SOME EXACT SOLUTIONS OF THE 2D EQUILIBRIUM EQUATIONS FOR A SMECTIC C

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A method for separation of variables in the two-dimensional (2D) equilibrium equation for smectic C with arbitrary splay and bend elastic constants is presented. The method takes advantages of the properties of analytic functions of complex variable. A number of applications of the obtained exact solutions are considered. In particular, these are the interaction between two "vortices" and between two "sources", i.e. line disclinations with unit charges, as well as between two spherical particles with identical charges in the cases of either tangential or homeotropic boundary conditions for the director at the particle surfaces.

1. Introduction

Last years, the researches of the orientational arrangement in thin freely suspended films of smectic C attract a large interest [1–5]. Such films consist of a number of smectic layers arranged in parallel to the free surface. In those layers, the average orientation of long axes of molecules is inclined by some angle θ with respect to the smectic layer normal. The projection of the average molecule orientation onto the layer plane forms a 2D field of molecular orientations which can be described by a 2D unit vector

$$\mathbf{c} = \cos\Phi \cdot \mathbf{e}_x + \sin\Phi \cdot \mathbf{e}_y, \qquad \Phi = \Phi\left(x, y\right). \tag{1}$$

The vector $\mathbf{c}$ is referred to as the $\mathbf{c}$-director of smectic C.

If the layers are not deformed across their thickness, the free elastic energy of smectic C, associated with a distortion of the $\mathbf{c}$-director field, looks like

$$F_C = \frac{L}{2}\int\limits_S \left[K_S\left(\mathrm{div}\mathbf{c}\right)^2 + K_B\left(\mathrm{rot}\mathbf{c}\right)^2\right]dS. \tag{2}$$

Here, the integration is carried out over the area S of the smectic layer, L is the film thickness, and K_S and K_B are the 2D longitudinal (splay) and transverse (bend), respectively, elastic constants. Note that relations (1) and (2), where the substitutions $\mathbf{c} \to \mathbf{n}$, $K_S \to K_{11}$, and $K_B \to K_{33}$ are made, are similar by their form to the expressions, which correspond to 2D plane deformations of the director $\mathbf{n}$ in a nematic liquid crystal [6, 7]. Therefore, to describe the equilibrium field of $\mathbf{c}$-director in smectics C theoretically, the methods developed earlier and the solutions obtained for nematics can be used. However, using these methods and solutions, one has to take into account that, unlike nematics, the directions $\mathbf{c}$ and $-\mathbf{c}$ of the director in smectics are not equivalent. This means, in particular, that no linear defects with half-integer topological charges can be observed in smectics C.

At arbitrary values of the elastic constants K_S and K_B, the equilibrium equation for smectic C (and a nematic in the 2D geometry) is a nonlinear one:

$$\Delta\Phi + \kappa\Lambda\left(\Phi\right) = 0,$$

$$\Lambda\left(\Phi\right) = \left(-\Phi_x^2 + 2\Phi_{x,y} + \Phi_y^2\right)\sin 2\Phi +$$

$$+\left(\Phi_{x,x} + 2\Phi_x\Phi_y - \Phi_{y,y}\right)\cos 2\Phi. \tag{3}$$

Here, $\kappa = (K_B - K_S)/(K_B + K_S)$, Δ is the Laplacian, and $\Phi_{\alpha,\beta} = (\partial^2\Phi/\partial\alpha\partial\beta)$. A general method to integrate this equation has not been developed yet. At the same time, there is a certain class of exact solutions for nematics, which are described by harmonic functions $\Phi = \Phi(x, y)$. Those solutions include, in particular, distortions around linear disclinations with charges $m = 1$ and 2 [7], as well as combinations of orientational distortions which are given by linear disclinations with unit charges [8]. Despite that those solutions have been known for rather a long time, the general method that would allow one to obtain all solutions of the given class in the framework of a unique procedure is still absent in the literature.

In this work, the indicated method for separation of variables in a nonlinear 2D equilibrium equation for nematics and smectics C is presented. The method is based on the application of analytical functions of the complex argument $\xi = x + iy$. In addition, a number of applications of the solutions obtained are considered. Those applications were not discussed at the description of nematics, where linear disclinations with integer topological charges – in particular, with $m = 1$ – are unstable with respect to the decay or leakage into the third dimension [6]. In smectics C, such defects are stable and experimentally observable (see, e.g., works [2, 3]).

