCY HEAT WAVES IN MAGNETIC FLUIDS

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On the basis of molecular-kinetic theory, we study the frequency dependences of the velocity and the absorption of heat waves under the action of an inhomogeneous magnetic field. We deduced the dynamical expressions for the velocity and the absorption coefficient of heat waves. Their asymptotic behavior at low and high frequencies is studied with regard for the contribution of a structural relaxation. We performed a numerical study by the example of a magnetic fluid on the basis of kerosene and solid particles of magnetite Fe_3O_4. The results of numerical calculations are in a satisfactory agreement with the theoretical results for simple fluids.

The dense arrangement of particles and the presence of an interaction between them imply that each particle of a fluid is in the field of action of the rest particles. As a result, the motions and the interactions of particles of a fluid acquire the collective character. Hence, the character of these collective oscillations will affect the nonequilibrium processes running in fluids. Collective oscillations in fluids arise as the natural oscillations of a system which are described by the equations of generalized hydrodynamics obtained on the basis of molecular-kinetic theory. In correspondence with the preservation laws, there are five types of natural oscillations. At a given value of the wave vector $\vec{k}$, the natural oscillations are set by two longitudinal acoustic modes, two transverse shear modes, and one longitudinal heat mode.

The velocity and the absorption coefficient of sound in various magnetic fluids (MF) were experimentally studied in works [1,2]. However, the heat modes in fluids, in particular in MF, are insufficiently investigated both theoretically and experimentally. Despite the available experimental results on the measurement of the static coefficient of heat conductivity [3], there exist no complex physical studies of thermoelastic properties of MF which would allow one to clarify the influence of collective and individual modes of thermal motion on the process of heat transfer. Such studies would give possibility to judge the character of relaxation processes and their contribution to the velocity and the absorption coefficient of waves in MF.

The above-presented discussion implies that the study of the propagation of heat waves in magnetic fluids within molecular-kinetic theory seems to be essential. Just such a study is the purpose of the present work.

We start from the equations of the generalized hydrodynamics of MF [4] derived on the basis of molecular-kinetic theory. These equations coincide, by their form, with the macroscopic expressions of the preservation laws of mass, momentum, and energy. However, the momentum flow tensor $P^{\alpha\beta}(\vec{q},t)$ and the heat flow vector $S^{\alpha}(\vec{q},t)$ which enter these laws are determined microscopically with the help of the one- and two-particle distribution functions (DF) and the other molecular parameters of MF. The heat equation or the energy preservation law [4] takes the form

$$\rho C_v \frac{\partial T(\vec{q}_1,t)}{\partial t} + T\left(\frac{\partial P_o}{\partial T_0}\right)_\rho div\vec{\vartheta} + \frac{\partial S^{\alpha}(\vec{q}_1,t)}{\partial q_1^{\alpha}} = 0, \quad (1)$$

where $S^{\alpha}(\vec{q},t) = S_k^{\alpha} + \frac{\sigma^3}{4}\int\left[\Phi(r)\delta^{\alpha\beta} - \frac{r^{\alpha}r^{\beta}}{r}\frac{d\Phi(r)}{dr}\right]\times \times J_2^{\beta}(\vec{q}_1,\vec{r},t)d\vec{r}$ – heat flow vector; C_v – heat capacity at a constant volume; T_0 and $T(\vec{q},t)$ – equilibrium and nonequilibrium temperatures, respectively; P_0 – equilibrium pressure; $\vartheta(\vec{q},t)$ – mean velocity of MF particles; σ – diameter of particles; $\Phi(|r|)$ –

intermolecular interaction potential, whose explicit form depends on the choice of a model of MF; ρ – density of MF; $S_k^\alpha = \frac{1}{2}\int \frac{\tilde{p}_1^2 \tilde{p}_1^\alpha}{m^2} f(\vec{x}_1, t) d\vec{p}_1$ – kinetic part of the heat flow vector, $\tilde{p}_1^\alpha = p_1^\alpha - mv^\alpha(\vec{q}_1, t)$, m – mass of a particle, $\vec{x}_1 = (\vec{q}_1, \vec{p}_1)$; and $J_2^\beta(\vec{q}_1, \vec{r}, t)$ – binary flow of MF particles which is described by the Smoluchowski equation [4].

