ENERGY GAIN AT TWO-BEAM COUPLING IN THE ORIENTATIONALLY INHOMOGENEOUS LIQUID CRYSTAL

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The two-beam energy transfer at the director diffraction grating in a flexoelectric liquid crystal cell is theoretically studied. The grating is recorded by a space-charge electric field penetrating into the liquid crystal from photorefractive substrates of the cell. In the geometric optics approach, the system of coupled wave equations is derived, and the signal gain coefficient is calculated. It is shown that the orientational inhomogeneity of a liquid crystal significantly influences the value and the type of the gain coefficient dependence on the grating spacing.

1. Introduction

Photorefractive effects well-known in solid inorganic crystals [1,2] have got an additional development in liquid crystals (LCs) where the high refractive index modulation, ∼0.2, has been achieved due to the reorientation of the LC director. In particular, the energy transfer in a two-wave mixing geometry due to the photorefractive effect intensively studied in photoconductive nematic LCs [3–5] and in the LC cells with adjacent polymer photorefractive or photoconducting layers [6–8]. These systems have generated the two-beam coupling with gain coefficients more than two orders of magnitude larger than those in solid inorganic photorefractive crystals. Recently, it was shown [9,10] that, in hybrid cells which contain LC with photorefractive substrates, the Bragg diffraction regime can be reached if LC is flexoelectric. Effectively working in the case of the Bragg regime, the coupled wave theory was developed for isotropic media [11]. Later on, it was extended to the case of anisotropic materials [12] and non-uniform gratings [13]. In a hybrid cell, the LC director reorients inhomogeneously so that the recording of a diffraction grating and the two-beam coupling take place, in this case, in inhomogeneous anisotropic media. To describe the energy gain at the two-beam coupling in hybrid cells, we use the geometric optics approach and show that the exponential gain coefficient essentially depends on the director spatial profile. The paper is organized as follows. In Section 2, we obtain the dielectric tensor of a hybrid LC cell. In Section 3, the geometric optics approach is used to take the influence of an orientational inhomogeneity of the LC medium on the propagation of signal and pump light waves into account. Coupled equations for light beams are obtained in Section 4. In Section 5, we derive the expression for the signal gain coefficient in a flexoelectric liquid crystal and present the results of numerical calculations. In Section 6, we present brief conclusions.

2. Liquid Crystal Dielectric Tensor in Hybrid Cell

Consider a nematic LC which is placed between two transparent photorefractive layers and bounded by the planes $z = -L/2$ and $z = L/2$. This hybrid cell is illuminated by two intersecting coherent light beams $\mathbf{E}_1 = A_1\mathbf{e}_1 \exp(i\mathbf{k}_1\mathbf{r} - i\omega t)$ and $\mathbf{E}_2 = A_2\mathbf{e}_2 \exp(i\mathbf{k}_2\mathbf{r} - i\omega t)$. The bisector of the beams is directed along the z-axis, the wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$ and the the polarization vectors $\mathbf{e}_1$, $\mathbf{e}_2$ of the beams lay in the xz-plane. The beams form a light intensity interference pattern

$$I = (I_1 + I_2)\left[1 + \frac{1}{2}\left(m \exp(iqx) + \text{c.c.}\right)\right], \qquad (1)$$

where $m = 2\cos(2\delta) A_1 A_2^* / (I_1 + I_2)$, δ is the two beams intersection half-angle, $I_1 = A_1 A_1^*$ and $I_2 = A_2 A_2^*$ are the intensities of incident beams, and $q = k_{1x} - k_{2x}$ is the wave number of the intensity pattern [1].

The light intensity pattern (1) induces a space charge in photorefractive layers. The fundamental component of the space-charge density is modulated along the x-axis with period equal to $2\pi/q$ and creates the space-charge electric field $\mathbf{E_{sc}}(q)\frac{m}{2}\exp(iqx)$ + c.c. in the photorefractive medium, where $\mathbf{E}_{\rm sc}(q)$ depends on the physical properties and the geometry of a photorefractive material [1]. The space-charge electric field penetrates into LC, by recording the director grating in the LC bulk [9]. Supposing the LC director to be in the xz-plane, we describe the director spatial profile in the cell by the angle $\vartheta(x,z)$ between the director and the $x-$axis so that $n=(\cos\vartheta(x,z),\,0,\sin\vartheta(x,z))$. It is convenient to present $\vartheta(x,z)$ as

