SEMICLASSICAL MODEL OF DIPOLE PYGMY-RESONANCE IN NUCLEI WITH NEUTRON EXCESS

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PACS 21.60.Ev

To study the dipole pygmy-resonance in nuclei with neutron excess, it is proposed to use a semiclassical model based on the dynamics in the phase space. The properties of the neutron skin of a nucleus are presented by a surface layer of moving neutrons. It is obtained that the strength function of the spherical layer has a resonance with the maximum energy corresponding to the energy of the dipole pygmy-resonance observed in the ^{208}Pb nucleus.

1. Introduction

Recently, both theoreticians and experimenters show a special interest in the investigation of the properties of the dipole pygmy-resonance [1–3]. This resonance is observed in nuclei with neutron excess in the neighborhood of the neutron binding energy and exhausts 3–6% of the energy weighted sum rule [4–6]. The dipole pygmy-resonance represents oscillations of the neutron excess with respect to the neutron-proton core, i.e. it is directly connected with properties of the neutron skin. The information on the thickness of the neutron skin can be obtained from various processes, in particular the dipole response of nuclei at the inelastic scattering of alpha-particles [7, 8]. The dipole pygmy-resonance was investigated in quantum approaches like the random phase approximation [9–12]. In order to study collective excitations in nuclei, we proposed a semiclassical approach based on the dynamics in the phase space [13]. In [14], this approach was generalized for the study of isovector dipole excitations in nuclei with neutron excess.

In the given work, we consider a semiclassical model of the dipole pygmy-resonance in nuclei with neutron excess using the kinetic approach [13,14]. We will investigate the response of the surface layer of interacting neutrons. In such a system, there arises a collective dipole mode that will represent oscillations of excess neutrons with respect to the neutron-proton core of the nucleus. The semiclassical model of the dipole pygmy-resonance will be described in Section 2. In Section 3, we find the dipole response function of the spherical neutron layer taking the residual interaction between neutrons in the separable approximation into account. Section 4 will be devoted to the consideration of properties of the dipole pygmy-resonance in spherical heavy nuclei using the dipole excitation strength distribution in our model.

2. Kinetic Model

It is known that, in a nucleus with neutron excess, a part of neutrons is located in the surface region of the nucleus forming the so-called neutron skin [8,15]. The dipole pygmy-resonance represents oscillations of these surface neutrons with respect to the core. Thus, in order to investigate the properties of the dipole pygmy-resonance, let us consider a neutron-proton asymmetric system (N neutrons and Z protons). It is supposed that our system consists of a neutron-proton core formed by A nucleons and a surface spherical layer including $N_{\rm sl}$ neutrons (see Fig. 1). The internal radius of the layer $R_{\rm c}$ that represents the radius of the core is defined as

$$R_{\rm c} = R - \Delta. \tag{1}$$

Here, $R = r_0 A^{1/3}$ denotes the average radius of the system, where $r_0 = \left(\frac{4\pi}{3}\rho_0\right)^{-1/3}$, ρ_0 is the density of the nuclear matter, and $= N + Z$ is the number of nucleons in the system. The quantity Δ in Eq.(1) stands for the difference between the average radius of the nucleus R and the core radius $R_{\rm c}$. It is much smaller than both R and $R_{\rm c}$. Equation (1) is used in what follows in order

