ABOUT THE INFLUENCE OF PLASMA PARTICLES COLLISIONS ON THE EFFECTIVE GRAIN POTENTIAL

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Kinetic solutions of the problem of the effective potential of a macroparticle charged by plasma currents are analyzed in detail. Calculations are performed taking the dependence of the ionic charging cross-section on the ion velocity into consideration. Analytical expressions for the potential are obtained for arbitrary values of ion mean free path. It is shown that the analytical approximations are in agreement with the numerical calculations, and the conditions for applications of these approximations are found.

Theoretical studies of effective grain potentials in a plasma still remain an important problem of dusty plasma theory. In view of very large electric charges, the effective grain potentials are usually studied by using the numerical solution of the appropriate nonlinear boundary-value problem or microscopic simulations. As a result of such studies, many features of effective grain potentials have been established. However, for the theoretical description of various interesting phenomena observed in dusty plasma (dusty crystal formation, dust-acoustic wave propagation, generation of nonlinear dust structures, *etc.*), it is highly desirable to have analytical relations for the effective potentials. Obviously, the most general solution of this problem can be obtained within the framework of the kinetic description of plasma processes. The advantage of such a description is that it allows one to study the effective grain potentials at arbitrary values of plasma parameters and to find the dependence of these potentials on the kinetic characteristics of plasma processes (notice that dusty plasma is an open system, and thus the effective potentials depend not only on the thermodynamic parameters, but also on the kinetic properties of a surrounding plasma). In particular, since the distance between the grains can be comparable with the ionic mean free path in the most practically important cases, the kinetic treatment is the only way to give a consistent description of the effective potentials. This explains the great interest in the kinetic solution of the problem.

For the first time, the kinetic calculations of the effective grain potentials were performed in [1, 2] using the point sink model in order to describe the electron and ion absorption by a grain. This model was primarily introduced for analytical calculations of the effective potentials in the limiting cases of the collisionless Vlasov plasmas and strongly collisional plasma [3–5]. A similar idea was also used in [6, 7] in the case of fluid plasma description. The consistent foundation of the point sink model was done in [8, 9].

In [1, 2], the general relations for the effective potentials were obtained, and the analytical estimates for strongly collisional and weakly collisional plasmas were presented. Later on, some part of results presented in [1, 2] was reproduced in [10], and the attempt to give a more detailed analysis was undertaken. However, the estimates presented in [10] were done within a rather rough approximation, namely, the velocity-dependent charging cross-section was replaced by some effective cross-section independent of the plasma particle velocity. The purpose of the present contribution is to perform analytical calculations of the effective potentials with regard to the velocity dependence of the charging cross-section.

We start from the relations for the effective potential of a stationary grain obtained in [1, 2]. Neglecting the effects of the absorption of electrons by a grain on the electron velocity distribution function (the widely used approximation), one obtains

$$\phi(r) = \frac{qe^{-k_D r}}{r} + i4\pi e_i n_i \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{kr}}}{k^2+k_D^2} \times$$

$$\times \frac{\int \frac{v\sigma(v) f_{0i}(v)}{\mathbf{kv}-i\nu_i} d\mathbf{v}}{1 + i\nu_i \int \frac{\Phi_i(\mathbf{v})}{\mathbf{kv}-i\nu_i} d\mathbf{v}} = \Phi_D(r) + \Phi_S(r). \quad (1)$$

Here, $k_D^2 = k_{Di}^2 + k_{De}^2$, $k_{D\sigma}^2 = 4\pi e_\sigma^2 n_\sigma / T_\sigma$, $\Phi_i(\mathbf{v})$ is the distribution function generated in the course of collisions (in what follows, we assume $\Phi_i(\mathbf{v})$ to be Maxwellian), $\sigma(v) = \pi a^2 (1 + v_{0i}^2/v^2)$ is the ionic charging cross-section, $v_{0i}^2 = 2|e_i q|/(a m_i)$, ν_i is the ion transport frequency, f_{0i} is the unperturbed ion velocity distribution function, a is the grain radius, and the rest of notations is traditional.

Notice that, after the integration over the angle between $\mathbf{k}$ and $\mathbf{v}$, Eq. (1) is reduced to representation (3) in [10]. However, such a representation is not suitable for the analytical calculations at small ν_i.

