NONLINEAR DYNAMICS OF A UBITRON OSCILLATOR WITH DELAYED FEEDBACK

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UDC 535

The theory of a ubitron oscillator with delayed feedback has been built. The starting currents of the oscillator have been determined. The generation modes have been studied at various magnitudes of the relativistic electron beam current.

1. Introduction

Much attention has been given recently to powerful microwave generators, where coaxial electrodynamic structures (EDSs) are applied. Interest in the oscillators of this type is caused by a number of their advantages as compared with microwave oscillators constructed on the basis of cylindrical EDSs. First of all, the limiting vacuum current is substantially larger in coaxial EDSs than that in cylindrical ones [1]. This circumstance allows the output microwave power to be elevated, provided that the dimensions of a microwave generator are the same. Among various coaxial microwave oscillators (MILO, vircators, carcinotrons, gyrotrons, and others), coaxial ubitrons based on permanent magnets are distinguished advantageously [2–6]. A periodic magnetic field in such devices is applied to fulfil two functions. It is used, on the one hand, for the radial focusing of a high-power relativistic electronic beam (REB) and, on the other hand, for the microwave excitation.

Two schemes of a ubitron oscillator are possible. In the first one, the microwave generation occurs on the back wave. In such an oscillator, the feedback is distributed. In the second scheme, the self-excited oscillator includes a nonlinear ubitron amplifier and an external circuit for linear (or nonlinear) feedback. This circuit is used to supply the signal of the nonlinear amplifier output to its input. This work deals with the study of the nonlinear dynamics of a ubitron oscillator with external feedback. Note that the results of the theory of lumped and distributed oscillators with external feedback are expounded, for instance, in monography [7].

2. Physical Model. Basic Equations

The electrodynamic system of a nonlinear ubitron amplifier is a piece of the coaxial line with the internal and external radii a and b, respectively. A thin-walled tubular REB is injected into the coaxial line. The coaxial magnetic undulator consists of two systems of periodically located permanent ring magnets with longitudinal magnetization. The first periodic magnetic system is located in the central metal pipe of the coaxial line, and the second one envelops the external metal pipe. Between permanent magnets are located ring polar tips fabricated of a soft magnetic material, e.g., iron. The internal and external magnetic systems are so oriented that two permanent magnets with oppositely directed magnetizations are located facing each other. A signal from the output of the nonlinear ubitron amplifier is applied to its input through an external feedback circuit. Let the thickness of ring magnets be equal to $L_m = L_w/4$, where L_w is the period of the magnetic system. In addition, let the strengths of magnetic fields produced by ring magnets be identical and equal to H_w. Then, the components of the undulator magnetic field are described by the relations [3,5,6]

$$H_z = H_w \sum_{n=1}^{\infty} [a_n \sin(nq_w\xi) + b_n \cos(nq_w\xi)] F_n^{(0)},$$

$$H_r = -H_w \sum_{n=1}^{\infty} [a_n \cos(nq_w\xi) - b_n \sin(nq_w\xi)] F_n^{(1)}, \qquad (1)$$

where $a_n = \dfrac{\cos(\pi n) - 1}{\pi n}$, $b_n = \dfrac{2}{\pi n}\sin\left(\dfrac{\pi n}{2}\right)$, $\xi = z/a$, $q_w = k_w a$, $k_w = 2\pi/L_w$, $\rho = r/a$, $F_n^{(0)}(nq_w\rho) = f_n I_0(nq_w\rho) - g_n K_0(nq_w\rho)$, $F_n^{(1)}(nq_w\rho) = f_n I_1(nq_w\rho) + g_n K_1(nq_w\rho)$, $g_n = \dfrac{I_0(nq_w s) + I_0(nq_w)}{\Delta_n}$, $f_n = \dfrac{K_0(nq_w s) + K_0(nq_w)}{\Delta_n}$, $\Delta_n = I_0(nq_w)K_0(nq_w s) - I_0(nq_w s)K_0(nq_w)$, $s = b/a$, and $I_0(x)$ and $K_0(x)$ are the modified cylindrical functions. The numerical analysis of the expressions for components of magnetic field (1) showed that the longitudinal component of the magnetic field H_z changes its sign, if the radius varies within the interval $b > r > a$, and becomes zero approximately at the middle point of the coaxial line $r = (a+b)/2$. The radial component H_r has an extremum at this point (a maximum or a minimum, depending on the longitudinal coordinate value). Note that the transportation of a tubular beam in coaxial magnetic undulators with such a design was considered in works [5,6]. The beam electrons oscillate along the azimuthal direction. Therefore, the REB excites electromagnetic waves of the TE type. For symmetric waves of this type, the dependences of longitudinal wave numbers on the frequency ω look like