$$\vartheta(x,z)=\theta_0(z)+\alpha(x,z),$$

$$\alpha(x,z)=\theta(z)\exp(iqx)+\text{c.c.},\qquad(2)$$

where $\theta_0(z)$ is induced by the initial director pre-tilt at the cell surfaces, and $\alpha(x,z)$ is caused by the director interaction with the spatially modulated space-charge field. Further, we put $|\alpha(x,z)|\ll\theta_0(z)\ll 1$. We suppose the infinitely strong director anchoring at the cell substrates so that the next boundary conditions for the director angles must be satisfied:

$$\theta_0(z=-L/2)=\theta_1,\quad \theta_0(z=L/2)=\theta_2,$$

$$\theta(z=\mp L/2)=0.\qquad(3)$$

The LC dielectric tensor at an optical frequency, $\varepsilon_{ij}=\varepsilon_\perp\delta_{ij}+\varepsilon_a n_i n_j$, can be written, up to the second order in the director angle $\vartheta(x,z)$, as

$$\hat\varepsilon=\begin{pmatrix}\varepsilon_\parallel-\varepsilon_a\vartheta^2(x,z) & 0 & \varepsilon_a\vartheta(x,z)\\ 0 & \varepsilon_\perp & 0\\ \varepsilon_a\vartheta(x,z) & 0 & \varepsilon_\perp+\varepsilon_a\vartheta^2(x,z)\end{pmatrix},\qquad(4)$$

where $\varepsilon_a=\varepsilon_\parallel-\varepsilon_\perp$ and $\varepsilon_\parallel,\varepsilon_\perp$ are the principal values of the dielectric tensor. Substituting $\vartheta(x,z)$from (2) into (4) and neglecting small terms of the second order in the angle $\theta(z)$one, we can rewrite the dielectric tensor in the form

$$\hat\varepsilon=\hat\varepsilon_1(z)+[\hat\varepsilon_2(z)\exp(iqx)+\text{c.c.}],\qquad(5)$$

where

$$\hat\varepsilon_1(z)=\begin{pmatrix}\varepsilon_\parallel-\varepsilon_a\theta_0^2(z) & 0 & \varepsilon_a\theta_0(z)\\ 0 & \varepsilon_\perp & 0\\ \varepsilon_a\theta_0(z) & 0 & \varepsilon_\perp+\varepsilon_a\theta_0^2(z)\end{pmatrix},$$

$$\hat\varepsilon_2(z)=\varepsilon_a\theta(z)\begin{pmatrix}-2\theta_0(z) & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 2\theta_0(z)\end{pmatrix}.\qquad(6)$$

The first term in (5) corresponds to the orientationally inhomogeneous LC in the absence of a photorefractive electric field; the second term describes a change of the dielectric tensor due to the periodic modulation of the director by the photorefractive electric field.

In what follows, we consider the propagation of light beams in an inhomogeneous LC with the dielectric function given by formula (5).

3. Geometric Optics Approximation

The equation for a monochromatic light wave $\mathbf{E}(r)e^{-i\omega t}$ in LC takes a form

$$\text{rotrot}\mathbf{E}-\frac{\omega^2}{c^2}\hat\varepsilon(x,z)\mathbf{E}=0,\qquad(7)$$

where $\hat\varepsilon(x,z)$ is given by (5). As $\theta(z)\ll\theta_0(z)$, we can consider the second term in (5) as a perturbation and solve the problem, by putting, at first, $\hat\varepsilon=\hat\varepsilon_1(z)$.

Using the geometric optics approach which is valid in the limits $\lambda=\frac{2\pi c}{\omega}\ll\frac{2\pi}{q},\ L$, we can put the light wave electric field $E(r)$ in (7) in the form $\mathbf{E}(r)=A^{(0)}e\,e^{k_0\psi(r)}$ and obtain an equation for the eikonal $\psi(\mathbf{r})$ [14],

$$\det\left\|p^2\delta_{ij}-p_ip_j-\varepsilon_{1ij}\right\|=0,\qquad(8)$$

where $p=\nabla\psi(\mathbf{r})$ and $k_0=\frac{\omega}{c}$.