It turns out to be convenient to evaluate the integrals over the velocity space in Eq. (1) using the relation

$$\int d\mathbf{v} \frac{f(\mathbf{v})}{\mathbf{kv}-i\nu} = i\int_0^\infty d\tau e^{-\nu\tau} \int d\mathbf{v} f(\mathbf{v}) e^{-i\mathbf{kv}\tau}, \quad (2)$$

where $f(\mathbf{v})$ is an arbitrary regular function. So doing, we obtain

$$\Phi_S(r) = \frac{2i}{\pi} e_i n_i \int_0^\infty dx \frac{\sin x}{x^2+\xi_D^2} \frac{F(\xi/x)}{V(\xi/x)}, \quad (3)$$

where

$$F(y) = -\sigma_0 \left(\frac{d}{d\beta} - u_{0i}^2\right) \frac{1}{\sqrt{2\beta}} \times$$

$$\times \int_0^\infty dt\, e^{-yt\sqrt{2\beta}} \frac{e^{-\frac{t^2}{2}}}{t} \operatorname{erf}\left(\frac{it}{\sqrt{2}}\right)\Bigg|_{\beta=1/2}, \quad (4)$$

$$V(y) = 1 - \left(\frac{\pi}{2}\right)^{\frac{1}{2}} y\, e^{\frac{y^2}{2}} \operatorname{erfc}\left(\frac{y}{\sqrt{2}}\right), \quad (5)$$

$$\sigma_0 = \pi a^2, \quad u_{0i}^2 = \frac{2|e_i q|}{a T_i}, \quad \xi_D = k_D r,$$

$$\xi = \frac{r}{l_i} = \frac{r\nu_i}{s_i}, \quad s_i^2 = \frac{T_i}{m_i}. \quad (6)$$

Here, l_i is the ion mean free path.

In order to obtain further analytical estimates, some approximations for $F(y)$ and $V(y)$ are needed, but the difficulty is that the argument of these functions takes both small and large values in Eq. (3). Let us divide the integration domain in Eq. (3):

$$\Phi_S(r) = \frac{2i}{\pi} e_i n_i \left[\int_0^\xi dx \frac{\sin x}{x^2+\xi_D^2} \frac{F(\xi/x)}{V(\xi/x)} + \right.$$

$$\left. + \int_\xi^\infty dx \frac{\sin x}{x^2+\xi_D^2} \frac{F(\xi/x)}{V(\xi/x)} \right] = I_1 + I_2. \quad (7)$$

So, one can use the series expansion for both integrands. In the first integral, $\xi/x \gg 1$, and, in the second one, $\xi/x \ll 1$.

We start from the consideration of I_2. As is seen, the function $F(y)$ has finite value at $y = 0$, but its first derivative does not exist at $y = 0$. This means that $F(y)$ cannot be expanded in the power series in y in the vicinity of zero. In order to describe the behavior of $F(y)$ at small y, it is convenient to represent it in the form

$$F(y) = F(0) + \delta F(y), \quad (8)$$

where

$$F(0) = \frac{i\pi}{2}\sigma_0(1 + u_{0i}^2), \quad (9)$$

$$\delta F(y) = -\frac{i\sigma_0}{\sqrt{2\pi}} \left(\frac{d}{d\beta} - u_{0i}^2\right) \int_0^y dq e^{q^2\beta} \operatorname{Ei}(-q^2\beta)\Bigg|_{\beta=1/2}. \quad (10)$$

At $y \ll 1$,

$$\delta F(y) \approx -\frac{i\sigma_0}{\sqrt{2\pi}} \left(\frac{d}{d\beta} - u_{0i}^2\right) y(\gamma + \ln\beta - 2 + 2\ln y), \quad (11)$$

$$\frac{F(y)}{V(y)} \approx (F(0) + \delta F(y)) \left(1 + \left(\frac{\pi}{2}\right)^{1/2} y\right). \quad (12)$$

In I_1 of Eq. (7) at $y \gg 1$,

$$\frac{F(y)}{V(y)} \approx i\sigma_0\sqrt{\frac{2}{\pi}}(2+u_0^2)y. \tag{13}$$

In the case $\xi \ll 1$ ($r \ll l_i$), the contribution of I_1 can be neglected. For analytical calculations, it is convenient to expand the lower limit of integration in I_2 to the zero. In fact, we return to Eq. (3). However, the approximations (11), (12) break down in the extended domain, so we need to examine their behavior at large y, which corresponds to the small x in Eq. (3),

$$\frac{\sin x}{x^2+\xi_D^2}\frac{F(\xi/x)}{V(\xi/x)} = O(\ln x),\ \text{at}\ x \to 0, \tag{14}$$

it does not coincide with the correct result given by Eq. (13):

$$\frac{\sin x}{x^2+\xi_D^2}\frac{F(\xi/x)}{V(\xi/x)} = O(1),\ \text{at}\ x \to 0. \tag{15}$$

This incorrect behavior may cause a significant deviation of our analytical estimates.