$$k_n(\omega) = \sqrt{k_0^2 - \frac{\lambda_n^2}{a^2}}, \qquad (2)$$

where $k_0 = \omega/c$, c is the speed of light, λ_n is a root of the transcendental equation $J_1(\lambda_n s)N_1(\lambda_n) - J_1(\lambda_n)N_1(\lambda_n s) = 0$, $J_1(x)$ and $N_1(x)$ are the Bessel and Neumann functions, respectively, and $s = b/a$. In particular, for $s = 2$, we have $\lambda_1 = 3.2$ and $\lambda_2 = 6.31$. The electron beam is parametrically synchronous with the electromagnetic wave, provided that the condition

$$\omega = (k_n(\omega) + k_w)v_0, \qquad (3)$$

where v_0 is the longitudinal velocity of the beam, holds true.

For the sake of simplicity, we neglect the beam thickness. Then, the dynamics of the oscillator with external feedback is described by a system of nonlinear equations. The system includes a non-stationary equation for the electromagnetic wave amplitude excitation,

$$\frac{dC}{d\xi} + \frac{dC}{d\tau} = -i\mu G \frac{1}{2\pi}\int_0^{2\pi} d\theta_0 \frac{\exp(i\theta)}{p_z}, \qquad (4)$$

and the equation of motion for beam electrons in the Lagrange variables,

$$p_z\frac{dp_z}{d\xi} = -i\frac{h_w}{4q_w\beta_0}GCe^{-i\theta} - i\frac{q_0\gamma_0}{p_0^2}\frac{I_b}{I_A}Qe^{-i\theta}\rho_\omega + \text{c.c.},$$

$$p_z\frac{d\gamma}{d\xi} = -\frac{ih_w}{4q_w}GCe^{-i\theta} - i\frac{q_0}{p_0}Q\frac{I_b}{I_A}e^{-i\theta}\rho_\omega + \text{c.c.},$$

$$\frac{d\theta}{d\xi} = q_0\left(\frac{\gamma}{p_z} - \frac{\gamma_0}{p_0}\right), \qquad (5)$$

where $\theta = \omega t_l - (k_n + k_w)z$ is the phase coordinate, $t_l(t_0, z)$ the Lagrange time, t_0 the time of electron entering into the interaction region, z the longitudinal coordinate, v_g the group speed, $\tau = (v_g/a)(1 - v_g/v_0)^{-1}(t - z/v_0)$, $\beta_0 = v_0/c$, $q_0 = k_0 a$, $q_n = k_n a$, γ is the relativistic factor of the particle, γ_0 is its initial value, $p_0 = \beta_0\gamma_0$, $C = eaA_n/mc^2$ is the dimensionless amplitude, A_n the dimensional amplitude, $\mu = \dfrac{k_0 a^2 h_w I_b}{q_w k_n N_n I_A}$, $h_w = \dfrac{2\sqrt{2}eaH_w}{\pi mc^2}$, I_b is the beam current, $I_A = mc^3/e = 17$ kA,