As the tensor ε_{1ij} depends only on the coordinate z, we can seek $\psi(\mathbf{r})$ in the form

$$\psi(\mathbf{r})=s_xx+s_yz+\varphi(z),\qquad(9)$$

where we can put $s_y=0$, by taking into account that the wave propagates in the xz-plane. Then, substituting (9) into (8) and solving Eq. (8), we arrive at

$$\varphi(z)=\pm\int\sqrt{\varepsilon_{1yy}-s_x^2}dz\qquad(10)$$

for the ordinary wave and

$$\varphi(z)=\int\frac{-\varepsilon_{1xz}s_x\pm\sqrt{(\varepsilon_{1xz}^2-\varepsilon_{1xx}\varepsilon_{1zz})(s_x^2-\varepsilon_{1zz})}}{\varepsilon_{1zz}}dz\qquad(11)$$

for the extraordinary wave. The signs plus and minus before the square root correspond to the forward- and back-traveling waves, respectively.

Further, we will consider only the case of extraordinary waves. Then the signal and pump light beams in LC can be written in the geometric optics approach as $\mathbf{E}_1=A_1^{(0)}\mathbf{e}_1\exp i(s_{1x}x+\varphi_1(z)-\omega t)$ and $\mathbf{E}_2=A_2^{(0)}\mathbf{e}_2\exp i(s_{2x}x+\varphi_2(z)-\omega t)$, respectively, where $\varphi_{1,2}(z)$ is defined by formula (11) with the substitution of s_x by s_{1x} or s_{2x}.

Putting the signal and pump light beams in the upper photorefractive substrate, $\mathbf{E}_1^{\text{Ph}}=\mathbf{E}_{01}\exp(i\mathbf{k}_{01}\mathbf{r}-i\omega t)$

and $E_2^{\rm Ph} = \mathbf{E}_{02} \exp(i\mathbf{k}_{02}\mathbf{r} - i\omega t)$ to be given, we can write

$$\mathbf{k}_{01} = k_0 n_{\rm Ph} (\sin\delta,\, 0,\, \cos\delta)\,,$$

$$\mathbf{k}_{02} = k_0 n_{\rm Ph} (-\sin\delta,\, 0,\, \cos\delta)$$

$$\mathbf{E}_{01} = E_{01} (\cos\delta,\, 0,\, -\sin\delta)\,,$$

$$\mathbf{E}_{02} = E_{02} (\cos\delta,\, 0, \sin\delta)\,, \tag{12}$$

where δ is an angle of the wave vectors with the z-axis, $n_{\rm Ph}$ is a light refraction index of the photorefractive medium (the last is considered isotropic).

Satisfying the boundary conditions for the light beam electric fields at $z = -L/2$,

$$D_n|_{z=-L/2} = D_n^{\rm Ph}\big|_{z=-L/2}\,, \quad E_\tau|_{z=-L/2} = E_\tau^{\rm Ph}\big|_{z=-L/2}\,, \tag{13}$$

where $\mathbf{D}$ is the electric displacement vector, we can obtain the following expressions for the amplitude and polarization vectors of light beams in the LC cell:

$$A_{1,2}^{(0)} = E_{01,2}\frac{\cos\delta}{e_{1,2x}}, \quad s_{1,2x} = \pm\sin\delta,$$

$$e_{1,2x} = \frac{1}{\sqrt{1 + \left(\frac{n_{\rm Ph}^2 \tan\delta}{\varepsilon_{1zz}}\right)^2}}, \quad e_{1,2z} = \frac{1}{\sqrt{1 + \left(\frac{\varepsilon_{1zz}}{n_{\rm Ph}^2 \tan\delta}\right)^2}}. \tag{14}$$

Using formulas (14) and (11), we can also write expressions for $\varphi_{1,2}(z)$ and the eikonal $\psi_{1,2}(\mathbf{r})$ as a whole for each light beam in the LC.