Substitution of Eqs. (11), (12) into Eq. (3) gives the following estimate at $r/l_i \ll 1$:

$$\Phi_S(r) = -\frac{e_i n_i}{k_D r}\sigma_0(1+u_{0i}^2)\Big\{Q(k_D r) - \frac{\varepsilon_1}{k_D l_i}Q^+(k_D r) +$$

$$+\frac{\varepsilon_1}{k_D l_i}\left[(1-e^{-k_D r})\ln\frac{r}{l_i} + e^{-k_D r}\ln k_D r\right] +$$

$$+\frac{1}{k_D l_i}\left[\varepsilon_2(1-e^{-k_D r})+\varepsilon_3\right]\Big\}, \tag{16}$$

where

$$Q(y) = \frac{1}{2}\left[e^{-y}\mathrm{Ei}(y) - e^{y}\mathrm{Ei}(-y)\right], \tag{17}$$

$$Q^+(y) = \frac{1}{2}\left[e^{-y}\mathrm{Ei}(y) + e^{y}\mathrm{Ei}(-y)\right], \tag{18}$$

$$\varepsilon_1 = \left(\frac{2}{\pi}\right)^{1/2}\frac{u_{0i}^2}{1+u_{0i}^2} \simeq 0.8,$$

$$\varepsilon_2 = \left(\frac{\pi}{2}\right)^{3/2} - \varepsilon_1\left[\frac{1}{u_{0i}^2} - \left(\frac{\gamma}{2} - 1 - \frac{1}{2}\ln 2\right)\right] \simeq$$

$$\simeq \left(\frac{\pi}{2}\right)^{3/2} + \frac{\varepsilon_1}{2}(\gamma - 2 - \ln 2) \simeq 1.12,$$

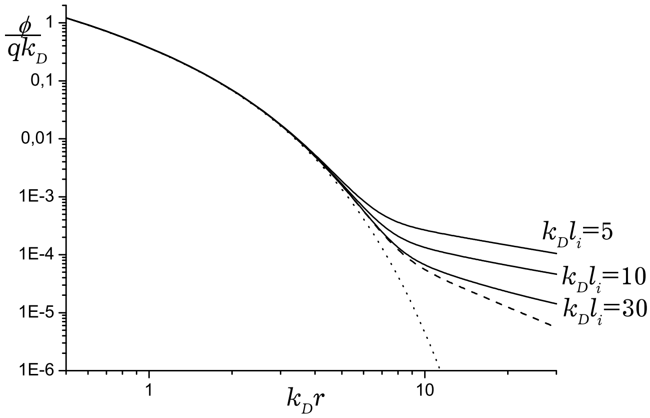

Fig. 1. Normalized effective grain potential plotted from Eq. (19). Dashed line corresponds to the case of collisionless plasma. Dotted line represents the Debye–Hückel potential

$$\varepsilon_3 = \varepsilon_1\gamma \simeq 0.46,$$

γ is the Euler constant.

The normalized form of the effective potential is

$$\frac{\phi(r)}{qk_D} = \frac{e^{-\xi_D}}{\xi_D} + \frac{(ak_D)(2+t/z)}{4\xi_D(1+t)}\Big\{Q(\xi_D) -$$

$$-\frac{\varepsilon_1}{\tilde{l}_i}Q^+(\xi_D) + \frac{\varepsilon_1}{\tilde{l}_i}\left[(1-e^{-\xi_D})\ln\frac{\xi_D}{\tilde{l}_i} + e^{-\xi_D}\ln\xi_D\right] +$$

$$+\frac{1}{\tilde{l}_i}\left[\varepsilon_2(1-e^{-\xi_D})+\varepsilon_3\right]\Big\}, \tag{19}$$

where $\tilde{l}_i = k_D l_i$.