Assuming the flows to be independent, i.e. $(\partial P_0/\partial T_0)_\rho = 0$, we present Eq. (1) as

$$\rho C_v \frac{\partial T(\vec{q}_1, t)}{\partial t} + \frac{\partial S^\alpha(\vec{q}_1, t)}{\partial q_1^\alpha} = 0. \tag{2}$$

As known [5], the heat flow is defined, in the general case, by the expression

$$\vec{S}(\vec{q}_1, t) = i\vec{k}\int_0^t \int_{-\infty}^{\infty} \lambda(\vec{q}_1 - \vec{q}\,', t - t')T'(\vec{q}\,', t')d\vec{q}\,'dt'. \tag{3}$$

After the Fourier-transformation with respect to the time and coordinates, Eq. (2) yields

$$-i\omega\rho_0 C_v T'(\omega, \vec{k}\,) + ik^\alpha S^\alpha(\omega, \vec{k}\,) = 0, \tag{4}$$

where $T'(\omega, \vec{k}\,)$ – Fourier transform of the nonequilibrium temperature, and ω – frequency of the process.

By applying the convolution of a function by the Fubini theorem [6], we reduce Eq. (3) to the form

$$S^\alpha(\omega, \vec{k}\,) = i\vec{k}\tilde{\lambda}(\omega, \vec{k}\,)T'(\omega, \vec{k}\,), \tag{5}$$

where $\tilde{\lambda}(\omega, \vec{k}\,)$ – complex-valued dynamical coefficient of heat conductivity. Assuming the MF to be spatially homogeneous, we write Eq. (5) as

$$S^\alpha(\omega, \vec{k}\,) = i\vec{k}\tilde{\lambda}(\omega)T'(\omega, \vec{k}\,). \tag{6}$$

According to [7], we have

$$\lim_{\omega\to\infty} i\omega\tilde{\lambda}(\omega) = \tilde{Z}(\omega). \tag{7}$$

With regard for this relation, formula (6) takes the form

$$S^\alpha(\omega, \vec{k}\,) = \frac{k^\alpha}{\omega}\tilde{Z}(\omega)T'(\omega, \vec{k}\,), \tag{8}$$

where $\tilde{Z}(\omega) = Z(\omega) - i\omega\lambda(\omega)$; $Z(\omega)$ – dynamical thermal modulus of elasticity; and $\lambda(\omega)$ – dynamical coefficient of heat conductivity.

The microscopic formulas for $Z(\omega)$ and $\lambda(\omega)$, which describe the thermoelastic properties of MF in a wide interval of variations of the thermodynamical parameters and the frequencies were obtained in [8,9]. There, the asymptotic behavior of $Z(\omega)$ and $\lambda(\omega)$ at low and high frequencies with regard for the contribution of various relaxation processes under the action of an external inhomogeneous magnetic field was studied as well.

Solving jointly Eqs. (4) and (8) for $T'(\omega, \vec{k}\,)$, we get the frequency spectrum of the heat modes in the following form:

$$\tilde{\omega}^2 = \frac{1}{\rho C_p}\tilde{Z}(\omega)k^2 + \frac{\lambda^2(\omega)}{4\rho^2 C_p^2}k^4. \tag{9}$$

If we set $\tilde{\omega} = \omega - i\gamma$, where ω is the frequency and γ is the damping coefficient of the heat modes in MF, relation (9) yields

$$\omega^2 = \frac{1}{\rho C_p}Z(\omega)k^2 + \frac{\lambda^2(\omega)}{4\rho^2 C_p^2}k^4,$$

$$\gamma = \frac{1}{2\rho C_p}\lambda(\omega)k^2. \tag{10}$$

Let us analyze the asymptotic behavior of ω and γ at low and high frequencies.

In the hydrodynamic mode, where $\omega\tau \ll 1$ ($\omega \to 0, k \to 0$), $Z(\omega) = \Lambda\omega^{3/2}$ and $\lambda(\omega) = \lambda - \Lambda\omega^{1/2}$ [8, 9]. Hence, relation (10) yields

$$\omega = (\Lambda/\rho C_p)^2 k^4 \to 0,$$

$$\gamma = (1/2\rho C_p)\lambda k^2.$$

In the low-frequency limit, the heat waves decay proportionally to the square of the wave number k. The damping of the heat modes depends on the heat conductivity coefficient.

