INSTANTANEOUS DETECTION OF THE COMPLEX AMPLITUDE OF COHERENT OPTICAL WAVE FIELDS

A. KHIZHNYAK, V. MARKOV

MetroLaser Inc.
(8, Chrysler, Irvine, CA, 92618)

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Conventional optical detectors such as CCDs, CMOS image sensors, or photographic films, being well-suited for the detection of high-quality images with a wide dynamic range of intensity variances, do not allow the detection of the complex amplitude of an optical wave field. Still the complete information on such fields is described not by its intensity, but by its complex amplitude. This paper is concerned with the instantaneous detection of the complex amplitude of an optical wave field by using an advanced phase-shifting interferometry technique. The proposed method allows the detection of the complete information on the optical wave field and the retrieval of its complex amplitude at an arbitrary position along its propagation path.

1. Introduction

Optical wave fields are electromagnetic by their nature, and their propagation is described by a system of Maxwell equations. In the scalar approximation, the complete information on the wave can be expressed through the complex amplitude function $\mathrm{A}(\mathbf{r}) = \mathrm{A}_0(\mathbf{r})\exp[i\varphi(\mathbf{r})]$, where $\mathrm{A}_0(\mathbf{r}) = \sqrt{\mathrm{I}_0(\mathbf{r})}$, $\mathrm{I}_0(\mathbf{r})$ is the intensity distribution in the cross-section normal to the wave's propagation vector, and the argument $\varphi(\mathbf{r})$ is related to the phase distribution on the wavefront. Knowledge of the complex amplitude is essential as it enables the retrieval of the structure of an optical wave in an arbitrary region along the propagation path of a light beam.

The detection of optical signals is based on the non-coherent interaction between the radiation and the detector's substance, no matter in which state the latter is: solid state, liquid or gaseous. Such an interaction results in changes of the detector's (physical) properties that are easy to measure, including electroconductivity, temperature, refractive index, *etc.* Since the response of a detector is proportional to the number of absorbed photons or, in other words, to the intensity of an optical wave field, the field phase remains undetermined in the process of detection. This makes it impossible to directly detect and reconstruct the complex amplitude of the optical wave field. Unlike the amplitude, the phase of an optical wave field doesn't incur an absolute value and is estimated though an indirect measurement technique. The major role of the phase of an optical wave is to define the relationship between various points across its wavefront or between two interacting wavefronts (interferometry).

The intensity distribution in an interference pattern contains the information on the product of complex amplitudes of interacting waves (usually called the reference and object waves), i.e. this pattern holds the information about the complex amplitudes of both waves. Since the reference wave is usually a well-known function, the interferometric technique enables the retrieval of the complex amplitude of the object wave.

The first half of the past century was marked by the significant progress in understanding the fundamental aspects of the wavefront formation [1] and synthesis [2–4]. Some of the studies of that time, especially by M. Wolfke [2], came very close to the solution of the problem of synthesis of a complete wavefront. This period culminated with the invention of holography by D. Gabor [5], the method that, by using the detection of the interference pattern of an object and the reference wave, allows the posterior reconstruction of the object wave. Initially, the recording of the hologram was achieved by recording this interference pattern on a sufficiently high-resolution photographic medium. Later on in some of the applications, the photographic film was replaced by optically nonlinear materials. This enabled one to record not only steady-state but dynamic (real time) holograms as well. Finally, the progress in the technology of coupled-charge devices (CCDs) or complementary metal-oxide semiconductors (CMOS) enabled one to use these types of the arrays as the reversible holographic medium and stimulated the establishment of a new branch of holography – electronic holography often used as an effective tool for the real-time non-destructive inspection.

The direct detection of an interference pattern with CCD/CMOS arrays and its straightforward transfer to the computer for the posterior data processing substantially simplified the retrieval of a complex amplitude, yet it remains a computationally extensive process. With this technique, the retrieval of a complex amplitude is equivalent to the physical process of diffraction of the reference wave on a diffraction grating formed as a product of recording the interference pattern in the light-sensitive medium. Obviously, the retrieval of an exact spatial pattern of the complex amplitude requires an adequate accuracy in the detection of interference patterns, since the spatial distribution of the intensity in such a pattern holds a detailed information on the wavefront to be retrieved. Furthermore, the solution of the diffraction problem – the key technique for the retrieval of a complex amplitude – requires intensive computational resources, because high-frequency varying integrands functions should be calculated. As a result, so far this technique didn't find a wide application for the detection of complex amplitudes [6].