The plots of Eq. (19) are presented in Fig. 1. We have used the following parameters for argon plasma: $T_i/T_e = 0.01$, $k_D a = 0.01$. The value of dimensionless grain charge $z = |e_i q|/aT_e$ is dependent on the ion mean free path and could be estimated from analytical expressions obtained for collisional plasma, from numerical simulation, or from experimental data [11]. Considering the grain as a small spherical probe, one can judge z as an arbitrary parameter. Thus, in order to distinguish the fine structure of the potential, disregarding the effects concerning a grain charge variation with collisionality, we have used the simple well-known equation $\sqrt{T_i m_i/T_e m_e}e^{-z} = t + z$ (collisionless plasma with the OML approximation) to calculate z.

The second term in Eq. (19) is proportional to ak_D. So, for $ak_D \ll 1$, the second term gives its contribution only when the first term is small, and a deviation from

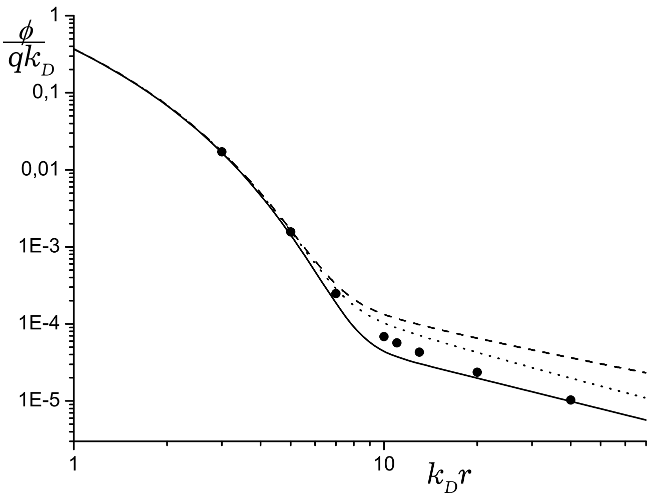

Fig. 2. Normalized effective grain potential, $k_D l_i = 10$. Our analytical results Eq. (24) (solid line), Eqs. (16), (19) (dashed line), the expression for the potential obtained in [10] (dotted line), and the result of numerical calculations of Eq. (3) (solid circles)

the Debye–Hückel potential appears only at several units of $k_D r$ (see Fig. 1). Hence, only the asymptotic behavior of $\Phi_S(r)$ at $k_D r \gg 1$ is important, and Eq. (16) is reduced to

$$\Phi_S(r) = -\frac{e_i n_i}{k_D r}\sigma_0 u_{0i}^2 \times$$

$$\times \left\{ \frac{1}{k_D r} + \frac{\varepsilon_1}{k_D l_i} \ln \frac{r}{l_i} + \frac{1}{k_D l_i}(\varepsilon_2 + \varepsilon_3) \right\}. \quad (20)$$

Direct plotting has shown that there is no considerable difference between Eq. (16) and Eq. (20) at $ak_D = 0.01$ and an arbitrary value of l_i.

The range of reasonable values of $l_i k_D$ is from the infinity (collisionless plasma) to several units (this limitation is caused by the usage of the OML cross-section). Equation (16) is valid for $l_i \gg r$; so, in the domain of $r \approx 10 r_D$ where the difference with the Debye–Hückel potential is considerable, the acceptable values of ionic mean free path is several dozens of r_D. But, for such a value of l_i, the difference with the collisionless case is slight (see Fig. 1). At $k_D l_i \gg 1$, Eq. (16) yields

$$\Phi_S(r) = -\frac{e_i n_i}{k_D r}\sigma_0 (1 + u_{0i}^2) Q(k_D r), \quad (21)$$

which is identical, to within designations, to Eq. (65) in [4] for the collisionless plasma. In contrast to Eq. (6) in [1], which differs, at $k_D l_i \gg 1$, from the collisionless solution by a factor of $(1 + z/t)/(1 + 2z/t) \approx 1/2$ at $t \ll 1$.