$$N_n = \int_a^b r dr \Phi_n^2(r)$$

is the wave norm,

$$F_1(g_w\rho_b) \equiv F_1^{(1)}(q_w\rho_b),$$

$$\Phi_n(\rho) = J_1(\lambda_n\rho) - \frac{J_1(\lambda_n)}{N_1(\lambda_n)}N_1(\lambda_n\rho),$$

$$G = \Phi_n(\rho_b)F_1(q_w\rho_b), \qquad (6)$$

$\rho_b = r_b/a$, r_b is the beam radius,

$$Q = I_0(\sigma\rho_b)K_0(\sigma) - I_0(\sigma)K_0(\sigma\rho_b)\times$$

$$\times\frac{I_0(\sigma\rho_b)K_0(\sigma s) - I_0(\sigma s)K_0(\sigma\rho_b)}{I_0(\sigma)K_0(\sigma s) - I_0(\sigma s)K_0(\sigma)}, \qquad (7)$$

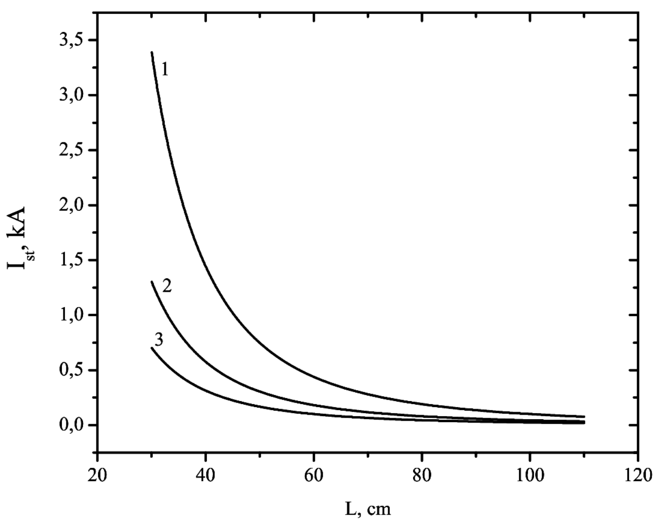

Fig. 1. Dependences of the starting current on the oscillator length: $H_w = 2$ (*1*), 3 (*2*), and 4 kOe (*3*). The oscillator parameters are $a = 2$ cm, $b = 4$ cm, $r_b = 3$ cm, $L_w = 3.92$ cm, $R = 0.3$

$$\sigma = q_0/p_0, \quad \rho_\omega = \frac{1}{2\pi}\int\limits_0^{2\pi} d\theta_0 \exp(i\theta), \quad \theta_0 = \omega t_0.$$

The self-consistent system of Eqs. (4) and (5) is to be appended by the boundary conditions for the equations of motion,

$$\theta(\xi = 0) = \theta_0, \quad \frac{d\theta}{d\xi}(\xi = 0) = 0, \tag{8}$$

and the equation for wave amplitude feedback [8],

$$C(\tau, 0) = Re^{i\triangle\varphi}C(\tau - T_d, l_g), \tag{9}$$

where R is the amplitude transfer ratio, $\triangle\varphi$ the phase shift of the signal in the feedback, $l_g = L/a$, L is the oscillator length, $T_d = l_g\dfrac{v_0/\bar{v}_g - 1}{v_0/v_g - 1}$, and $\bar{v}_g$ is the group velocity in the feedback circuit. The system of Eqs. (4) and (5) yields

$$|C(\xi = l_g, \tau)|^2 - |C(\xi = 0, \tau)|^2 + \frac{\partial}{\partial\tau}\int\limits_0^{l_g} d\xi|C(\xi, \tau|^2 +$$

$$+\Lambda\frac{1}{2\pi}\int\limits_0^{2\pi}(\gamma(\xi = l_g, \tau, \theta_0) - \gamma_0) = \text{const}, \tag{10}$$

which reflects the conservation law of energy. Here, $\Lambda = 4\dfrac{\omega a^2 I_b}{k_n c N_n I_A}$. The first two summands on the left-hand side of Eq. (10) make allowance for the difference between the energy fluxes at the output and input of the oscillator, the third term describes the variation of energy in the oscillator volume. In the stationary mode of generation, this term equals zero. At last, the last summand is a difference between the REB kinetic energy fluxes at the output and input of the oscillator.