Though the detection of the amplitude of an optical wave field can be easily achieved, the phase retrieval remains the concern. An alternative solution to this problem can be found by using the so-called transport-of-intensity equation (TIE), a partial differential equation which relates the phase of an optical disturbance to its intensity and the spatial intensity derivative [7, 8]. The solution of this equation allows one, under certain conditions, to uniquely determine the phase of an optical wave field, given the results of measurements of its intensity and the intensity derivative over some plane. Although this technique is currently under the development for such applications as X-ray phase imaging, optical microscopy, and adaptive optics, its practical implementation remains questionable, and the reliability of the obtained data yet has to be proved for the complex amplitude retrieval.

Finally, the retrieval of a complex amplitude can be performed by using the phase-shifting interferometry (PSI), the technique in which at least four interferograms of the same optical field are recorded sequentially with a controlled phase shift between two interfering waves [9]. However, the time-delayed detection of these four interferograms as required by the standard realization of the PSI technique restricts its application to the analysis of a steady-state complex amplitude and makes it unacceptable for the non-stationary processes, such as pulsed optical wave fields. Below, we show that the use of a novel realization of the PSI method, PhaseCam$^{\mathrm{TM}}$, allows the instantaneous detection of the complex amplitude of an optical wave and consider some of the potential applications of this technique.

2. Detection of the Complex Amplitude of an Optical Wave Field

It is a well-established fact that the characterization of the optical wavefront of a laser beam (for convenience, we will call it the object beam) can be achieved with phase-shifting interferometry [9] for at least three known-valued phase shifts between object and reference waves. Evidently, if the characterization of a pulsed object wave is of interest (for a pulse laser radiation), these measurements should be performed within a single shot. The possibility of the instantaneous detection of a sequence of phase-shifted interference patterns of a CW laser beam has been demonstrated earlier by using MetroLaser PSI – PhaseCam$^{\mathrm{TM}}$ [10]. A typical example of the characterization of the aspherical wavefront by using the PhaseCam$^{\mathrm{TM}}$ is shown in Fig. 1.

Thus, the setting illustrated in Fig. 2 allows the simultaneous detection and measurement of four interference patterns with the successive $\pi/2$ phase shift valued from 0 to 2π between them. The received output signals are

$$I_n\left(x,y\right) = I_{Ob}\left(x,y\right) + I_{Rf}\left(x,y\right) +$$

$$+\sqrt{I_{Ob}\left(x,y\right) I_{Rf}\left(x,y\right)}\cos\left[\Delta\Phi\left(x,y\right) + \delta_n\right] \times$$

$$\times\Delta\Phi\left(x,y\right) = \varphi_{Ob}\left(x,y\right) - \varphi_{Rf}\left(x,y\right);$$

$$\delta_n = \left(n-1\right)\pi/2\,; \quad n = 1,2,3,4. \tag{1}$$

In Eq. (1), $I_{Ob}(x,y)$ and $I_{Rf}(x,y)$ are the intensity distributions of the object and reference beams at the detection plane; $\Delta\Phi(x,y)$ is a phase difference between the object and reference wave; and n is the number of a quadrant in Fig. 2.

As the system detects four interference patterns at once, the first two terms om the right-hand side of Eq. (1) are equal for all four frames, thus producing the subtraction, as it is seen from the following expression:

$$A\left(x,y\right) = \left(I_1\left(x,y\right) - I_3\left(x,y\right)\right) +$$

$$+i\left(I_2\left(x,y\right) - I_4\left(x,y\right)\right) = 4E_{Ob}\left(x,y\right)E_{Rf}\left(x,y\right). \tag{2}$$

Here, $E_{Ob}(x,y)$ and $E_{Rf}(x,y)$ are, respectively, the complex amplitudes of the object and reference beams.