4. Coupled Equations for Light Beams

To solve Eq. (7) for the electric field of light beams taking the second term in formula (5) into account, we suppose that this term is small and can be treated as a perturbation. Therefore, we seek a solution to Eq. (7) in the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r}) =$$

$$= A_1(z)\mathbf{e}_1 \exp(i\mathbf{k}_1\mathbf{r}) + A_2(z)\mathbf{e}_2 \exp((i\mathbf{k}_2\mathbf{r}))\,. \tag{15}$$

Here, $\mathbf{k}_{1,2} = k_0(s_{1,2x},\, 0,\, \varphi_{1,2}(z)/z)$ and $A_{1,2}(z) = A_{1,2}^{(0)} + A_{1,2}^{(1)}(z)$, where $A_{1,2}^{(1)}(z)$ are considered to be slowly varying functions. Then, by applying the slowly varying amplitude approximation, we can neglect the second-order derivatives of $A_{1,2}(z)$. After some vector algebra, we arrive from (7) to the equation

$$\{[\nabla A_1 \times [K_1 \times e_1]] + [K_1 \times [\nabla A_1 \times e_1]]\} e^{ik_1 r} +$$

$$+ \{[\nabla A_2 \times [K_2 \times e_2]] + [K_2 \times [\nabla A_2 \times e_2]]\} e^{ik_2 r} +$$

$$+ ik_0^2 \left(\hat\varepsilon_2(z) e^{iqx} + \hat\varepsilon_2^*(z) e^{-iqx}\right) \times$$

$$\times \left(A_1 e_1 e^{ik_1 r} + A_2 e_2 e^{ik_2 r}\right) = 0, \tag{16}$$

where we used the designation $\mathbf{K}_{1,2} = \mathbf{k}_{1,2} + z\frac{\partial k_{1,2z}}{\partial z}\mathbf{e}_z$, and $\mathbf{e}_z$ is a unit vector along the z-axis.

For the Bragg diffraction regime, we can neglect the waves $A_1\mathbf{e}_1 e^{i(k_1 r + qx)}$, $A_2\mathbf{e}_2 e^{i(k_2 r - qx)}$ in (16) and get the equation

$$\Big\{ [\nabla A_1 \times [K_1 \times e_1]] + [K_1 \times [\nabla A_1 \times e_1]] +$$

$$+ ik_0^2 \hat\varepsilon_2(z) e_2 A_2 e^{i\Delta k_z z} \Big\} e^{ik_1 r} +$$

$$+ \Big\{ [\nabla A_2 \times [K_2 \times e_2]] + [K_2 \times [\nabla A_2 \times e_2]] +$$

$$+ ik_0^2 \hat\varepsilon_2^*(z) e_1 A_1 e^{-i\Delta k_z z} \Big\} e^{ik_2 r} = 0, \tag{17}$$

where $\Delta k_z \equiv \Delta k_z(z) = k_{2z} - k_{1z} = (\varphi_2(z) - \varphi_1(z))/z$, and we suppose that $|\Delta k_z| \ll k_0$.

In accordance with the method of coupled waves, we can obtain the following system of two coupled equations from (17) after simple algebraic transformations:

$$\nabla A_1 [\mathbf{K}_1 - \mathbf{e}_1(\mathbf{K}_1\mathbf{e}_1)] - i\frac{k_0^2}{2}\mathbf{e}_1\hat\varepsilon_2(z)\mathbf{e}_2 A_2 e^{i\Delta k_z z} = 0,$$

$$\nabla A_2 [\mathbf{K}_2 - \mathbf{e}_2(\mathbf{K}_2\mathbf{e}_2)] - i\frac{k_0^2}{2}\mathbf{e}_2\hat\varepsilon_2^*(z)\mathbf{e}_1 A_1 e^{-i\Delta k_z z} = 0. \tag{18}$$

In particular, in the case of anisotropic homogeneous media, the system of equations (18) reduces to that obtained in [12].

Taking into account that $A_{1,2}(z) = A_{1,2}^{(0)} + A_{1,2}^{(1)}(z)$, where $A_{1,2}^{(1)}(z)$ are small values of the same order as $\hat{\varepsilon}_2(z)$, and neglecting small terms of the second order in (18), we obtain

$$[K_{1z} - e_{1z}(K_{1x}e_{1x} + K_{1z}e_{1z})]\frac{\partial A_1^{(1)}}{\partial z} =$$

$$= i\frac{k_0^2}{2}\mathbf{e}_1\hat{\varepsilon}_2(z)\mathbf{e}_2A_2^{(0)}e^{i\Delta k_z z},$$

$$[K_{2z} - e_{2z}(K_{2x}e_{2x} + K_{2z}e_{2z})]\frac{\partial A_2^{(1)}}{\partial z} =$$

$$= i\frac{k_0^2}{2}\mathbf{e}_2\hat{\varepsilon}_2^*(z)\mathbf{e}_1A_1^{(0)}e^{-i\Delta k_z z}. \quad (19)$$

We now use Eqs. (19) to calculate the energy gain in the LC cell.