3. Starting Currents of Self-Excited Oscillator

The frequency spectrum of the excited oscillations is determined by the transcendental equation

$$(1 - Re^{-i\triangle T_d + i\delta_1 l_g})\frac{\delta_3 - \delta_2}{(\delta_2^2 + P)(\delta_3^2 + P)} +$$

$$+(1 - Re^{-i\triangle T_d + i\delta_2 l_g})\frac{\delta_1 - \delta_3}{(\delta_3^2 + P)(\delta_1^2 + P)} +$$

$$+(1 - Re^{-i\triangle T_d + i\delta_3 l_g})\frac{\delta_2 - \delta_1}{(\delta_1^2 + P)(\delta_2^2 + P)} = 0, \tag{11}$$

where $\delta_i (i = 1, 2, 3)$ are the roots of cubic equation

$$(\delta + \triangle)(\delta^2 - K_b^2) + \Pi A(\delta + \frac{q_0}{p_0\gamma_0}) = 0, \tag{12}$$

$P = Q\dfrac{I_b}{I_A p_0^5}$, $A = \dfrac{h_w G}{4q_w p_0^4 \beta_0}$, $K_b^2 = \dfrac{q_0^2}{p_0^5}\dfrac{I_b}{I_A}Q$, and $\Pi = \mu G$. When deriving Eq. (11) for the spectrum, we assumed that there is no beam modulation at the nonlinear amplifier input and the perturbation of the longitudinal velocity of electrons in the beam is equal to zero. In addition, we used relation (9) for the feedback circuit. Equation (11) is valid, if there is no phase shift in the feedback circuit.

The starting currents of the ubitron oscillator were obtained by solving numerically the system of Eqs. (11) and (12). In so doing, we fixed the following oscillator parameters: the internal radius of the coaxial line $a = 2$ cm, its external radius $b = 4$ cm, the period of the undulator magnetic field $L_w = 3.92$ cm, the beam energy $U = 490$ keV, the beam radius $r_b = 3$ cm, and $T_d = 1.5l_g$. For those values of magnetic undulator parameters and the REB energy, the microwave generation frequency is $f = 7.94$ GHz.

Figure 1 demonstrates the dependences (in dimensional units) of the starting current on the oscillator length at various magnetic field strengths. As the oscillator length increases, the starting current

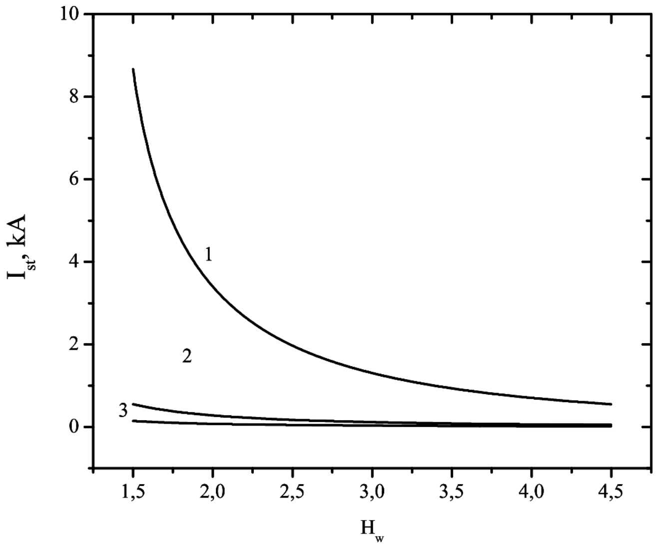

Fig. 2. Dependences of the starting current on the magnetic field strength: L = 30 (*1*), 50 (*2*), and 70 cm (*3*). The oscillator parameters are a = 2 cm, b = 4 cm, r_b = 3 cm, H_w = 3 kOe. $R = 0.3$