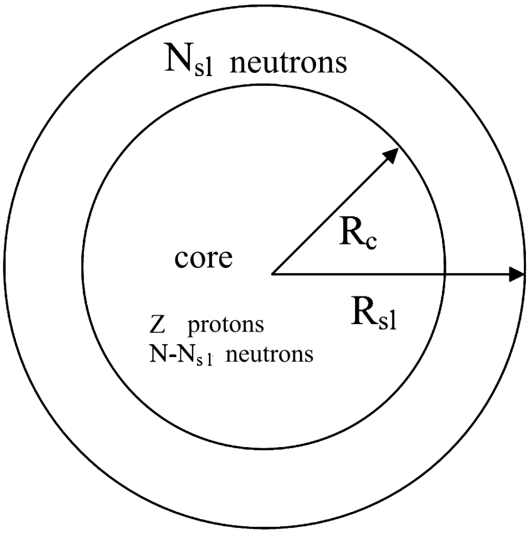

Fig. 1. Diagram of the model of pygmy-resonance

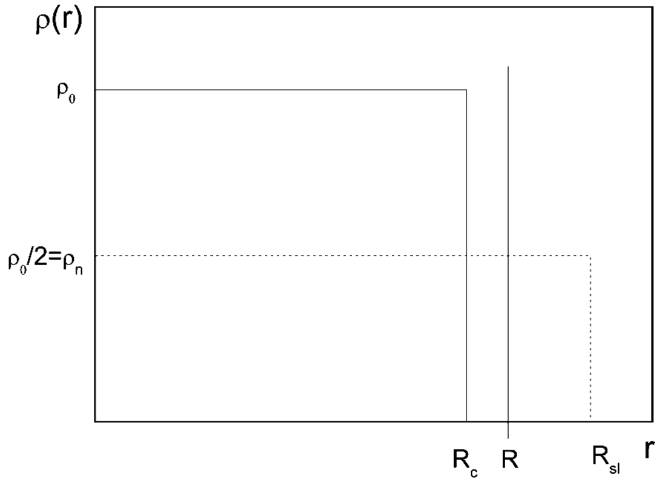

Fig. 2. Nucleon density distribution in the sharp surface approximation. Solid curve – density of the neutron-proton core, the dashed curve – neutron density

to determine the quantities in the terms of the average radius of the system (see, for example, (4)).

The external radius of the layer $R_{\rm sl}$ is related to the thickness of the neutron skin t:

$$R_{\rm sl} = R_{\rm c} + t. \tag{2}$$

In this case, the number of neutrons in the surface layer $N_{\rm sl}$ is determined by the expression

$$N_{\rm sl} = \frac{4\pi}{3}(R_{\rm sl}^3 - R_{\rm c}^3)\frac{\rho_0}{2}, \tag{3}$$

if it is considered that the neutron density in the surface layer is equal to a half of the nuclear matter density, see Fig. 2.

Substituting (1) and (2) in (3) and taking relation $\Delta \ll R$ into account, the number of neutrons in the surface layer $N_{\rm sl}$ is presented to within the terms $(\Delta/R)^2$ as

$$N_{\rm sl} = \frac{3}{2}\left(\rho_0\frac{4\pi}{3}\right)^{1/3} A^{2/3}t. \tag{4}$$

This expression coincides with that obtained in [16].

It is worth noting that Eq. (4) was derived assuming that the neutron density in the spherical layer was equal to $\rho_0/2$ like in the case of a symmetric neutron-proton system. Using a more realistic value of the density equal to $(N/A)\rho_0$, we obtain an additional factor $2N/A$ in Eq. (4), which is less than or equal to 1.2 for heavy nuclei. With the help of Eq. (4), one can estimate that it will result in an increase of the number of neutrons in the spherical layer by 1-2 particles and thus will insignificantly influence the magnitude and distribution of the dipole excitation strength in the layer.

Neutron oscillations in the surface layer with respect to the core will be studied varying the distribution function $\delta n(\mathbf{r},\mathbf{p},t)$. We suppose that this function obeys the linearized kinetic Vlasov equation in the middle of the spherical layer at $R_{\rm c} < r < R_{\rm sl}$

$$\frac{\partial}{\partial t}\delta n(\mathbf{r},\mathbf{p},t) + \frac{\mathbf{p}}{m}\frac{\partial}{\partial \mathbf{r}}\Big[\delta n(\mathbf{r},\mathbf{p},t) -$$

$$- \frac{dn^0(\varepsilon)}{d\varepsilon}\left[\delta U(\mathbf{r},t) + V(\mathbf{r},t)\right]\Big] = 0 \tag{5}$$

and satisfies the condition of mirror reflection of particles from the stationary surface at $r = R_{\rm sl}$ and $r = R_{\rm c}$

$$\left[\delta n(\mathbf{r},\mathbf{p}_\perp,p_r,t) - \delta n(\mathbf{r},\mathbf{p}_\perp,-p_r,t)\right]\Big|_{\substack{r\,=\,R_{\rm sl}\\ r\,=\,R_{\rm c}}} = 0, \tag{6}$$