2. Variable Separation Method

Let us obtain the solution of Eq. (3) from the class of harmonic functions that satisfy the system of equations

$$\Delta\Phi = 0, \qquad \Lambda(\Phi) = 0. \tag{4}$$

In this case, the functions $\Phi(x, y)$ will be the solutions of equilibrium equation (3) both in the two-constant approximation $K_S = K_B$ and in the general case $K_S \neq K_B$. The essence of the variable separation method, which corresponds to those solutions, is as follows.

Let us introduce new independent variables $\{\xi = x+iy$ and $\bar{\xi} = x - iy\}$ which are widely used in solving the problems of the plane theory of elasticity [9]. Then, taking into consideration that the differentiation operators look like

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \bar{\xi}}, \ \frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial \xi} - \frac{\partial}{\partial \bar{\xi}}\right)$$

in terms of the new variables, let us rewrite the general equilibrium equation (3) and express the trigonometric functions in terms of exponents. As a result, we obtain

$$2\Phi_{\xi,\bar{\xi}} + \kappa\Big[\left(\Phi_{\xi,\xi} + i\Phi_{\xi}^2\right)\exp(2i\Phi) +$$

$$+\left(\Phi_{\bar{\xi},\bar{\xi}} - i\Phi_{\bar{\xi}}^2\right)\exp(-2i\Phi)\Big] = 0. \tag{5}$$

Here, $\Phi = \Phi(\xi, \bar{\xi})$. This equation has the same nonlinear character, as the general equation of equilibrium (3) does. The first and second equations in system (4) correspond to the condition that, respectively, the first term in expression (5) and the expression in square brackets are equal to zero. We intend to seek the real-valued harmonic function $\Phi(\xi, \bar{\xi})$ as the imaginary part of an analytical function $G(\xi)$. In so doing, it is convenient to present $\Phi(\xi, \bar{\xi})$ in the form

$$\Phi(\xi, \bar{\xi}) = \frac{1}{2i}\left[G(\xi) - \bar{G}(\bar{\xi})\right] = \frac{1}{i}\ln\left(\frac{\Xi(\xi)}{\bar{\Xi}(\bar{\xi})}\right). \tag{6}$$

Expression (6) determines the angle $\Phi(\xi, \bar{\xi}) = \arg G(\xi)$ with an accuracy to a constant $2\pi k$, $k = 0, \pm1, \pm2, \ldots$. To avoid the ambiguity, we consider the function $G(\xi) = 2\ln\Xi(\xi)$ in a one-sheet domain at $k = 0$.

Expression (6) obviously satisfies the Laplace equation and gives rise to the separation of variables both in the second equation of system (4) and in general equation (3). As a result, we have

$$\frac{-i\kappa}{\Xi^2(\xi)\,\bar{\Xi}^2(\bar{\xi})}\left[\Xi^3(\xi)\frac{\partial^2\Xi(\xi)}{\partial\xi^2} - \bar{\Xi}^3(\bar{\xi})\frac{\partial^2\bar{\Xi}(\bar{\xi})}{\partial\bar{\xi}^2}\right] = 0. \tag{7}$$

Whence it follows that

$$\Xi^3(\xi)\frac{\partial^2\Xi(\xi)}{\partial\xi^2} = \bar{\Xi}^3(\bar{\xi})\frac{\partial^2\bar{\Xi}(\bar{\xi})}{\partial\bar{\xi}^2} = -A. \tag{8}$$

Here, the constant A must be real-valued, because Eqs. (8) are valid for two complex conjugate functions, which requires that the condition $A = \bar{A}$ be obeyed.

Now, let us discuss all possible solutions $\Xi(\xi)$ of Eqs. (8) and construct the functions $\Phi(x, y)$.

