ALIVE OPTICAL VORTICES: PHASE DISLOCATIONS IN DYNAMIC RANDOM LIGHT FIELDS

V.I. VASIL'EV, V.V. PONEVCHINSKY, M.S. SOSKIN

Institute of Physics, Nat. Acad. Sci. of Ukraine
(46, Nauky Ave., Kyiv 03680, Ukraine; e-mail: marats@vortex.kiev.ua)

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This article deals with the space-time dynamics of phase dislocations in a nonstationary generic scalar random light field which has not been studied yet. We investigate experimentally the light field that appears as a result of the photoinduced light scattering in $LiNbO_3$:Fe crystal. The elaborated original method of phase reconstruction makes it possible to observe all stages of wave front transformations during the optical vortices evolution, including their nucleation and annihilation. These processes are described correctly by a suggested mathematical model for a moving phase dislocation curve.

1. Introduction

Wave field singularities (points with undefined parameters) are widespread in solid-state physics, physics of liquid crystals, hydrodynamics, acoustics, quantum mechanics, etc. [1, 2]. The most intensive investigation of singularities is carried out in optics. Singular beams appear as a result of interference, diffraction, scattering, propagation of light in nonlinear medium, laser generation, etc. The full destructive interference is the common condition for the phase dislocation existence [1]. In 2D, singularities are the points in a light field cross-section, where the amplitude has zero value, and the phase is undefined [3]. These points make up curves in 3D. Following the analogy with periodicity defects in crystals, these lines are called phase dislocations. The constant phase surface is a helicoid, and there is a vorticity of the Poynting vector around the dislocation line. The phase of the light wave varies over $2\pi m$ for one closed contour revolution around the dislocation line. The signed integer m is called the topological charge. Generic optical fields contain optical vortices with charges $m = \pm 1$. The two-dimensional pattern of an optical vortex (obtained in an observation plane) is a complex structure which consists of a point defect (core) and a vortical phase distribution in its neighbourhood (local structure) [2]. Optical vortices always nucleate (annihilate) in pairs (two vortices with opposite charges) due to the conservation law of topological charge.

There are a lot of papers devoted to the evolution of a spatial configuration of phase dislocations during an observation plane movement [4–6]. This movement causes a displacement of vortices, their nucleation as a pair, when a phase dislocation touches the observation plane, and their annihilation, when the pair leaves this plane. The other area of researches (the parametric dynamics) deals with changes of a field pattern in the fixed observation plane caused by the alteration of some parameters of an optical system such as changing the Gaussian beam waist [7] or the lens entrance aperture [8, 9].

All previous researches did not observe the generic space-time dynamics of optical vortices in non-stationary speckle fields. Such fields are generated when light interacts with changeable random scattered media. They appear, for instance, when a light beam passes through the turbulent atmosphere or a flow of fluid or when a laser beam is used for the diagnostics of a rough surface shift, etc. Such investigations are of great significance for fundamental science and practical applications.

2. Experimental Technique and Results

We were first who suggested [10,11] to use the photo-induced light scattering by photorefractive crystals as a model of non-stationary speckle field. The propagation of a laser beam through the photorefractive medium induces a continuously varying speckle pattern structure [12]. The speed of development of photo-induced microinhomogeneities in a crystal depends on the incident beam intensity. Its appropriate value gives a suitable speed of the field evolution for a detailed investigation of the nucleation (annihilation) of vortices.

The experimental interferometry setup is shown in Fig. 1. The interference pattern is registered with a CCD camera. The obtained pixel map is used to reconstruct the phase distribution (Fig. 2). By opening the shutters in turn, we get the interference pattern with horizontal Φ_x and vertical Φ_y fringes. As the scattering evolution is a very slow process (it takes ~ 30 min), we can be sure