where $n_0(\varepsilon) = \theta(\varepsilon - \varepsilon_{\rm F})$ is the equilibrium neutron distribution function in the spherical layer, $\varepsilon_{\rm F}$ is the Fermi energy of the nuclear matter, $\mathbf{p} = (\mathbf{p}_\perp, p_r)$ denotes the momentum of the particle, $p_r = mv_r$, $\mathbf{p}_\perp = (0, p_\theta, p_\varphi)$.

As we are interested in dipole excitations of neutrons in the surface layer, the external field $V(\mathbf{r},t)$ in Eq. (5) will be taken in the form

$$V(\mathbf{r},t) = \beta\delta(t)rY_{10}(\theta). \tag{7}$$

The variation of the mean field $\delta U(\mathbf{r},t)$ in Eq. (5) caused by the interaction between neutrons is given by the expression [13]

$$\delta U(\mathbf{r},t) = \frac{2}{h^3}\int d\mathbf{r}'d\mathbf{p}'u(\mathbf{r},\mathbf{r}')\delta n(\mathbf{r}',\mathbf{p}',t). \tag{8}$$

The interaction between neutrons in the middle of the layer will be calculated in the separable approximation:

$$u(\mathbf{r},\mathbf{r}') = \kappa\sum_M rr'Y_{1M}(\theta,\phi)Y^*_{1M}(\theta',\phi'). \tag{9}$$

Here, κ stands for the interaction parameter that can be determined with the help of the polarization sum rule, see [17],

$$\kappa = F_0\frac{40\pi}{9}\frac{\varepsilon_{\rm F}}{AR^2},$$

where F_0 is the isoscalar Landau parameter.

3. Dipole Response Function of the Spherical Layer

The dipole response function is determined in the following way (see, for example, [18]):

$$R(\omega) = \frac{1}{\beta}\int d\mathbf{r}r\delta\rho(\mathbf{r},\omega), \tag{10}$$

where $\delta\rho(\mathbf{r},\omega)$ is the Fourier transform of the neutron density variation induced by the external field (7) with respect to time:

$$\delta\rho(\mathbf{r},\omega) = \frac{2}{h^3}\int d\mathbf{p}\delta n(\mathbf{r},\mathbf{p},\omega).$$

Using the solution of the kinetic equation (5) with the boundary conditions (6), one can obtain the collective response function [13]

$$R_{\rm sl}(\omega) = \frac{R^0_{\rm sl}(\omega)}{1-\kappa R^0_{\rm sl}(\omega)}, \tag{11}$$

where $R^0_{\rm sl}(\omega)$ is the semiclassical single-particle dipole response function of the neutron spherical layer

$$R^0_{\rm sl}(\omega) = \frac{2}{h^3}\pi(p_{\rm F}R_{\rm sl})^2\sum_{N=\pm1}\sum_{n=-\infty}^{\infty}\int\limits_{R_{\rm c}/R_{\rm sl}}^{1} d\lambda\lambda\times$$

$$\times\omega_{nN}(\lambda)T(\varepsilon_{\rm F},\lambda)\frac{(Q_{nN}(\lambda))^2}{\omega-\omega_{nN}(\lambda)}. \tag{12}$$

Here, $\lambda = \frac{l}{p_{\rm F}R_{\rm sl}}$ denotes the dimensionless angular momentum of the particle (l is its angular momentum), $Q_{nN}(\lambda) = (-1)^n\frac{v^2_{\rm F}}{R_{\rm sl}\omega^2_{nN}(\lambda)}$ are the dipole radial matrix elements in the semiclassical approximation [13]; $\omega_{nN}(\lambda) = \frac{2\pi n}{T(\varepsilon_{\rm F},\lambda)} + N\frac{\Gamma(\lambda)}{T(\varepsilon_{\rm F},\lambda)}$ denote the dipole single-particle frequencies, $T(\varepsilon_{\rm F},\lambda) = \frac{2R_{\rm sl}}{v_{\rm F}}\sqrt{1-\lambda^2}$ and $\Gamma(\lambda) = 2\arccos\lambda$ are the periods of the radial and "angular" motion of the particles, respectively.