In the limit of high frequencies, where $\omega\tau \gg 1$ ($\omega \to \infty$), $Z = Z_\infty$ and $\lambda(\omega \to \infty) \sim (Z_\infty/\omega) \to 0$, relation (10) gives

$$\omega^2 = (1/\rho C_p)Z_\infty k^2,$$

$$\gamma = (1/2\rho C_p)(Z_\infty/\omega)k^2 \to 0,$$

where Z_∞ – high-frequency thermal modulus of elasticity. In the high-frequency limit, the frequency spectrum of the heat modes depends only on the high-frequency thermal modulus of elasticity.

According to [3], the heat transfer at high frequencies has the wave nature. Therefore, we neglect the second

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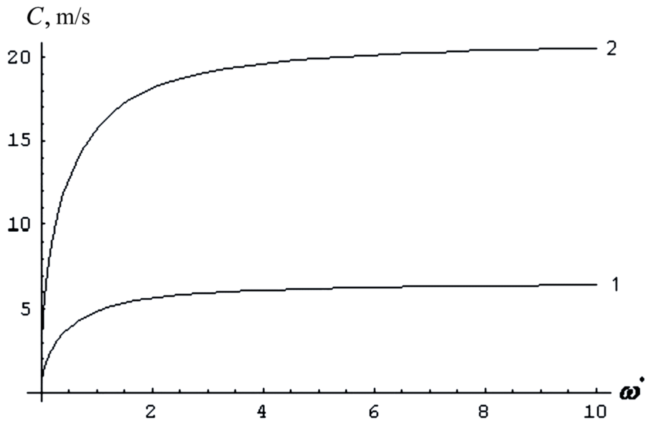

Fig. 1. Velocity of heat waves versus the reduced frequency at *1* – $|\vec{\nabla}H| = 10^2$ A/m^2, *2* – $|\vec{\nabla}H| = 10^3$ A/m^2

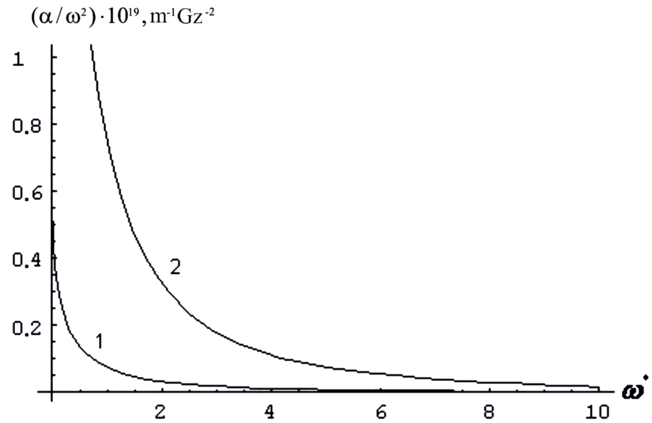

Fig. 2. Coefficient of absorption of heat waves versus the reduced frequency at *1* – $|\vec{\nabla}H| = 10^2$ A/m^2, *2* – $|\vec{\nabla}H| = 10^3$ A/m^2

term in the first relation in (10). By solving this equation for k with regard for the relation $k = k_0 - i\alpha$, we obtain the square of the high-frequency velocity and the absorption coefficient of heat waves per wavelength in MF:

$$C^2 = (\rho C_p)^{-1} Z(\omega),$$

$$\alpha(\omega) = (\omega^2/2\rho\, C_p C_0^3)\lambda(\omega). \tag{11}$$

According to (11), the velocity of heat waves depends on the thermal modulus of elasticity $Z(\omega)$, whereas the absorption coefficient depends on the heat conductivity coefficient $\lambda(\omega)$.