3. Exact Solutions in the Class of Harmonic Functions

As will be shown below, there are only five different harmonic functions $\Phi(x, y)$ that define inhomogeneous structures of the director field. The first three solutions correspond to distortions which describe a twist around separate disclination lines [7]. In particular, at $A = 0$, Eq. (8) is satisfied by a linear function $\Xi(\xi) = B\xi - C$, where B and C are arbitrary complex constants. From here, we obtain the first solution (6), which describes a distortion around a separate disclination line with the charge $m = 2$:

$$\Phi = \frac{1}{i}\ln\left(\frac{B\xi - C}{\bar{B}\bar{\xi} - \bar{C}}\right) = -i\Big[\ln\left(\frac{\xi - \xi_0}{\bar{\xi} - \bar{\xi}_0}\right) +$$

$$+\ln\left(\frac{B}{\bar{B}}\right)\Big] = 2\mathrm{arctg}\left(\frac{y - y_0}{x - x_0}\right) + \Phi_0. \tag{9}$$

The disclination axis passes through the point $\xi_0 = C/B = x_0 + iy_0$. The disclination phase $\Phi_0 = -i\ln(B/\bar{B})$ is arbitrary.

If the constant $A \neq 0$, by multiplying Eq. (8) for the function $\Xi(\xi)$ by $\Xi^{-3}(\partial\Xi/\partial\xi)$, we obtain

$$\frac{\partial\Xi(\xi)}{\partial\xi}\frac{\partial^2\Xi(\xi)}{\partial\xi^2} + \frac{A}{\Xi^3(\xi)}\frac{\partial\Xi(\xi)}{\partial\xi} =$$

$$= \frac{\partial}{\partial\xi}\left[\frac{1}{2}\left(\frac{\partial\Xi(\xi)}{\partial\xi}\right)^2 - \frac{A}{2\Xi^2(\xi)}\right] = 0. \quad (10)$$

If the first integral of this equation equals zero, the expression $\Xi = \pm\sqrt{2\sqrt{A}\xi - E}$ is valid, where E is a complex constant. Then, if $A = (\alpha/2)^2 > 0$, we obtain the second solution

$$\Phi = \frac{1}{2i}\ln\left(\frac{\alpha\xi - E}{\alpha\bar{\xi} - \bar{E}}\right) = -\frac{i}{2}\ln\left(\frac{\xi - \xi_0}{\bar{\xi} - \bar{\xi}_0}\right) =$$

$$= \mathrm{arctg}\left(\frac{y - y_0}{x - x_0}\right), \quad (11)$$

which corresponds to a "source" located at the point $\xi_0 = E/\alpha = x_0 + iy_0$, i.e. to a plane disclination with the charge $m = 1$ and the phase $\Phi_0 = 0$.

If the constant $A = (\alpha/2)^2 < 0$, the solution is

$$\Phi = \frac{1}{2i}\ln\left(\frac{i\alpha\xi - E}{-i\alpha\bar{\xi} - \bar{E}}\right) =$$

$$= -\frac{i}{2}\ln\left(-\frac{\xi - \xi_0}{\bar{\xi} - \bar{\xi}_0}\right) = \mathrm{arctg}\left(\frac{y - y_0}{x - x_0}\right) \pm \frac{\pi}{2}, \quad (12)$$

where $\xi_0 = -iE/\alpha = x_0 + iy_0$. It describes a distortion in the form of a "vortex" that is created by a separate disclination line with the charge $m = 1$ and the phase $\Phi_0 = \pm\pi/2$. This function corresponds to the third independent solution of the system of equations (4).

Note that solutions (9), (11), and (12) correspond to simple distortions of the director field, which can be described with the help of harmonic functions. Actually, they depend only on the polar coordinate $\varphi = \arctan[(y - y_0)/(x - x_0)]$, and just this approach was used for their derivation in work [7]. More complicated solutions were demonstrated in work [8]. However, since the manner of report in work [8] was specific, the derivation procedure remained beyond the scope of discussion. Here, on the basis of the above-presented variable separation method, we can obtain those solutions in an explicit form.