Let us consider another limiting case $\xi \gg 1$. Here, we neglect I_2 in Eq. (7) and continue the upper limit in I_1 to the infinity. The behavior of approximation (13) in the extended domain at $y \ll 1$ is

$$\frac{\sin x}{x^2 + \xi_D^2} \frac{F(\xi/x)}{V(\xi/x)} = O\Big(\frac{\sin x}{x^3}\Big), \text{ at } x \to \infty, \quad (22)$$

and the correct behavior of the integrand in Eq. (3), which is given by Eq. (12), is

$$\frac{\sin x}{x^2 + \xi_D^2} \frac{F(\xi/x)}{V(\xi/x)} = O\Big(\frac{\sin x}{x^2}\Big), \text{ at } x \to \infty. \quad (23)$$

In this case, the difference is not so critical as compare to Eqs. (14), (15).

The general relations give

$$\Phi(r) = -\frac{e_i n_i}{r k_D l_i}\sigma_0 \left(\frac{\pi}{2}\right)^{1/2} (2 + u_{0i}^2)(1 - e^{-k_D r}). \quad (24)$$

We have performed the direct numerical calculations of Eq. (3) to examine our analytical calculation (see Fig. 2).

As is seen, Eq. (20) is different from the result obtained disregarding the velocity dependence of the charging cross-section (Eq. (6) in [10]). Namely, the numerical factor at the Coulomb-like part is different from that in [10] (their ratio is $2(\varepsilon_2+\varepsilon_3)/0.6\pi^{3/2} \sim 0.94$), and the new term proportional to $\ln(r/l_i)/r$ is present in Eq. (20). Thus, Eq. (6) in [10] does not describe qualitative details of the potential behavior at $r < l_i$ due to a specific distribution of the charge density perturbation caused by the ion absorption by grains. In particular, the primary (unscreened) charge density perturbation is described by

$$\rho_i^{(0)}(r) = \frac{i e_i n_i}{2\pi^2 r^2} \int_0^\infty dx \sin x \frac{F(\xi/x)}{V(\xi/x)} \simeq$$

$$\simeq -\frac{e_i n_i}{4\pi r^2}\sigma_0 (1+u_{0i}^2) \left\{ 1 + \frac{r}{l_i}(\varepsilon_2 + \varepsilon_3) + \frac{r}{l_i}\varepsilon_1 \ln \frac{r}{l_i} \right\}. \quad (25)$$

The total (screened) charge density at $r < l_i$ can be estimated as

$$\rho_i^{tot}(r) = -\frac{e_i n_i}{4\pi r^2}\sigma_0 (1 + u_{0i}^2) \times$$

$$\times \left\{ 1 - k_D r Q(k_D r) \left(1 + \frac{\varepsilon_1}{k_D l_i}\right) + \right.$$

$$\left. + \frac{r}{l_i} e^{-k_D r} (\varepsilon_2 + \varepsilon_1 \ln(k_D l_i) \right\}. \quad (26)$$

It is essential to point out once more that the conditions of application of the obtained estimates (16) and (24) for weakly collisional and strongly collisional limits, respectively, are determined by the ratio of the distance from the charge to the ionic mean free path r/l_i. Just this ratio controls the type of the approximation for $\Phi_S(r)$ (the part of the potential associated with ion absorption by a grain). It can be shown rigorously, but it also follows from the approximations of the integrand ($\nu_i/ks_i \ll 1$ (that means $r \sim 1/k \ll s_i/\nu_i = l_i$) for the weakly collisional limit and $\nu_i/ks_i \gg 1$ ($r \gg l_i$) for the strongly collisional limit). In the formal treatment used in [1, 2, 10] and in this paper, both cases can be arbitrary.

Thus, the collisions can considerably influence the grain potential. In particular, they can lead to the Coulomb-like behavior at large distances, but such Coulomb-like asymptotics can be dominant at $r > l_i$ only. In the opposite case $r < l_i$, the Coulomb-like terms are generated, but the collisionless contribution remains decisive.

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ПРО ВПЛИВ ЗІТКНЕНЬ ПЛАЗМОВИХ ЧАСТИНОК
НА ЕФЕКТИВНИЙ ПОТЕНЦІАЛ ПОРОШИНКИ

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Р е з ю м е

Детально проаналізовано кінетичні розв'язки задачі про ефективні потенціали макрочастинки, що заряджається плазмовими струмами. Розрахунки виконано з урахуванням залежності іонного перерізу заряджання від швидкості іонів. Отримано аналітичні вирази для потенціалів при довільних значеннях довжини вільного пробігу іонів. Показано, що аналітичні наближення узгоджуються з проведеними нами точними числовими розрахунками, та встановлено сфери застосування таких аналітичних наближень.