From expression (12), one can see that a contribution into the single-particle response function is made by orbits of particles with the angular momentum lying between $l_{\min} = p_{\rm F}R_{\rm c}$ and $l_{\max} = p_{\rm F}R_{\rm sl}$.

4. Dipole Pygmy-Resonance

In order to study the properties of the dipole pygmy-resonance, let us consider the distribution of the dipole excitation strength in our model. It is described by the strength function determined in the following way:

$$S_{\rm sl}(E) = -\frac{1}{\pi}{\rm Im}R_{\rm sl}(E), \tag{13}$$

where $E = \hbar\omega$.

An important integral characteristic of the strength of multipole excitations is presented by the energy weighted sum rule determined with the help of the response function

$$m^1 = \int\limits_0^\infty dEE\left[-\frac{1}{\pi}{\rm Im}R(E)\right]. \tag{14}$$

The dipole energy weighted sum rule for a nucleus consisting of nucleons has a form [19]

$$m^1_{L=1} = \frac{3}{8\pi}\frac{\hbar^2}{m}A. \tag{15}$$

Using the response function (11) and definition (14), one can find the dipole energy weighted sum rule for the spherical neutron layer to within the $(t/R)^2$ terms in the form

$$m^1_{\rm sl} \approx \sqrt{2}\left(\frac{t}{R}\right)^{3/2}m^1_{L=1}. \tag{16}$$

Then the contribution of the excitations of the surface neutron layer with the thickness t to the dipole energy weighted sum rule (15) is given by the expression

$$\frac{m^1_{\rm sl}}{m^1_{L=1}} \approx \sqrt{2}\left(\frac{t}{R}\right)^{3/2}. \tag{17}$$

The results of numerical calculations coincide with those obtained with the help of expression (17).

Numerical calculations of the strength function (13) were performed for a system with the number of neutrons $N = 126$ and the number of protons $Z = 82$, which corresponds to the ^{208}Pb nucleus. In the calculations, we used the standard values of the nuclear parameters: $r_0 = 1.12$ fm, $\varepsilon_F = 40$ MeV, $m = 1.04$ MeV $(10^{-22}\ \mathrm{s})^2/\mathrm{fm}^2$, and $F_0 = -0.42$, see [14].

Numerical calculations of the strength function were performed for various numbers of neutrons in the layer. Three cases were considered. In the first one, it was supposed that all excess neutrons are located in the surface layer and oscillate with respect to the neutron-proton symmetric core. That is, the number of neutrons in the surface layer is equal to $N_{\mathrm{sl}}^{\mathrm{max}} = N - Z = 44$. The results of the calculations for this case are presented in Fig. 3 by the dashed curve. One can see that the strength distribution has a resonance structure. The maximum of the distribution lies at an energy of 7.8 MeV and exhausts 7.5% of the dipole energy weighted sum rule (15).

The experimental value of the neutron skin thickness for the ^{208}Pb nucleus is equal to ~ 0.2 fm [7,8]. From expression (4), it follows that the number of neutrons in the surface layer corresponding to this neutron skin is equal to $N_{\mathrm{sl}} = 9$. The expression for the number of neutrons in the neutron skin obtained in [16] coincides with our expression (4). However, the number of neutrons in the neutron skin for the ^{208}Pb nucleus obtained in [16] is equal to 18, as the neutron skin thickness was calculated with the help of the direct variational method and amounts to 0.4 fm.