Let us come back to relation (10). If its first integral is equal to a complex constant $P/2 \neq 0$, the function $\Xi(\xi)$ has the form

$$\Xi = \pm\sqrt{P(\xi - Q)^2 - (A/P)}, \quad (13)$$

where Q is another complex constant. At $A = \alpha^2 > 0$, we obtain the fourth solution

$$\Phi = \frac{1}{2i}\ln\left(\frac{P(\xi - Q)^2 - (\alpha^2/P)}{\bar{P}(\bar{\xi} - \bar{Q})^2 - (\alpha^2/\bar{P})}\right) =$$

$$= -\frac{i}{2}\ln\left(\frac{(\xi - \xi_1)(\xi - \xi_2)}{(\bar{\xi} - \bar{\xi}_1)(\bar{\xi} - \bar{\xi}_2)}\right) + i\ln\left(\frac{\xi_1 - \xi_2}{|\xi_1 - \xi_2|}\right) =$$

$$= \mathrm{arctg}\left[(y - y_1)/(x - x_1)\right] +$$

$$+ \mathrm{arctg}\left[(y - y_2)/(x - x_2)\right] - \Phi_0,$$

$$\xi_1 = Q + (\alpha/P) = x_1 + iy_1,$$

$$\xi_2 = Q - (\alpha/P) = x_2 + iy_2,$$

$$\Phi_0 = \mathrm{arctg}\left[(y_1 - y_2)/(x_1 - x_2)\right]. \quad (14)$$

Function (14) corresponds to a distortion created in smectic C by two "sources", i.e. two plane disclinations with charges $m_1 = m_2 = 1$ which are located at the points ξ_1 and ξ_2. In this case, the total phase Φ_0 of disclinations is defined in the plane ξ by a direction that passes through the indicated positions of the disclination lines ξ_1 and ξ_2 (see Fig. 1,*a*).

The fifth solution is also found making use of function (13) at $A = (i\alpha)^2 < 0$. As a result, we have

$$\Phi = \frac{1}{2i}\ln\left(\frac{P(\xi - Q)^2 + (\alpha^2/P)}{\bar{P}(\bar{\xi} - \bar{Q})^2 + (\alpha^2/\bar{P})}\right) =$$

$$= -\frac{i}{2}\ln\left(-\frac{(\xi - \xi_1)(\xi - \xi_2)}{(\bar{\xi} - \bar{\xi}_1)(\bar{\xi} - \bar{\xi}_2)}\right) + i\ln\left(\frac{\xi_1 - \xi_2}{|\xi_1 - \xi_2|}\right) =$$

$$= \mathrm{arctg}\left[(y - y_1)/(x - x_1)\right] +$$

$$+ \mathrm{arctg}\left[(y - y_2)/(x - x_2)\right] - \Phi_0,$$

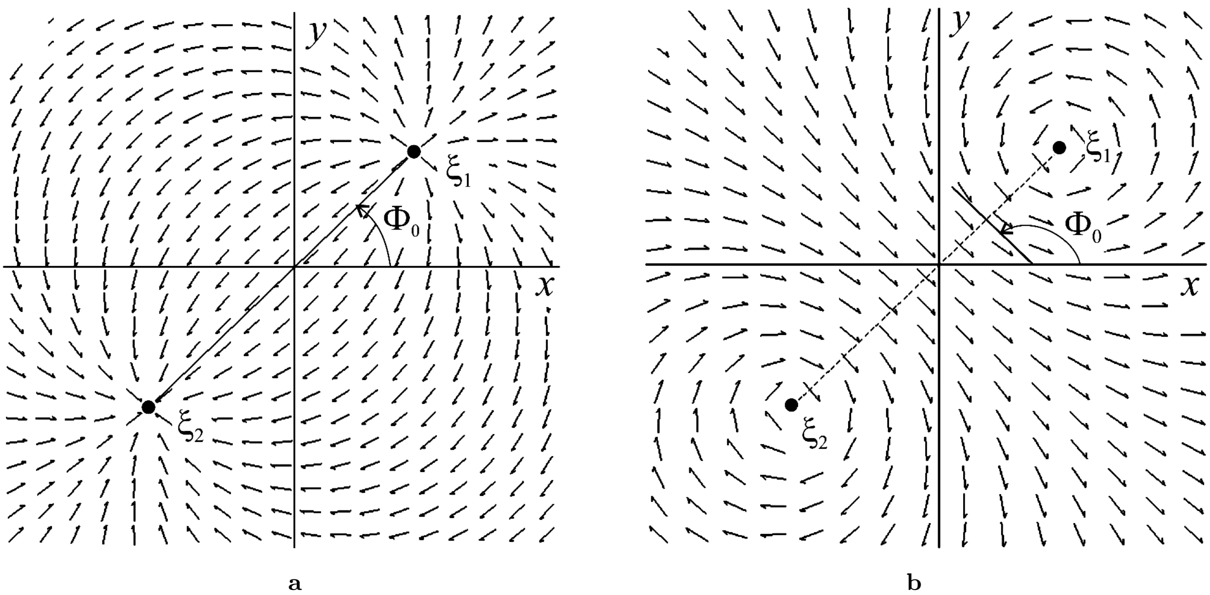

Fig. 1. Distributions of the **c**-director for exact solutions: (*a*) solution (14) for two "sources" and (*b*) solution (15) for two "vortices"