It is evident from Eq. (2) that a trivial combination of the measured intensity distributions in four interference patterns can be used to calculate the product of the reference and object complex amplitude. Consequently, having the known well-defined complex amplitude of the

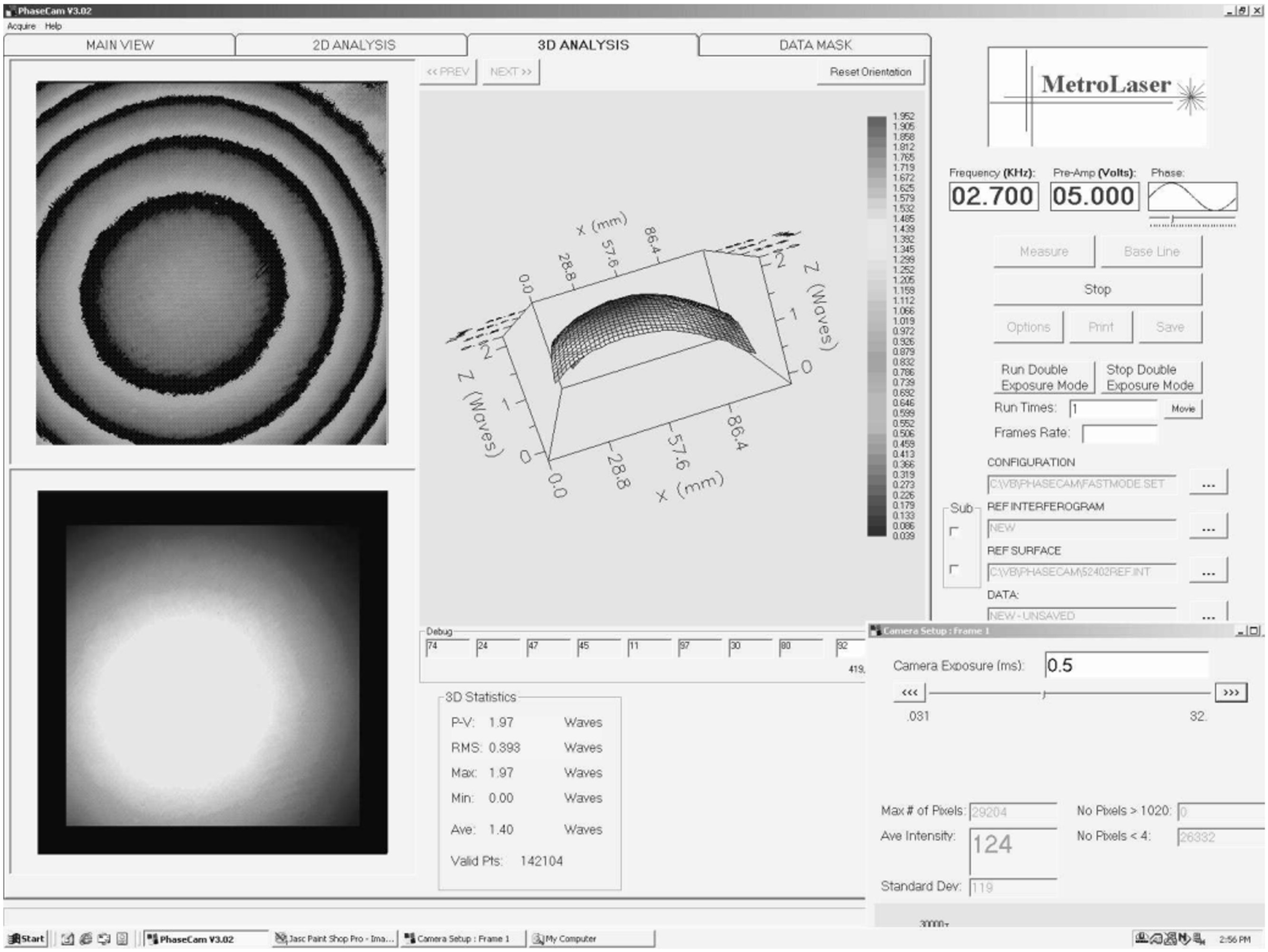

Fig. 1. User Interface of PhaseCam. Left column illustrates the spatial structure of a wrapped (top) and unwrapped (bottom) wavefronts, the central picture shows the 3D shape of the wavefront

reference wave, one can derive the complex amplitude of the object wave.