The solid curve in Fig. 3 shows the results of calculations performed for the number of neutrons in the layer $N_{\mathrm{sl}} = 9$. In this case, the obtained maximum of the distribution lies at an energy of 8.6 MeV and exhausts $\sim$1% of the energy weighted sum rule (15). The dotted curve presents the results of calculations performed for the number of neutrons in the layer $N_{\mathrm{sl}} = 18$, which corresponds to work [16]. In this case, the obtained maximum of the distribution lies at an energy of 8.4 MeV and exhausts $\sim$ 2% of the energy weighted sum rule (15). The dipole pygmy-resonance for the ^{208}Pb nucleus is observed experimentally at an energy of $\sim$8 MeV and exhausts $\sim$ 6% of the energy weighted sum rule [6]. Thus, in order to coordinate our calculations with the experimental data, it is necessary to consider a more realistic model. It is worth noting that, in the considered model, the surfaces restricting the layer are supposed to be stationary. However, it is known that the dynamic surface effects significantly influence the distribution of the dipole excitation strength [17]. In addition, the neutron-proton core is also considered to be stationary, i.e. its excitations are not taken into account.

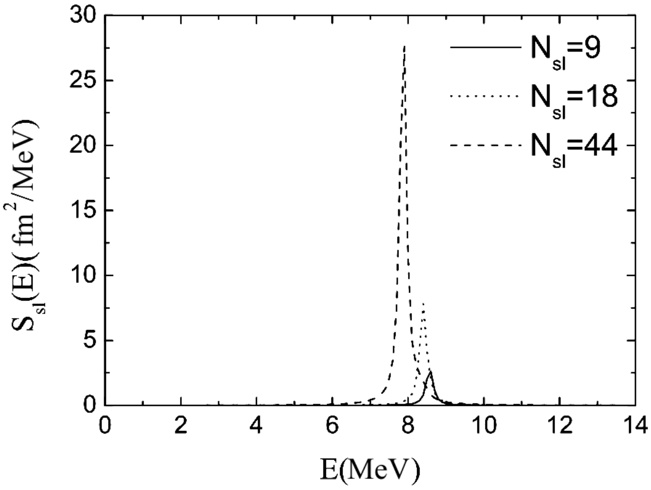

Fig. 3. Distribution of the dipole excitation strength in the spherical layer with various numbers of neutrons N_{sl}: solid curve – $N_{\mathrm{sl}} = 9$; dashed curve – $N_{\mathrm{sl}} = 18$; dotted curve – $N_{\mathrm{sl}} = 44$

It is also worth noting that, as we consider dipole excitations of neutrons in the layer with respect to the stationary core, the system does not shift as a whole. Thus, the dipole response of the neutron layer in our model is not reduced to the motion of the center of mass of the system.

5. Conclusions

In the given work, the dipole pygmy-resonance in nuclei with neutron excess is considered in the framework of the semiclassical model based on the dynamics in the phase space. The properties of the neutron skin of a nucleus are presented by the surface layer of moving neutrons.

A dipole response function of the spherical layer is obtained using the residual interaction between nucleons in the separable approximation.

The strength function of the spherical layer is calculated numerically for a system with the number of neutrons $N = 126$ and that of protons $Z = 82$. It is found that the strength function of the spherical layer has a resonance with the maximum energy corresponding to the energy of the dipole pygmy-resonance observed in the ^{208}Pb nucleus. However, the strength of the obtained resonance is lower than the observed strength of the pygmy-resonance in the ^{208}Pb nucleus. Possibly, it is related to the fact that the considered model did not take into account the dynamic surface effects and the core dynamics. One can expect that the allowance for dynamic effects of the surface and the core will allow one to obtain more realistic properties of the dipole pygmy-resonance and, in addition, to simultaneously describe the giant dipole resonance.

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Received 09.12.08.
Translated from Ukrainian by H.G. Kalyuzhna

НАПІВКЛАСИЧНА МОДЕЛЬ ДИПОЛЬНОГО ПІГМІ-РЕЗОНАНСУ В ЯДРАХ ІЗ НАДЛИШКОМ НЕЙТРОНІВ

В.І. Абросімов, О.І. Давидовська

Р е з ю м е

Для вивчення дипольного пігмі-резонансу в ядрах із надлишком нейтронів запропоновано напівкласичну модель, що спирається на динаміку в фазовому просторі. Властивості нейтронної шкіри ядра представлено поверхневим шаром, в якому рухаються нейтрони. Знайдено, що силова функція сферичного шару має резонанс з енергією максимуму, що відповідає енергії дипольного пігмі-резонансу, який спостерігається в ядрі ^{208}Pb.