$$\xi_1 = Q + (i\alpha/P) = x_1 + iy_1,$$

$$\xi_2 = Q - (i\alpha/P) = x_2 + iy_2,$$

$$\Phi_0 = \mathrm{arctg}\,[(y_1 - y_2)/(x_1 - x_2)] \pm (\pi/2)\,. \quad (15)$$

This solution differs from solution (14) by the phase Φ_0, which is now defined by a direction that is perpendicular to a straight line connecting the points ξ_1 and ξ_2. Such a difference generates a new configuration of the director field, which corresponds to two "vortices" (see Fig. 1,*b*). This structure is the last among possible distortions of the director field which are described by harmonic functions.

4. Application of Exact Solutions

Below, we consider – in general formulation ($K_S \neq K_B$) and making use of solutions (14) and (15) – a number of problems which were solved for 2D nematics only in the two-constant approximation [6].

Application 1. Interaction between two "vortices". Let us find the energy and the force of interaction between two linear defects with the charges $m_1 = m_2 = 1$ and the phase $\Phi_0 = \pi/2$. Let the distance between disclinations be equal to $d = 2h$, and let they be located at points with coordinates $(h, 0)$ and $(-h, 0)$. Then, the function $\Phi(x, y)$ obtained from expression (15) looks like

$$\Phi = \mathrm{arctg}\,(y/(x-h)) + \mathrm{arctg}\,(y/(x+h)) + \pi/2. \quad (16)$$

To calculate energy (2), we use the bipolar coordinates $\{x = \sinh\tau/(\cosh\tau - \cos\sigma), y = \sin\sigma/(\cosh\tau - \cos\sigma)\}$. Then, after the integration over the specimen with dimensions $2D$, except for the regions with radius ρ near the disclination lines (nuclei), and in the approximation $\rho \ll d \ll D$, we obtain

$$F = \frac{L}{2}\iint \frac{K_B \sinh^2\tau + K_S \sin^2\sigma}{(\cosh\tau - \cos\sigma)^2}\, d\tau d\sigma =$$

$$= 2\pi K_B L \ln\frac{D}{\rho} + 2\pi K_S L \ln\frac{D}{d}. \quad (17)$$

Here, the first term on the right-hand side corresponds to the own energy of two separate "vortices", and the second one to the energy of their interaction. The interaction force is found by differentiating expression (17) with respect to d. We obtain

$$f = -\frac{\partial F}{\partial d} = \frac{2\pi K_S L}{d}, \quad (18)$$

which corresponds to a repulsion between linear defects ($f < 0$).

Hence, in contrast to the two-constant approximation [6], we obtained a rather interesting result: for two "vortices", the own energies of which depend only on the bend elastic constant K_B, the energy and the force of their interaction (repulsion) depend only on the splay elastic constant K_S.

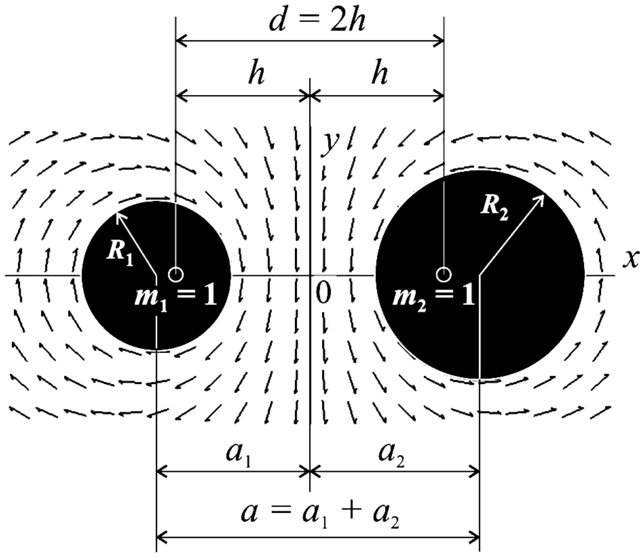

Fig. 2. Distribution of the **c**-director around spherical droplets under tangential boundary conditions

Application 2. Interaction between a "vortex" and a hard wall.