The wavefront structure is resulted from Eq. (2) as

$$\Phi(x,y) = \arctan\left[\frac{\mathrm{Im}\left[E_{Ob}(x,y)\right]}{\mathrm{Re}\left[E_{Ob}(x,y)\right]}\right] =$$

$$= \arctan\left[\frac{I_2(x,y) - I_4(x,y)}{I_1(x,y) - I_3(x,y)}\right]. \qquad (3)$$

It follows from Eq. (3) that the spatial structure of the wavefront is completely determined by the intensity distribution in four quadrants. It is important to emphasize that the solution of expression (3) for the waves with the branch-points is undetermined at these points, thus resulting in difficulties with the retrieval of a true structure of the wavefront. On the other hand, both the real and imaginary parts of a complex amplitude are continuous functions that turn to zero at the branch points. The analysis of the complex amplitude in a vicinity of the branch point is simple and allows readily the retrieval of the complete wavefront. A typical example of the calculated three-dimensional shape of the wavefront is illustrated in Fig. 1.

As mentioned before, knowledge of the complex amplitude of an optical wave field is essential for the variety of applications, including, the characterization of the atmospheric turbulence [11, 12], 3D imaging [13], laser beam control [14], *etc.* Now, let us consider some practical applications of the above-discussed technique of direct detection and retrieval of the complex amplitude of an optical wave field.

3. Atmosphere Turbulence Characterization

The characterization of the atmospheric turbulence is traditionally made through the detection and analysis of a spatial distribution of the probe beam intensity [11,12], and it is commonly assumed that the detection of the wave field for this probe beam is an unrealizable task [11]. However, it is evident that should the complex amplitude of this beam be recorded and retrieved, for example, at the pupil plane, this would enable calculating all orders of the correlation functions, especially the mutual coherence function (MCF) or the two-point function [11]. The latter one is an essential parameter for estimating the characteristics of the turbulent atmosphere, because the solution for different atmospheric models, at various levels of approximations, is obtained by applying this function [11,12]. However, the use of standard detection methods, based on the recording of the light intensity distribution only, makes it impossible to measure the MCF, since such detection methods ignore the information on the lightwave front. Presumably, it should be the reason for the conclusion that a "2-point function is unphysical and cannot be directly measured" made in review [11]. This seems to become a commonly accepted opinion, and most efforts have been directed toward the analysis of the correlation features of the intensity distribution of a received signal, i.e., the 4-point correlation function, although the solution of the function doesn't exist, even in the Rytov approximation [12].

It is evident from Eq. (2) that a simple combination of the measured intensity distributions in four interference patterns can be used to calculate the product of the reference and object wave fields. Then with the known field distribution of the reference wave, one can derive the complex amplitude of the object wave. Since the object wave is deterministic and has nonzero constant amplitude, Eq. (2) yields

$$\langle E(\mathbf{r},L)\rangle = (\langle I_1(\mathbf{r})\rangle - \langle I_3(\mathbf{r})\rangle)+$$

$$+i(\langle I_2(\mathbf{r})\rangle - \langle I_4(\mathbf{r})\rangle)/(4\sqrt{I_{Ob}}), \tag{4}$$

i.e., the mean field is described by the average intensities detected in each quadrant. In the same way, the 2-point correlation function is expressed through the 2-point correlation function for all quadrants both in the auto and cross multiplicands:

$$\Gamma_2(\mathbf{r}_1,\mathbf{r}_2) = (\langle[\{I_1(\mathbf{r}_1) - I_3(\mathbf{r}_1)\} + i\{I_2(\mathbf{r}_1)-$$