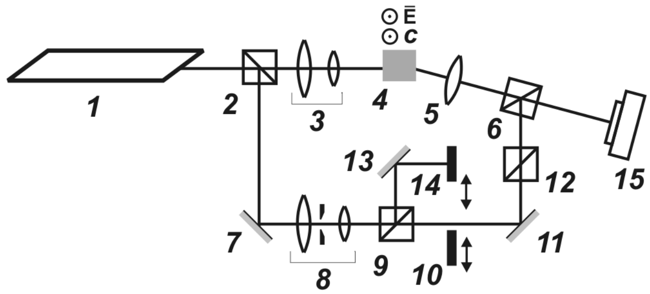

Fig. 1. Experimental setup. *1* – 15-mW He-Ne laser, *2, 6, 9, 12* – beam-splitters, *4* – $LiNbO_3$:Fe crystal, *3* – telescope, *8* – spatial filter, *5* – projecting lens, *7, 11, 13* – mirrors, *10, 14* – electromechanical shutters, *15* – CCD camera. Mirror *11* and beam-splitter *6* are regulated in such a way that the system of vertical fringes appears when shutter *10 (14)* is open (closed). The opposite position of the shutters makes the system of horizontal fringes

that there are no changes in the speckle field during the time of one cycle of the shutters movement (0.02 s). The Φ_x and Φ_y distributions give two different values of the optical vortex position. The difference between them is smaller than the interference fringe width, which defines an error of this method. The arithmetic mean of these two values is used as the true vortex position.

The modified Hilbert transformation method was used to reconstruct the phase distribution from an interference pattern in the range of [0, 2π] [13]. The phase unwrapping is based on both a discrete cosine transform algorithm for the potential phase component [14] and a computing method for the vortical phase component [15]. The unwrapped phase can be written as

$$\Phi(x,y) = IDCT\left(\frac{DCT(\rho(x,y))}{2\left[\cos\left(2\pi\frac{x}{L_x}\right)+\cos\left(2\pi\frac{y}{L_y}\right)-2\right]}\right)+$$

$$+\mathrm{Im}\left(Ln\left(\frac{\prod\limits_{k=1}^{K+}(x-x_k^+)-i\cdot(y-y_k^+)}{\prod\limits_{k=1}^{K-}(x-x_k^-)-i\cdot(y-y_k^-)}\right)\right),$$

where DCT (IDCT) is direct (inverse) discrete cosine transforms, $L_x$and L_y are the image sizes, i is the imaginary unity, x_k^+,y_k^+ and x_k^-,y_k^- denote the coordinates of optical vortices with positive and negative topological charges, and $K+$ and $K-$ – denote the numbers of these vortices. The quantity $\rho(x,y)$ is defined as

$$\rho(x,y) = W(\Phi_x(x+\Delta x,y)-\Phi_x(x,y))-$$

$$-W(\Phi_x(x+\Delta x,y-\Delta y)-\Phi_x(x,y-\Delta y))+$$

$$+W(\Phi_x(x,y+\Delta y)-\Phi_x(x,y))-$$

$$-W(\Phi_x(x,y)-\Phi_x(x,y-\Delta y)).$$

The operator $W(p)$ reduces the difference between phase values to the range of [0, 2π].

If any region of the field contains an optical vortex, $\oint\limits_C d\,\Phi(x,y) = \pm 2\,\pi$. Here, Φ is the phase distribution, and the path of integration C is a boundary of the domain, which is usually taken to be a square with two pixels on a side.

The original features of our method are a new way of the estimation of $\rho(x,y)$ and the determination of the coordinates of vortices by two interference patterns. Its application allows one to obtain a better accuracy of the detection of optical vortices and to increase the noise-to-signal ratio of the phase reconstruction in comparison with methods based on the analysis of one fringe pattern.

The experimentally obtained transformation of a phase structure which results from the nucleation of a pair of optical vortices is shown in Fig. 2,*a*–*d*. A quick change of the phase arises before the nucleation of vortices (Fig. 2,*a*–*c*). Then the pair of nucleated vortices appears (Fig. 2,*d*). The line of a phase discontinuity of the first kind is situated between two vortices. It is a contour of equal phase value determined by initial conditions. The annihilation and relaxation processes proceed in the reversed order (Fig. 2,*e*–*h*).

It is appropriate to describe the speckle field by using its gradient. The appearance of the phase gradient magnitude maximum precedes the nucleation of the pair of optical vortices (Fig. 3). The value of the maximum increases till the time moment of the nucleation of optical vortices (Fig. 3,*a*–*c*). After the annihilation, the value of the maximum is continually decreasing until its value becomes comparable with phase fluctuations of the speckle field (Fig. 3,*d*–*f*).