Let the plane of a hard wall or a hard membrane boundary of smectic C, at which the planar boundary conditions for the **c**-director are given, coincide with the plane $(x0z)$. One of the linear defects described in Application 1 is in the liquid crystal bulk, and the other is fictitious. Then, integrating expression (2) over the half-space occupied by the liquid crystal, we obtain – by analogy with formula (17) – that

$$F = \pi K_B L \ln \frac{D}{\rho} + \pi K_S L \ln \frac{D}{h}. \tag{19}$$

In this case, the force of disclination repulsion from the wall $f = -\partial F/\partial h$ coincides with expression (18), i.e. depends only on the splay elastic constant K_S.

Application 3. Interaction between two particles with charges $q_1 = q_2 = 1$: tangential fixation of the director at particles.

At temperatures close to the point of transition of smectic C into a less ordered liquid crystal phase (a nematic [4] or a cholesteric [5]), nuclei of the new phase are formed in the form of spherical droplets in membranes of smectic C. The droplet diameter registered in experiments [4, 5] ranged from 5 to 40 μm for the film thickness L less than or of the order of 0.5 μm. If a distortion of smectic layers near the droplet is neglected, nuclei of the new phase can be considered as hard spherical particles which are described by circles of the given radius in the film plane. In every smectic layer, the determination of the **c**-director field remains a two-dimensional problem at that. In a similar way, disk-shaped inclusions or particles in smectics C can be considered. Analogs of such inclusions are formed, when a film with nuclei is cooled down to the inverse transition temperatures of the nucleus substance into the smectic phase [4]. In this case, the height of nuclei measured along the normal to the film decreases, whereas their diameter measured in the film plane increases several times and reaches 100–150 μm. The internal structure of smectic inclusions that were formed is substantially different from the liquid crystal environment. Therefore, they can be considered as extraneous objects in smectics C.

Experiments show that, depending on the way of manufacturing a membrane of smectic C, nuclei (particles of the new phase) with topological charges $q = 0$ and 1 can be formed. If $q = 0$, an individual particle and the interaction between particles with zero charge are managed to be analytically described [1, 4] only in the two-constant approximation $K_S = K_B$. If $q = 1$, a particle with, e.g., tangential boundary conditions for the **c**-director at its surface is described by simple solution (12), and the interaction between particles with $q_1 = q_2 = 1$ can be analyzed in the general case $K_S \neq K_B$.

Let two particles with radii R_1 and R_2 be in smectic C (see Fig. 2). The distance between the particle centers is equal to α. In the general case, let us assume that $R_1 \neq R_2$. To describe the director field, we use function (16). In order that this function satisfy the tangential boundary conditions for the director at the particle surfaces, the points $x = h$ and $x = -h$, through which the axes of disclination lines pass, have to be mutually conjugate [9] with respect to circles with radii R_1 and R_2. This allows one to find the positions of the particle centers $x = -a_1$ and $x = a_2$ in the selected coordinate system and the relation between the distances $d = 2h$ and $a = a_1 + a_2$:

$$a_1 = \left(R_1^2 + h^2\right)^{1/2}, \qquad a_2 = \left(R_2^2 + h^2\right)^{1/2},$$

$$d = 2h = \frac{1}{a}\left[a^2 - (R_1 + R_2)^2\right]^{1/2}\left[a^2 - (R_2 - R_1)^2\right]^{1/2}. \tag{20}$$

In the bipolar coordinates, the particle surfaces are set by the relations $\tau_1 = -\text{arcsinh}(h/R_1)$ and $\tau_2 = -\text{arcsinh}(h/R_2)$. Then, integrating in formula (17) over a specimen with dimensions $2D$, except for the regions occupied by particles, and in the approximation

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$D \gg d \approx R_1 \approx R_2$, we obtain

$$F = \pi K_B L \left(\ln \frac{D}{R_1} + \ln \frac{D}{R_2} \right) + 2\pi K_S L \ln \frac{D}{d},$$

$$f = \frac{2\pi K_S L}{d}. \quad (21)$$

It is quite natural that we have obtained an analogy with the two-constant approximation: two particles with charges $q_1 = q_2 = 1$ interact (repulse each other) as two disclinations that are located at two points conjugate with respect to the circles which represent the surfaces of both particles. However, result (21) demonstrates that, under tangential boundary conditions at the particle surfaces, the character of this interaction depends only on the splay elastic constant K_S.