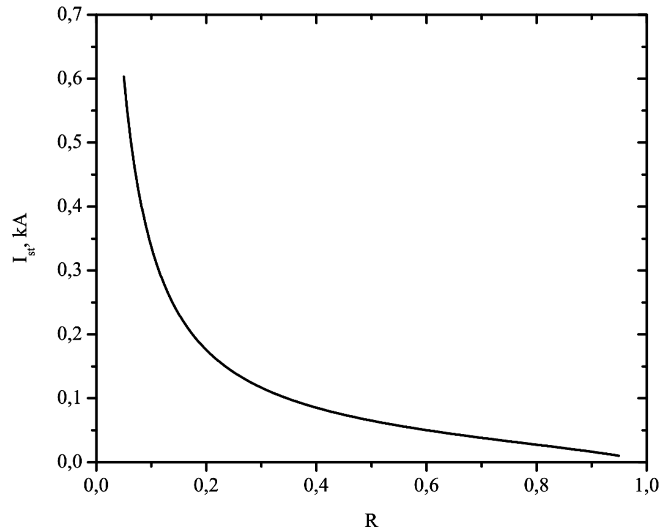

Fig. 3. Dependence of the starting current on the transfer ratio R. The oscillator parameters are a = 2 cm, b = 4 cm, r_b = 3 cm, $L = 70$ cm, $H_w = 4$ kOe

diminishes. In Fig. 2, the dependences of the starting current on the undulator magnetic field strength are depicted for three values of the oscillator length. As the strength of the magnetic field grows, the starting current also decreases. Figure 3 illustrates the functional dependence of the starting current on the coefficient of field transfer through the feedback circuit. As the transfer ratio R increases, the starting current decreases and vanishes at $R = 1$.

4. Results of Numerical Calculations

The solutions for the system of equations (4) and (5) with the initial and boundary conditions (8) and (9), respectively, were sought for the magnetic undulator and REB parameters given in the previous section, the strength of undulator magnetic field H_w =3 kOe, and various magnitudes of the beam current. We set the undulator length L = 70 cm and the transfer ratio $R = 0.3$.

The results of numerical calculations revealed that, depending on the electron beam current magnitude, the ubitron oscillator demonstrates different modes of microwave generation. If the electron beam current I_b = 0.2 kA, the stationary generation mode takes place. Figure 4 illustrates the time dependence of the wave amplitude $|C|$ at the output end of the ubitron oscillator ($\xi = l_g$). One can see that the wave amplitude grows

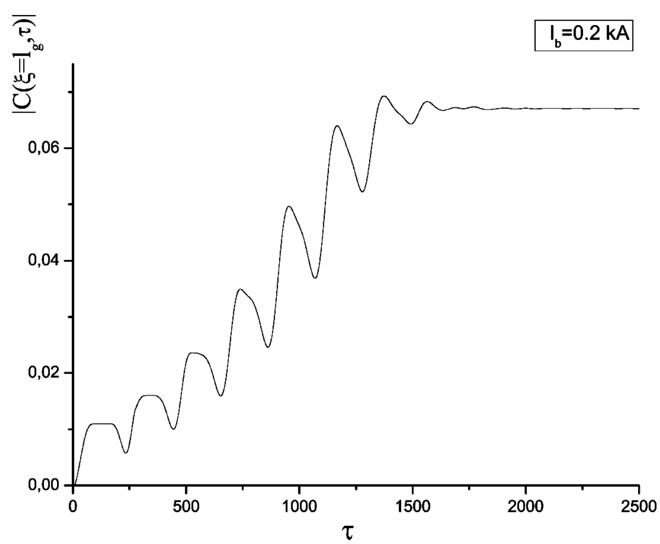

Fig. 4. Time dependence of the oscillation amplitude of $|C(\xi = l_g, \tau)|$ at the output end of the oscillator at $I_b = 0.2$ kA

nonmonotonously in time at the linear stage of generation [9]. The observed oscillations of the wave amplitude stems from the inhomogeneity of the field distribution along the coaxial system due to the presence of the beam. Such a non-uniform distribution of the field with a pronounced minimum circulates from the input to the output of the oscillator, which stimulates a nonmonotonous growth of the wave amplitude in time. At the nonlinear stage, after the transient processes terminate, the amplitude reaches a stationary value. In