3. Theoretical Model

A simple model of variable light field is a moving phase dislocation line. Let us take the (x,y) plane as the observation plane. If the phase dislocation moves with velocity v along the z-axis, the light field can be described by the equation

$$E(x,y,z,t) = x^2 + b\,y^2 - c+$$

$$+2\,i\,(1+b)\,(z+v\,t+\gamma_1\,x+\gamma_2\,y), \qquad (1)$$

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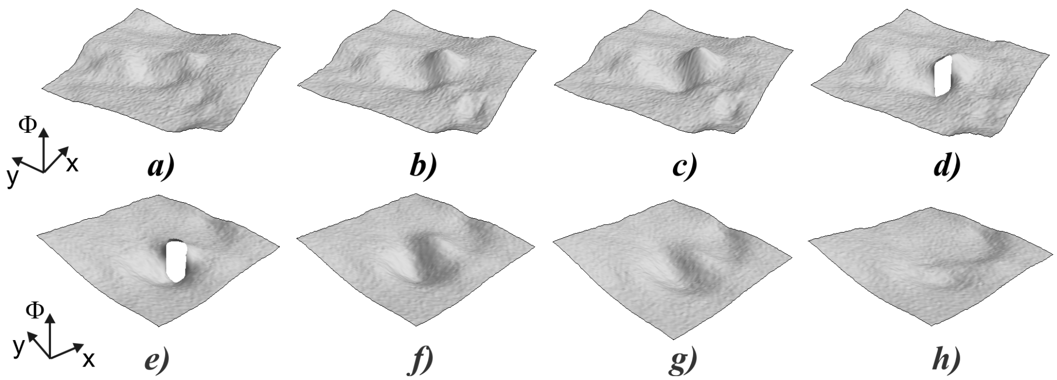

Fig. 2. Nucleation (*a–d*) and annihilation (*e–h*) of a pair of optical vortices. The plotted region is a square of 700 μm on a side. The value of the phase varies from 0 to 2π. The plots show the evolution stages with the time interval $\Delta t = 75$ s

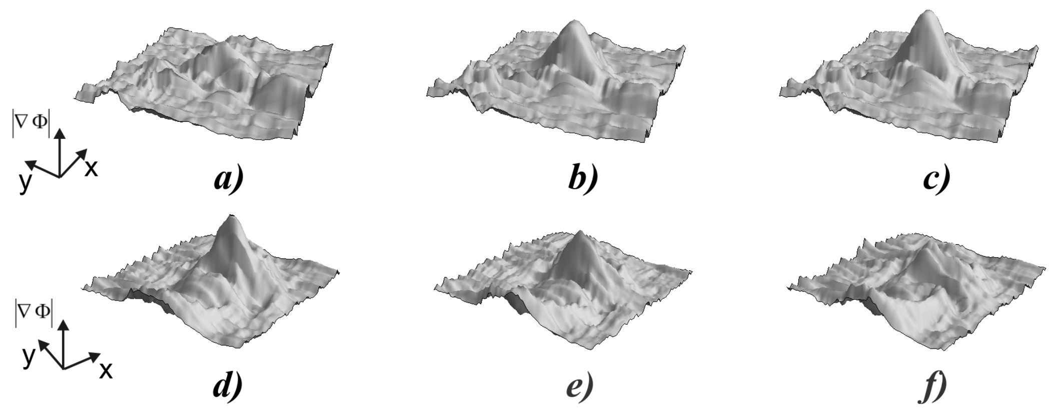

Fig. 3. Evolution of the phase gradient distribution during the nucleation (*a–c*) and annihilation (*d–f*) of a pair of optical vortices. The field regions plotted in (*a–c*) and (*d–f*) are the same as in Fig. 2,*a–c* and Fig. 2,*f–h*

where $b < 0$, $c < 0$, γ_1 and γ_2 define the tilting of the plane with a dislocation to the z-axis. It is a good approximation of a real phase dislocation in the speckle field [5] near a region of the nucleation or annihilation of optical vortices. If $v = 0$, we get the light field with the phase dislocation ring at rest [9].