To obtain a clearer representation of the dependence of C and α on the frequency, we performed the numerical calculations by the example of MF fabricated on the basis of kerosene and solid particles of magnetite Fe_3O_4. According to (11), $C(\omega)$ and $\alpha(\omega)$ are determined in terms of $Z(\omega)$ and $\lambda(\omega)$ and depend on both the interaction potential of particles $\Phi(r)$ and the equilibrium distribution function $g(r)$, whose explicit forms are determined by the choice of a model of MF.

Following [10], we chose the radial distribution function in the form

$$g(r, n, T) = y(r, \rho^*) \exp\left[-\frac{\Phi(r)}{kT}\right], \tag{12}$$

where $\rho^* = \frac{\pi}{6} n\sigma^3 = \frac{\pi}{6}\frac{\rho\sigma^3 N_0}{\mu}$ – reduced density of MF; N_0 – Avogadro number; and μ – molar mass.

Assuming the dipole-dipole interaction of magnetite particles to be weak, we choose, according to the model of MF proposed in [11], the interparticle interaction potential as

$$\Phi(|\vec{r}_{ij}|) = \Phi_j^H + \Phi^S(|r_{ij}|), \tag{13}$$

where $\Phi_j^H = -\frac{kTh(\vec{u}\vec{H})}{H}$ – potential interaction energy of the particle j with the external magnetic field $\vec{H}$ directed in parallel to the vector of the magnetic moment of particles $\vec{u}$; $h = \mu_0 mH/kT$ – Langevin parameter, m – magnetic moment of a magnetite particle, μ_0 – magnetic permeability of vacuum; $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ – vector of a relative displacement of the particles i and j; and $\Phi^S(r) = 4\varepsilon\left(r^{-12} - r^{-6}\right)$ – Lennard-Jones potential, where $r = |\vec{r}| = |\vec{r}_{ij}|/\sigma$ – dimensionless mutual distance. The value of ε determines the depth of the potential well defined by $\Phi(r)$. In the presence of an orienting magnetic field, it was evaluated as 0.9 kT [12].

Using relations (11) with regard for (12) and (13), we performed the numerical calculation of the velocity dispersion and the absorption coefficient of heat waves per wavelength for MF fabricated on the basis of kerosene and particles of magnetite Fe_3O_4, at $T = 298$ K, the MF density $\rho = 1340$ kg/m^3, concentration $\varphi = 0.12$, and $|\vec{\nabla}H| = 10^2 - 10^3$ A/m^2. The values of the concentration and the density are taken from [12]. The results of numerical calculations are given in Figs. 1 and 2.

In Fig. 1, we give $C(\omega)$ versus the reduced frequency $\omega^* = \omega\tau_1$ in the range $\omega^* = 10^{-7} \div 10$ (e.g., if $\tau_1 = 10^{-12}$ s, $\omega = 10^5 \div 10^{13}$ Hz).

According to Fig. 1, the region of the velocity dispersion $C(\omega)$ of heat waves in MF is very narrow and corresponds to that for classical fluids [13]. This result agrees qualitatively with those in [1, 2]. As seen from Fig. 2, the value of α/ω^2 sharply decreases at low reduced frequencies and tends to zero at high frequencies, which corresponds to results in [1, 2, 13].

Thus, the qualitative coincidence of the results of numerical calculations with the experimental data [1, 2] and the obtained order of the velocity and the absorption coefficient in MF confirm the correctness of the obtained formulas characterizing the thermoelastic properties of MF.

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ДО МОЛЕКУЛЯРНО-КІНЕТИЧНОЇ ТЕОРІЇ ПОШИРЕННЯ ВИСОКОЧАСТОТНИХ ТЕПЛОВИХ ХВИЛЬ В МАГНІТНИХ РІДИНАХ

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Р е з ю м е

На основі молекулярно-кінетичної теорії досліджено частотні залежності швидкості та поглинання теплових хвиль під впливом неоднорідного магнітного поля. Знайдено динамічні вирази для швидкості та коефіцієнта поглинання теплових хвиль. Досліджено їх асимптотичну поведінку при низьких і високих частотах з врахуванням внеску структурної релаксації. Проведено чисельні дослідження на прикладі магнітної рідини на основі гасу і твердих частинок магнетиту Fe_3O_4. Результати чисельних розрахунків добре узгоджуються з теоретичними результатами для простих рідин.