Application 4. Interaction between a particle with the charge $q = 1$ and a hard wall: tangential anchoring. The solution of this problem can be obtained using the results of Application 3. Let $R_1 = R_2 = R$; let the plane $y = 0$, where the tangential boundary conditions for the director are realized, coincide with the wall plane; and let the particle be located to the left (or to the right) of the wall. Then, taking into account the fact that the integration is carried out over the half of the region that was considered in Application 3, expression (20) and (21) yield

$$F = \pi K_B L \ln \frac{D}{R} + \pi K_S L \ln \frac{D}{2h},$$

$$f = \frac{\pi K_S L}{h} = \frac{2\pi K_S L}{d}, \quad a_1 = a_2 = \left(R^2 + h^2\right)^{1/2},$$

$$d = 2h = \left[a^2 - 4R^2\right]^{1/2}, \quad (22)$$

which corresponds to a repulsion of the particle from the wall with a force proportional to K_S.

Applications 5 to 8. These applications are described by solution (14) for two "sources". They can be derived from relations (17)–(22), by swapping $K_S \leftrightarrow K_B$. Then, Applications 1 to 4 will correspond to the problems of interaction between two "sources", between a "source" and a wall, between two particles with homeotropic boundary conditions for **c**-director, and between a particle and a wall with homeotropic anchoring of the director at the particle surface and the wall. For all those problems, the force of interaction between the objects considered depends only on the bend elastic constant K_B.

Applications 9 and 10. Here, it is pertinent to recall two more applications of solutions (14) and (15) which describe planar 2D distortions of a nematic in cylinder capillaries. These are the so-called planar-polar [10] and circular planar-polar (or planar bipolar) [8, 11] structures which correspond to the homeotropic and tangential, respectively, boundary conditions for the director at the capillary walls. The given structures were described in work [8] in detail. It was shown, in particular, that two "vortices" correspond to the planar-polar configuration in the complete plane ξ, and two "sources"located on the capillary surface to the circular planar-polar one. In smectics C, similar distortions of the **c**-director field are realized, e.g., inside disk-shaped smectic inclusions [4].

5. Conclusions

The nonlinear equation of equilibrium (3) for smectics C allows various variable separation methods to be used. In this work, one of those methods is presented, which allows all possible solutions of the given equation in the class of harmonic functions to be obtained. Among those solutions, the most interesting from the practical point of view are solutions (14) and (15). They correspond to distortions created in smectics C by disclinations in the form of two "sources"and two "vortices". The indicated solutions have at least ten applications. The results of applications demonstrate that the interaction force between various objects (disclinations, spherical or disk-shaped particles) in smectics C, which are described by solution (14) in the form of two "sources", depends only on the bend elastic constant K_B. For analogous objects, which solution (15) in the form of "vortices" corresponds to, the interaction force depends, in its turn, only on the splay elastic constant K_S. Comparing our results with those available in the literature and obtained in the two-constant approximation, the conclusions made are new and can be used, for instance, for experimental estimation of the elastic constants K_S and K_B.

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ДЕЯКІ ТОЧНІ РОЗВ'ЯЗКИ 2D-РІВНЯНЬ РІВНОВАГИ СМЕКТИКА C

С.В.Бурилов

Р е з ю м е

Представлено один з методів розділення змінних в 2D-рівнянні рівноваги смектика C при довільних значеннях констант пружності поздовжнього K_S та поперечного K_B вигинів. Метод засновано на використанні аналітичних функцій комплексного змінного. Розглянуто низку застосувань отриманих точних розв'язків, у тому числі питання про взаємодію двох "вихорів" і двох "джерел", тобто лінійних дисклінацій із зарядами, рівними одиниці, а також про взаємодію двох сферичних частинок з такими ж значеннями зарядів при тангенціальних і гомеотропних граничних умовах для директора на поверхні частинок.