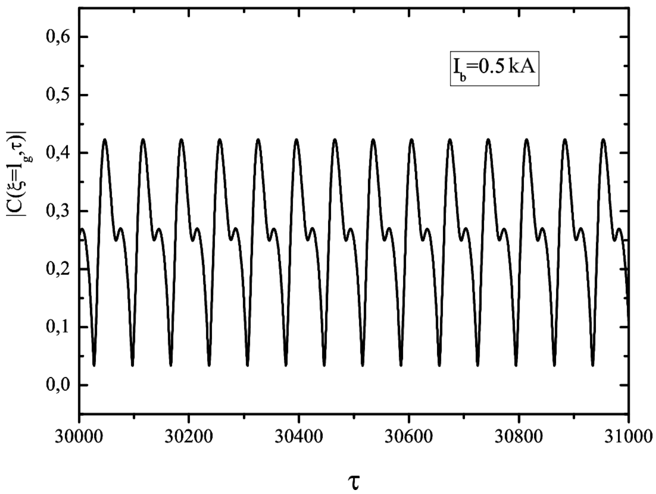

Fig. 5. The same as in Fig. 4, but for $I_b = 0.5$ kA

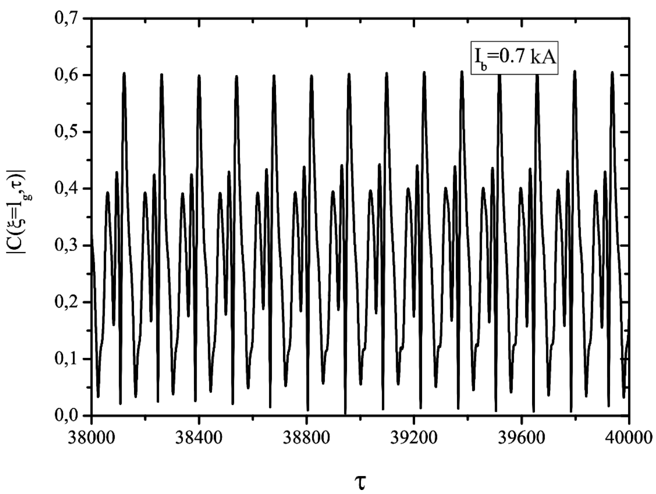

Fig. 6. The same as in Fig. 4, but for $I_b = 0.7$ kA

the stationary mode, the amplitude is distributed asymmetrically along the system. As the beam current increases above $I_b = 0.225$ kA, the stationary generation mode is quenched, and the mode of regular amplitude self-modulation is established. The frequency spectrum of the power distribution was calculated on the logarithmic scale by the formula

$$S(\omega) = \lg \frac{S_f(\omega)}{S_{\max}} + 4, \tag{13}$$

where $S_f(\omega)$ is the frequency spectrum on the linear scale, and $S_{\max}$ is the $S_f(\omega)$-maximum. For the current $I_b = 0.3$ kA, the stationary self-modulation oscillations are practically harmonic. The increase of the current up to 0.5 kA gives rise to a more complicated law of self-