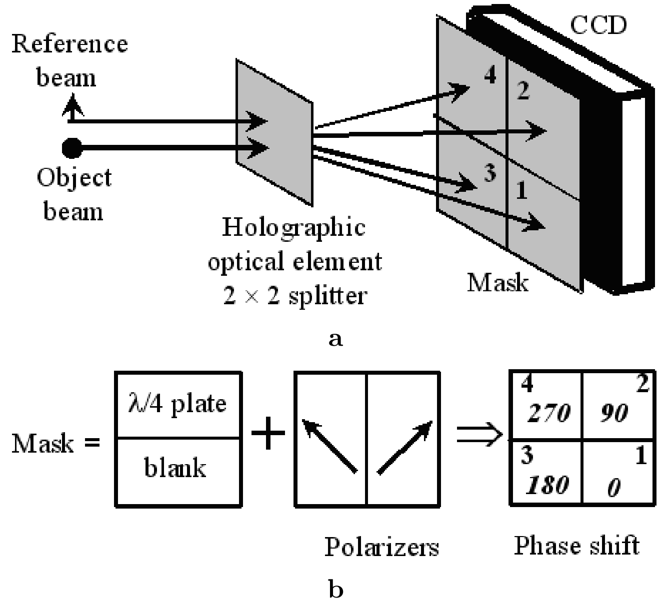

Fig. 2. *a* – the scheme for the instantaneous registration of four interference patterns; *b* – the design of the mask

$$-I_4(\mathbf{r}_1)\}][\{I_1(\mathbf{r}_2) - I_3(\mathbf{r}_2)\}-$$

$$-i\{I_2(\mathbf{r}_2) - I_4(\mathbf{r}_2)\}]\rangle)/(16I_{Ob}). \tag{5}$$

It follows from Eqs. (4) and (5) that the procedure of deriving the mean field and the MCF can be reduced to the computation of the correlation and cross-correlation functions of the intensity distributions in four quadrants. This procedure can be performed in a relatively simple way, i.e., the described system allows the real measurement of the MCF, contrary to the previously made assumption on the non-physical nature of the MCF.

4. MCF of a Spherical Wave

The following results serve to demonstrate the capabilities of the proposed approach in detecting and deriving the MCF of a spherical wave. In the case of a pure spherical wave with wavefront curvature radius R, the MCF is described by a simple expression

$$\Gamma_2\left(r_1,r_2\right) = E_0\left(r_1\right)E_0\left(r_2\right)\exp\left(ik\,\frac{r_1^2-r_2^2}{R}\right) =$$

$$= E_0\left(r_1\right)E_0\left(r_2\right)\left[\left(k\frac{r_1^2-r_2^2}{R}\right) + i\,\sin\left(k\frac{r_1^2-r_2^2}{R}\right)\right], \tag{6}$$

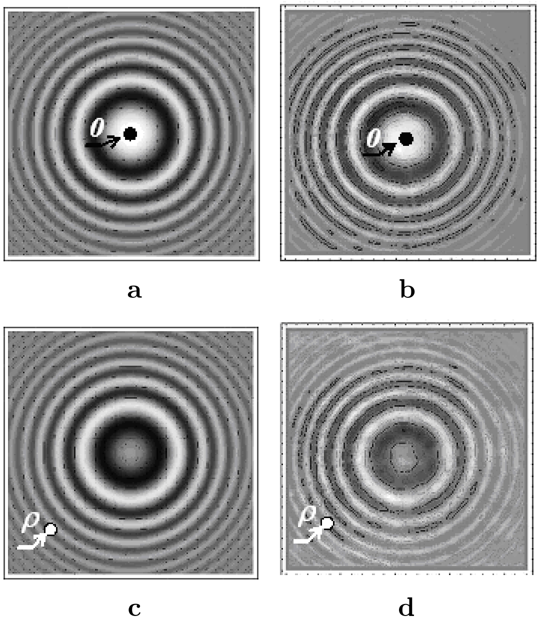

Fig. 3. Real part of the MCF for a spherical wave: *a*) and *c*) are simulated data, *b*) and *d*) are measured by PhaseCam. *a*) and *b*) are Γ_2(r,0); *b*) and *d*) are Γ_2(r, ρ), the location of the points 0 and ρ is marked in all the pictures

where R is the curvature radius of the spherical wave.

Figure 3 shows a collection of typical examples of the simulated and measured real parts of the MCF for a spherical wave. These figures clearly demonstrate that the PhaseCam-based technique allows measuring the MCF of coherent laser beams.