5. Exponential Gain Coefficient in Flexoelectric Liquid Crystal

The energy gain for a signal wave is defined by the formula

$$G = \frac{A_1(z = L/2)}{A_1(z = -L/2)} = \frac{A_1^{(0)} + A_1^{(1)}(z = L/2)}{A_1^{(0)}} =$$

$$= \frac{\tilde{A}_1^{(0)} + \tilde{A}_1^{(1)}(z = L/2)}{\tilde{A}_1^{(0)}}, \quad (20)$$

where we took into account that the amplitude $A_1^{(0)}$ is constant and $A_1^{(1)}(-L/2) = 0$ and introduced the relative quantities $\tilde{A}_1^{(0)} = \frac{A_1^{(0)}}{A_2^{(0)}}$ and $\tilde{A}_1^{(1)} = \frac{A_1^{(1)}}{A_2^{(0)}}$.

After the integration, the first equation in (19) yields

$$\tilde{A}_1^{(1)}(z = L/2) =$$

$$= i\frac{k_0^2}{2}\int_{-L/2}^{L/2}\frac{e_1\hat{\varepsilon}_2(z)e_2e^{i\Delta k_z z}}{[K_{1z} - e_{1z}(K_{1x}e_{1x} + K_{1z}e_{1z})]}\,dz. \quad (21)$$

To proceed further, we need to know the expression for the director angle $\theta(z)$ which determines the tensor $\hat{\varepsilon}_2(z)$. The expression for $\theta(z)$ was obtained in [15] in the case of a flexoelectric LC, but it is too bulky to write it here. Substituting $\hat{\varepsilon}_2(z)$ and $\theta(z)$ into (21), we reduce it to

$$\tilde{A}_1^{(1)}(z = L/2) = \tilde{A}_1^{(0)}J_1 + \left[\tilde{A}_1^{(0)} + \tilde{A}_1^{(1)}(z = L/2)\right]J_2, \quad (22)$$

where the coefficients J_1 and J_1 are presented in Appendix.

Taking formula (22) into account, we can rewrite expression (20) for the energy gain as

$$G = \frac{1 + J_1}{1 - J_2}. \quad (23)$$

Then the exponential gain coefficient is given by the formula

$$g = \frac{1}{L}\ln|G| = \frac{1}{L}\ln\left|\frac{1 + J_1}{1 - J_2}\right|. \quad (24)$$

For the numerical calculation of the gain coefficient (24), we use the parameter values from [10] for a hybrid cell filled with LC TL205, namely, the light wave length in air $\lambda = 0.532 \times 10^{-6}$ m, the light refraction indices of LC $n_0 = 1.527$ and $n_e = 1.744$, the low-frequency dielectric constants $\varepsilon_{\parallel} = 9.1$ and $\varepsilon_{\perp} = 4.1$, and the elastic constants $K_{11} = 1.73 \times 10^{-11}N$ and $K_{33} = 2.04 \times 10^{-11}N$. For the flexoelectric coefficients, we took typical values and put $e_{11} + e_{33} = 10^{-11}C\,\mathrm{m}^{-1}$, the dielectric permittivity of photorefractive layers $\varepsilon_{\mathrm{Ph}} \approx 200$, and $T = 300$ °K. For the space-charge electric field $E_{\mathrm{sc}}(q)$, we use the expression $E_{sc}(q) = E_d = iq\frac{k_bT}{e}$, where E_d is the so-called diffusion field [1], and e is the electron charge.