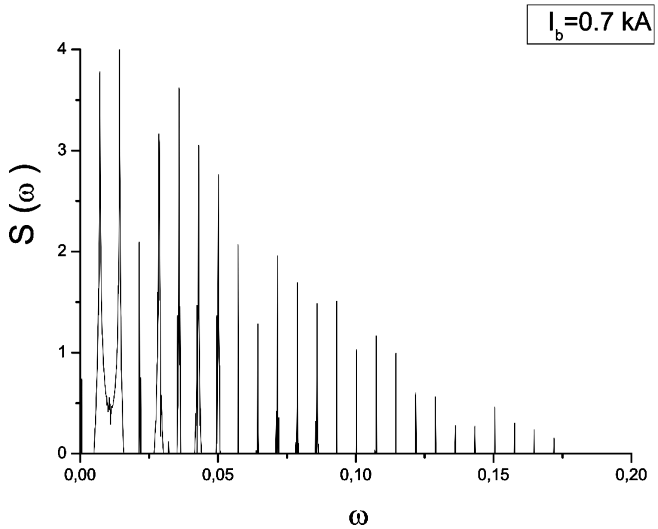

Fig. 7. Frequency spectrum of the self-modulation oscillation power, $I_b = 0.7$ kA

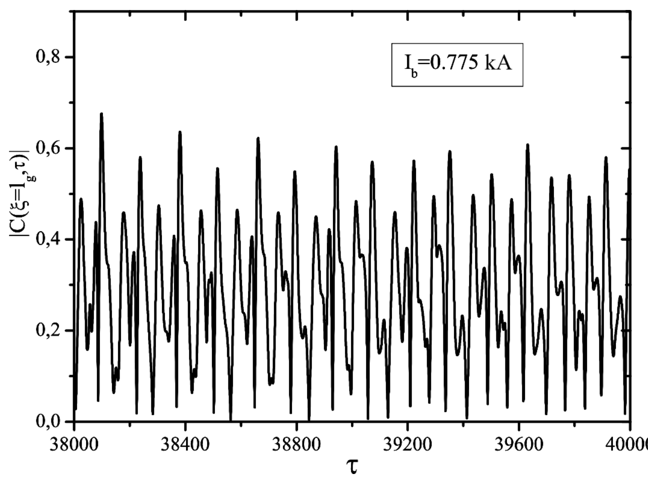

Fig. 8. Time dependence of the oscillation amplitude, $I_b = 0.775$ kA

modulation (Fig. 5). The frequency spectrum contains intensive multiple harmonics. Figures 6 and 7 illustrate the dynamics of microwave generation at the beam current $I_b = 0.7$ kA. One can see that a period-doubling bifurcation took place. The spectrum is enriched with multiple harmonics. At last, at the current $I_b = 0.775$ kA, the mode of chaotic self-modulation is established (Figs. 8 and 9). The frequency spectrum possesses a continuous component which is a background for peaks that correspond to multiple harmonics. The autocorrelation function first drops down rapidly and then oscillates. The latter mode corresponds to the lines in the frequency spectrum.