In a next collection of experiments, the simulation of an impact of the atmospheric turbulence effect on a transmitted laser beam was made by using a random phase screen with the Kolmogorov spectrum. The maximum phase shift introduced by this screen was of the order of 1.5 λ. For introducing a substantially stronger effect on the wave-function structure, we performed the experiments using a laser beam with the radius curvature of the spherical wavefront higher than that in Fig. 3. The pictures of the MCF of these measurements are shown in Fig. 4.

5. 3D Image Restoration

We will now analyze the possibility of the synthesis of a volumetric scene, by using the wave-function reconstructed from the object-scattered waves, and advantages of such an approach for the 3D imaging. The monochromatic wave in a free space is described by its

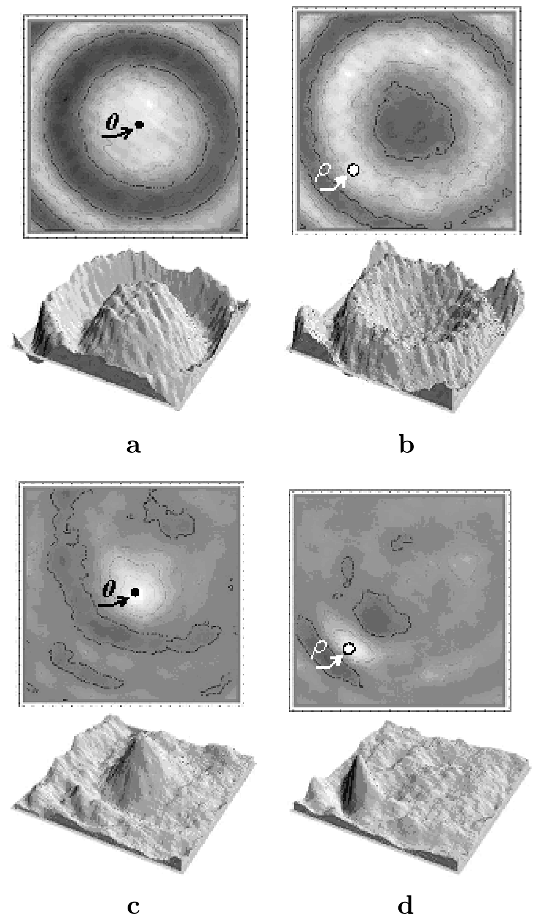

Fig. 4. Real part of the MCF for a pure spherical wave (*a* and *b*) and for the same wave after its distortion by a random phase screen (*c* and *d*)

electric field

$$\mathbf{E}(\mathbf{r},t) = \mathbf{e}u(x,y,z)\exp[i\omega_0 t], \tag{7}$$

where ω_0 is the carrier frequency; the vector $\vec{e}$ describes the state of polarization; $u(x,y,z)$ is the scalar complex amplitude of the wave-function.

If the function $u(x,y,z)$ is described for the plane $z=z_1$, then its angular spectrum is

$$F_u(k_x,k_y) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} u(x,y,z_1)\times$$

$$\times\exp[-i(k_x x + k_y y)]\,dx\,dy. \tag{8}$$

The angular spectrum in Eq. (8) defines the plane wave with transverse wave numbers k_x and k_y and the

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longitudinal wave number $k_z = \sqrt{k_0^2 - k_x^2 - k_y^2}$, where $k_0 = n{\cdot}\omega_0/c = 2\pi/\lambda_0$.

Using the angular spectrum F_u given by Eq. (8) makes it possible to compute the wave-function at an arbitrary cross section along the beam path:

$$u(x, y, z_2) = \iint\limits_{k_x, k_y} F\left[k_x, k_y\right] \times$$

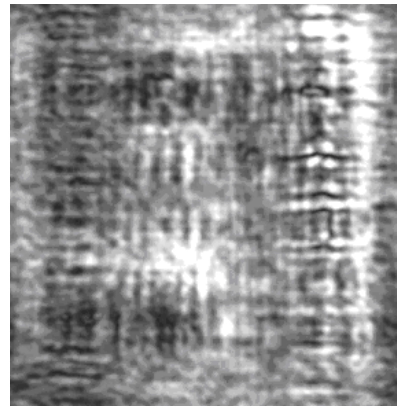
$$\times \exp\left[ik_0\sqrt{1 - \frac{k_x^2 + k_y^2}{k_0^2}}\left(z_2 - z_1\right)\right] \times$$