The wave function $E(x, y, z, t)$ satisfies the paraxial approximation of the scalar wave equation [16]. Its real $Re(E(x, y, z, t))$ and imaginary parts $\mathrm{Im}(E(x, y, z, t))$ are equal to zero on the dislocation line. We take a half-ring of the phase dislocation which touches (leaves) the observation plane for description of the nucleation (annihilation) of a pair of optical vortices. The variation of t changes a relative position of the phase dislocation half-ring and the observation plane (Fig. 4,*a–d*). When the dislocation line crosses the observation plane, two optical vortices with opposite charges nucleate (Fig. 4,*d*).

The phase distribution $\Phi(x, y)$ of the wave function in the plane of observation z_p depends on the phase of the complex function (1):

$$\Phi(x, y, t) = \mathrm{arctg}\left(\frac{2\ (1 + b)\,(z_p + v\,t + \gamma_1\,x + \gamma_2\,y)}{x^2 + b\,y^2 - c}\right).$$

The model gives the following time dependence for the phase gradient maximum $M(t)$:

$$M(t) =$$

$$= \max\left(\left|\nabla \mathrm{arctg}\left(\frac{2\ (1 + b)\,(z_p + v\,t + \gamma_1\,x + \gamma_2\,y)}{x^2 + b\,y^2 - c}\right)\right|\right). \tag{2}$$

This model can be also used to describe the dynamics of lines with circular polarization (C-lines) in vector light fields which always can be resolved into a sum of two fields with right- and left-hand circular polarizations.

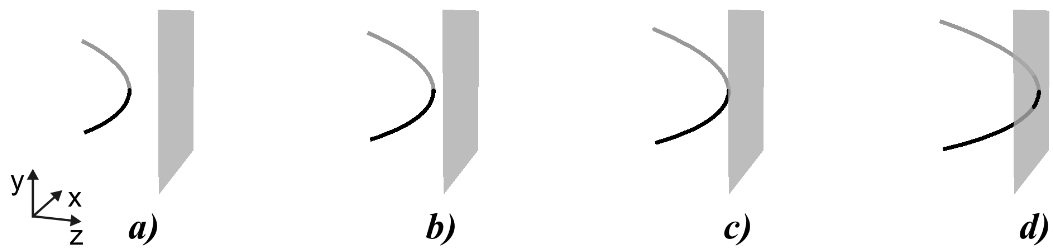

Fig. 4. Location of the phase dislocation curve relative to the observation plane (a-d). The black (grey) line is the line of the vortex with $m = +1(-1)$. The plotted field region coordinates are $-8< x <8$, $-8< y <8$, $10< z <13$. The observation plane is placed at $z_p = 5.5$. The model parameters are $b = 1$, $c = 1$, $\gamma_1 = 5$, $\gamma_2 = 0$, and $v = -0.5$ s^{-1}. Episode (a) corresponds to $t = -9$. Plots (b–d) are build with the increment of time $\Delta t = 4$ s. At $t = 0$, the phase dislocation curve touches the plane of observation

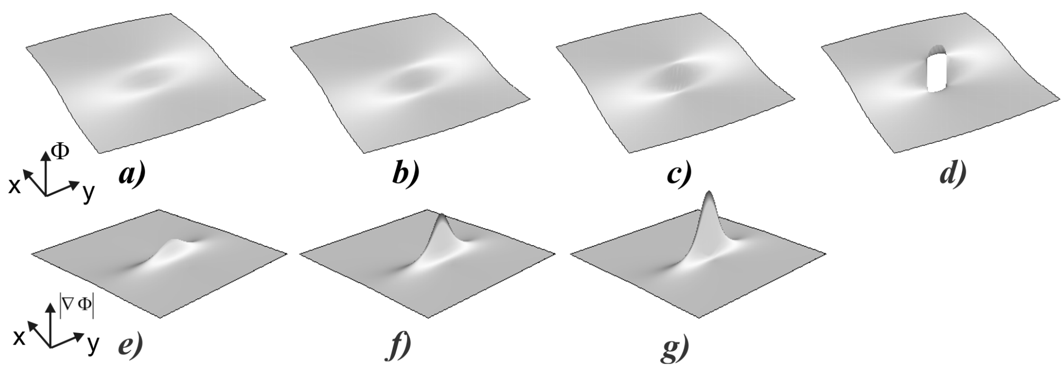

Fig. 5. Calculated phase distribution in the observation plane (a–d). The phase varies from 0 to 2π. The calculated distribution of the phase gradient magnitude in the observation plane is shown in (e–g). The model parameters are the same as in Fig. 4