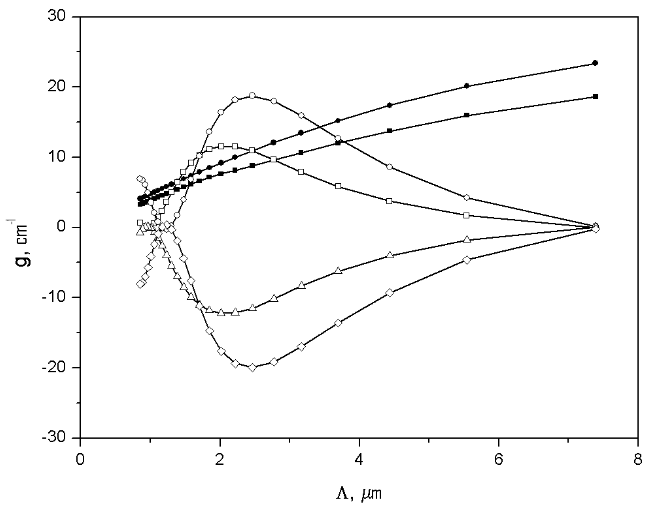
The results of numerical calculations of the gain coefficient g versus the grating spacing $\Lambda = 2\pi/q$ are plotted in the Figure for the LC cell thickness $L = 10$ μm and several values of the director pre-tilt angles θ_1 and θ_2 at the cell substrates.

It is seen from the Figure that the gain coefficient $g(\Lambda)$ significantly depends on the director pre-tilt angles which determine the director spatial profile $\theta_0(z) = \frac{\theta_1+\theta_2}{2} + \frac{\theta_2-\theta_1}{L}z$. In this case, $g(\Lambda)$ possesses an extremum in the area of small $\Lambda \approx 2$ μm, if the second term in the expression for $\theta_0(z)$ is small, and thus the inhomogeneity of the director spatial distribution is low. If the director spatial inhomogeneity is high, i.e. the second term in $\theta_0(z)$ is important, the gain coefficient $g(\Lambda)$ monotonically increases with Λ in the considered area of the grating spacing.

It is worth to note that the gain coefficient maximum in the area $\Lambda \approx 2$ μm was observed experimentally in a hybrid cell filled with LC mixture TL205 [10]. But, in the mixture, the LC molecules possess unequal dipoles, and the energy gain can also depend on the inhomogeneity of the spatial dipole distribution.

6. Conclusions

The two-beam energy transfer at the director diffraction grating recorded in a LC cell by the photorefractive space-charge electric field has been theoretically studied. We used the geometric optics approach to obtain the system of coupled wave equations in an orientationally inhomogeneous LC and calculated the small signal gain coefficient in the case of a flexoelectric LC. We have shown that the inhomogeneity of the director spatial distribution influences not only the gain coefficient but also its dependence on the grating spacing. We believe that the proposed method of derivation of the system of coupled wave equations based on the geometric optics approach can also be used for other inhomogeneous anisotropic media.



Signal gain coefficient versus the grating spacing at different values of the director pre-tilt angles. $\theta_1=\theta_2=-16°$-circles, $\theta_1=\theta_2=-12°$ – squares, $\theta_1=\theta_2=16°$- rhombuses, $\theta_1=\theta_2=12°$ – triangles, $\theta_1=-16°,\ \theta_2=16°$ – black circles, $\theta_1=-16°,\ \theta_2=8°$ – black squares

APPENDIX

$$J_1=\int\limits_{-L/2}^{L/2}a_1(z)dz,\quad J_2=\int\limits_{-L/2}^{L/2}a_2(z)dz, \tag{A1}$$

where

$$a_1(z)=-iE_{sc}(q)\cos(2\delta)\,\frac{\varepsilon_a k_0^2}{2}\,\frac{(e_{11}+e_{33})\,q}{K_{11}\left(\tilde q^2-q_1^2\right)}\times$$

$$\times\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}\,\frac{M_{12}e^{i\Delta k_z z}}{K_{1x}e_{1x}+K_{1z}e_{1z}}\,f_2(z)$$

$$a_2(z)=iE_{sc}(q)\cos(2\delta)\,\frac{\varepsilon_a k_0^2}{2}\,\frac{(e_{11}+e_{33})\,q}{K_{11}\left(\tilde q^2-q_1^2\right)}\times$$

$$\times\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}\,\frac{M_{12}e^{i\Delta k_z z}}{K_{1x}e_{1x}+K_{1z}e_{1z}}\,f_1(z), \tag{A2}$$