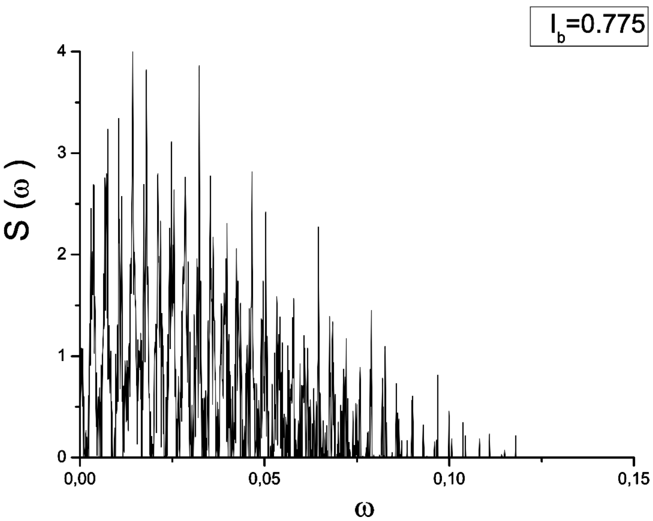

Fig. 9. The same as in Fig. 7, but for $I_b = 0.775$ kA

Let us discuss an issue concerning the ubitron oscillator efficiency. We confine ourselves to the case of the stationary generation mode. Let us determine the oscillator efficiency η as a ratio between the microwave power given by the oscillator and the initial power of the beam. Then $\eta = \dfrac{1-R^2}{\Lambda(\gamma_0 - 1)}|C(\xi = l_g)|^2$. The increase of the electron beam current is accompanied by the efficiency growth. For instance, the efficiency grows from $\eta = 13.2\%$ at the current $I_b = 0.15$ kA, to $\eta = 21.1\%$ at $I_b = 0.2$ kA, and to $\eta = 23.4\%$ at $I_b = 0.25$ kA.

5. Conclusions

In this work, the nonlinear dynamics of the microwave generation in a coaxial ubitron oscillator with external delayed feedback has been studied. We have obtained a self-consistent system of nonlinear equations which describes the process of generation of electromagnetic oscillations in such an oscillator. As a first step of the theory, we have studied the dependences of the starting current on the oscillator parameters, such as its length, the strength of the undulator magnetic field, and the transfer ratio. The growth of each of these parameters is accompanied by a reduction of the starting generation current. The qualitative character of the microwave generation in REB-based oscillators is governed by the beam current. At relatively low beam currents, the mode of stationary microwave generation takes place. If the beam current exceeds the bifurcation threshold $I_b = 0.3$ kA, the stationary generation mode gets quenched, and the mode of wave amplitude self-modulation is established in the system. If the beam current grows further, a period-doubling bifurcation is observed, and the mode of chaotic oscillation generation is ultimately established. In the chaotic mode, the frequency spectrum contains a series of peaks against a continuous component background. In the stationary generation mode, the efficiency $\eta = 23.4\%$, if the current $I_b = 0.25$ kA.

1. G.V. Sotnikov and T.Yu. Yatsenko, Zh. Tekhn. Fiz. **72**, N 5, 22 (2002).
2. R.M. Phillips, IRE Trans. Electron Devices **ED-7**, 231 (1960).
3. H.P. Freud, R.H. Jackson, D.E. Pershing, and J.M. Taccetti, Phys. Plasmas **1**, 1046 (1994).
4. A.J. Balkcum, D.B. McDermott, R.M. Phillips, and N.C. Luhmann, IEEE Trans. Plasma Science **20**, 548 (1998).
5. D.B. McDermott, A.J. Balkcum, R.M. Phillips, and N.C. Luhmann, Phys. Plasmas **2**, 4332 (2002).
6. V.A. Balakirev, A.V. Borodkin, Yu.V. Tkach, and T.Yu. Yatsenko, Radiotekh. Elektron. **3**, N 5, 1 (2007).
7. Yu.I. Neimark and P.S. Landa, *Stochastic and Chaotic Oscillations* (Kluwer, Dordrecht, 1992).
8. N.I. Aizatskii and A.O. Ostrovskii, *Preprint KhFTI 91-39* (KhFTI, Kharkiv, 1991) (in Russian).
9. V.A. Balakirev, N.I. Karbushev, A.O. Ostrovskii, and Yu.V. Tkach, *Theory of Cherenkov Amplifiers and Oscillators on Relativistic Electron Beams* (Kyiv, Naukova Dumka, 1993) (in Russian).

Received 26.12.07.
Translated from Ukrainian by O.I. Voitenko

НЕЛІНІЙНА ДИНАМІКА УБІТРОН-ГЕНЕРАТОРА
ІЗ ЗАТРИМАНИМ ЗВОРОТНИМ ЗВ'ЯЗКОМ

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Р е з ю м е

Побудовано теорію убітронного генератора із зовнішнім затриманим зворотним зв'язком. Визначено стартові струми генератора. Досліджено режими генерації мікрохвиль при різних значеннях струму релятивістського електронного пучка.