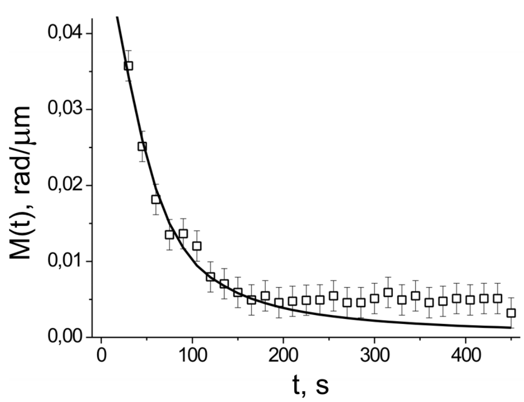

Fig. 6. Time profile of the phase gradient maximum $M(t)$ (2). The empty boxes show experimental data, the solid line is the theoretical dependence for $b = 1$, $c = 1$, $\gamma_1 = 5$, $\gamma_2 = 0$ and $v = 0.087$ s^{-1}

4. Discussion and Conclusion

The elaborated model of moving phase dislocation can be used for the interpretation of experimental results. The phase distribution for different positions of the phase dislocation half-ring is presented in Fig. 5,a–d). When the half-ring approaches the observation plane, the surface of the phase distribution gets an increasing fold (Fig. 5,a–c). The anisotropic region with a large value of the phase gradient $|\nabla \Phi(x, y, t)|$ is formed before the nucleation of vortices (Fig. 5,e–g). It is stretched in the line of the plane containing the dislocation. When the dislocation intersects the observation plane, two phase helicoids are born at the point, where $|\nabla \Phi(x, y, t)|$ takes its maximum (Fig. 5,d). They are twisted in opposite directions. The processes of nucleation and annihilation are completely reversible. The same fragment of the large phase gradient remains after the annihilation of vortices.

We use the averaged data of several annihilation events to get the time dependence of the phase gradient maximum (Fig. 6). The theoretically constructed function (2) is a good fit for the experimental data obtained right after the annihilation. The discrepancy between the theoretical function and the experimental data comes at $t > 200$ s. This happens because the model does not take other phase dislocations and other phase maxima and minima of random fields into consideration. The time moment $t = 200$ s can be considered as an estimation of the decomposition residual termination time. The same estimation can be used for the beginning of the formation of a nucleus.

As a result, a nucleus (a residual) which is the region of a large phase gradient exists before the nucleation (after annihilation) of a pair of optical vortices. This fact has its indirect proof in conclusions made in [6], where the propagation of a beam with singularities through the turbulent atmosphere was observed. The theoretically developed dependence can be used as a good approximation for the experimental data obtained near the nucleation (annihilation) region. The existence of residuals gives some possibilities to reconstruct the initial distribution of critical points and so to create the light wave systems with better noise immunity. It can be applied to the transmission of the data encoded with a distribution of critical points of a laser beam through the turbulent atmosphere.

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ЖИВІ ОПТИЧНІ ВИХОРИ: ФАЗОВІ ДИСЛОКАЦІЇ У ДИНАМІЧНОМУ ВИПАДКОВОМУ СВІТЛОВОМУ ПОЛІ

В.І. Василʼєв, В.В. Поневчинський, М.С. Соскін

Р е з ю м е

Роботу присвячено вивченню раніше не досліджуваній просторово-часовій динаміці фазових дислокацій у нестаціонарних випадкових світлових полях. Експериментально досліджено оптичне поле, що виникає в результаті фотоіндукованого розсіювання у $LiNbO_3$:Fe. Розроблена оригінальна методика відновлення фази інтерферограм, яка дозволяє дослідити процеси народження та анігіляції пар оптичних вихорів. Запропонована математична модель перетину фазовою дислокацією площини спостереження, яка коректно описує процес топологічної реакції.