$$f_1(z)=-\Bigg(1-i\left(\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}+\sqrt{\frac{\tilde\varepsilon_\perp}{\tilde\varepsilon_\|}}\right)\left[\frac{(\theta_2+\theta_1)}{2}+\right.$$

$$\left.+\frac{1}{4}(\theta_2-\theta_1)-\frac{\tilde q\,(\theta_2-\theta_1)}{L\left(\tilde q^2-q_1^2\right)}\right]\Bigg)\exp\left(q_1\left(z-L/2\right)\right)+$$

$$+\Bigg(1-i\left(\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}+\sqrt{\frac{\tilde\varepsilon_\perp}{\tilde\varepsilon_\|}}\right)\left[\frac{(\theta_2+\theta_1)}{2}-\right.$$

$$\left.-\frac{\tilde q\,(\theta_2-\theta_1)}{L\left(\tilde q^2-q_1^2\right)}+\frac{(\theta_2-\theta_1)\,z}{2L}\right]\Bigg)\exp\left(\tilde q\left(z-L/2\right)\right), \tag{A3}$$

$$f_2(z)=-\Bigg(1+i\left(\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}+\sqrt{\frac{\tilde\varepsilon_\perp}{\tilde\varepsilon_\|}}\right)\left[\frac{(\theta_2+\theta_1)}{2}-\right.$$

$$\left.-\frac{1}{4}(\theta_2-\theta_1)+\frac{\tilde q\,(\theta_2-\theta_1)}{L\left(\tilde q^2-q_1^2\right)}\right]\Bigg)\exp\left(-q_1\left(z+L/2\right)\right)+$$

$$+\Bigg(1+i\left(\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}+\sqrt{\frac{\tilde\varepsilon_\perp}{\tilde\varepsilon_\|}}\right)\left[\frac{(\theta_2+\theta_1)}{2}+\right.$$

$$\left.+\frac{\tilde q\,(\theta_2-\theta_1)}{L\left(\tilde q^2-q_1^2\right)}+\frac{(\theta_2-\theta_1)\,z}{2L}\right]\Bigg)\exp\left(-\tilde q\left(z-L/2\right)\right), \tag{A4}$$

$$M_{12}=e_1\cdot\begin{pmatrix}-2\theta_0(z) & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 2\theta_0(z)\end{pmatrix}\cdot e_2,$$

$$\theta_0(z)=\frac{\theta_1+\theta_2}{2}+\frac{\theta_2-\theta_1}{L}z, \tag{A5}$$

$$q_1^2=q^2\frac{K_{33}}{K_{11}}-(e_{11}+e_{33})\,\frac{(e_{11}+e_{33})}{K_{11}\tilde\varepsilon_\perp}\,\frac{(\theta_2-\theta_1)^2}{L^2},\quad \tilde q=q\sqrt{\frac{\tilde\varepsilon_\|}{\tilde\varepsilon_\perp}}. \tag{A6}$$

Here, e_{11}, e_{33}, K_{11}, K_{33} are the flexoelectric and elastic coefficients of LC, respectively; $\tilde\varepsilon_\|$ and $\tilde\varepsilon_\perp$ are the components of the LC static dielectric tensor along and perpendicularly to the director.

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ОБМІН ЕНЕРГІЄЮ ПРИ ДВОПРОМЕНЕВІЙ ВЗАЄМОДІЇ В ОРІЄНТАЦІЙНО НЕОДНОРІДНОМУ РІДКОМУ КРИСТАЛІ

М.Є. Певний, І.П. Пінкевич, В.Ю. Решетняк

Р е з ю м е

Теоретично розглянуто двопроменевий обмін енергією на дифракційній ґратці директора у флексоелектричній рідкокристалічній комірці. Ґратка записується електричним полем просторового заряду, яке проникає в рідкий кристал з фоторефрактивних підкладок комірки. В наближенні геометричної оптики отримано систему рівнянь для зв'язаних хвиль і розраховано коефіцієнт підсилення сигнальної хвилі. Показано, що орієнтаційна неоднорідність рідкого кристала суттєво впливає на величину і характер залежності коефіцієнта підсилення від періоду ґратки.