$$\times \exp\left[i\left(k_x x + k_y y\right)\right] dk_x dk_y. \quad (9)$$

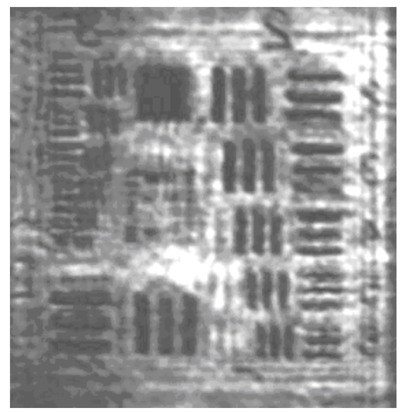
Relation (9) is, in essence, the inverse Fourier transform of the angular spectrum $F[k_x, k_y]$, by virtue of the propagation of a plane wave between the initial plane $z = z_1$ and the plane of interest z_2.

Thus, the detection of the complex amplitude at a particular plane enables one to reconstruct this function at any arbitrary plane along the direction of propagation of this wave. A three-step computation can be used for that: (1) fast Fourier transformation (FFT) of the complex amplitude at the detection plane (z_1); (2) multiplication of the derived spectrum by the phase-function defined by a phase shift of plane waves (parabolic, in the paraxial approximation); (3) inverse FFT of the modified angular spectrum. Obviously, such a process of image restoration involves a well-developed FFT routine, fast enough for a real-time operation.

Since the square of the complex amplitude absolute value is the intensity distribution at a given cross section, the measurement of the wave-function at an arbitrary plane allows the complete restoration of the object wave along its propagation path, i.e. the restoration of an image of the 3D object. An example of such an operation is illustrated in Fig. 5.



a



b

Fig. 5. Intensity distribution of the optical field transmitted through the Air Force resolution target (AFRT). The AFRT is located at about $L = 20$ cm from the PhaseCam detection plane. a) Intensity distribution at the detection plane; b) recalculated intensity distribution at the AFRT plane

6. Conclusion

This paper discusses an innovative approach for the detection of an optical wave function and the retrieval of its complex amplitude using an original implementation of the phase-shifting interferometry method. The proposed technique allows the instantaneous detection of the complex amplitude of an optical wave field, thus enabling one not only to compute a complete wavefront of a laser beam containing wavefront dislocations, but also the retrieval of its complex amplitude at an arbitrary plane along the propagation path, i.e. the restoration of the wavefront at this plane. If the laser beam control is of interest, such a possibility allows computing a precise topology of the adaptive optics mirror needed for the correction of a laser beam wavefront. Placing such an adaptive optics mirror at the position corresponding to the detected complex amplitude should allow the efficient beam control operation.

We should emphasize that, unlike the Shack–Hartmann system, the above-described sensor enables one to instantaneously perform the true authentic measurements of wavefronts of arbitrary complexity, including those with branch points.

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МИТТЄВА РЕЄСТРАЦІЯ КОМПЛЕКСНОЇ АМПЛІТУДИ КОГЕРЕНТНИХ ОПТИЧНИХ ХВИЛЬОВИХ ПОЛІВ

А. Хижняк, В. Марков

Р е з ю м е

Звичайні оптичні детектори, такі як прилади із зарядовим зв'язком, датчики зображення з комплементарною МОН-структурою, чи фотоплівки, хоча вони добре пристосовані для реєстрації високоякісних зображень з широким динамічним діапазоном за інтенсивністю, не дозволяють визначити комплексну амплітуду оптичного хвильового поля. Однак повна інформація про таке поле надається його комплексною амплітудою, а не інтенсивністю. В роботі розглянуто проблему миттєвої реєстрації комплексної амплітуди оптичного хвильового поля з використанням розвинутої техніки інтерферометрії з фазовим зміщенням. Запропонований метод дозволяє одержати повну інформацію про оптичне хвильове поле та відновити його комплексну амплітуду в будь-якій точці вздовж шляху його поширення.