OPTIMIZATION AND NEW APPLICATIONS OF A MAGNETIC TRAP FOR ULTRA-COLD NEUTRONS[1]

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We discuss some problems related to the physics and dynamic behavior of ultra-cold neutrons (UCNs) in a magnetic trap. First, we present the results of our computer simulations for a permanent-magnet neutron magnetic trap suggested by J.D. Bowman and P.L. Walstrom. We demonstrate how to optimize parameters in order to minimize the "cleaning time" in the trap. Next, we propose using a magnetic trap for UCNs as an ultra-sensitive method for measuring the neutron magnetic resonance. We also propose using this method to measure the decoherence time of neutron spins and to measure more accurately the neutron gyromagnetic ratio.

1. Introduction

The main objective of building a magnetic trap for UCNs is to study the free neutron decay, in particular, to more accurately measure the free neutron lifetime which is approximately 886 s [1]. The neutron decay

$$n \rightarrow p + e^{-} + \bar{\nu}_e \tag{1}$$

is a classical manifestation of the weak interaction. It is associated with the decay of the down (d) quark into the up (u) quark

$$d \rightarrow u + e^{-} + \bar{\nu}_e. \tag{2}$$

The neutron decay depends on the weak coupling constants. The data obtained from the neutron decay are widely used in astrophysics and particle physics. The creation of a neutron magnetic trap would enable a more accurate determination of the weak coupling constants, which will have an important impact on many fields of physics. The magnetic storage of neutrons was initially proposed by V.V. Vladimirskii [2]. W. Paul *et al.* [3] demonstrated the first neutron magnetic trap that was based on superconducting magnets. V.F. Ezhov *et al.* [4] reported the UCN storage in a magnetic trap made of permanent magnets. J.D. Bowman and S.I. Penttila [1] proposed an experiment to measure the neutron lifetime with accuracy of about 10^{-4} s. In their paper, they considered a vacuum quadrupole trap created by a superconducting quadrupole magnet. The magnetic field has a minimum in the trap. The magnetic force captures those neutrons with their magnetic moments parallel to the field and pushes neutrons with the opposite magnetic moment out of the trap. Because the neutron's gyromagnetic ratio is negative, the neutrons with spin "down" (relative to the magnetic field) are captured by the trap.

The neutron density is the highest at the center of a trap. A small number of neutrons are trapped near the edges of the trap. The neutron decay in the trap can be studied using scintillation detectors that detect electrons with the energies up to 0.78 MeV. The component of the neutron spin along the magnetic field in the trap is an adiabatic invariant. The depolarization of neutrons is expected to be negligible. (It is estimated to be 10^{-25} in 10^4 s.) The major problem with the neutron traps is associated with the so-called quasi-bound neutron orbits.

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Quasi-bound neutrons may have long escape times, much longer than a few seconds. They do not allow an accurate measurement of the neutron lifetime because the number of trapped neutrons becomes uncertain.

Later on, J.D. Bowman and P.L. Walstrom (BW) [5] proposed a new design for a magnetic neutron trap that would reduce the escape time. Instead of a superconducting magnet, they proposed using a permanent-magnet trap. The BW trap has a special geometrical surface (see Fig. 1) covered with small magnets which form a Halbach-type array. It is a special arrangement of permanent magnets that augments the magnetic field on one side of the device while cancelling the field to near zero on the other side (outside of the trap). This trap is expected to be cleaned to the level about 10^{-5} during its "cleaning time" $\tau_{\rm cl}$ which is expected to be much smaller than the neutron lifetime τ_n equal to 886 s.

The first objective of our work is to optimize the BW trap parameters in order to reduce the cleaning (escape) time $\tau_{\rm cl}$ for unbound neutrons. (In other words, for the minimal time $\tau_{\rm cl} << \tau_n$, there have to be no neutrons in the trap that can overcome a potential barrier and escape from the trap. There stay only "cold" (trapped) neutrons, so that the neutron lifetime can be precisely measured ahead of time.) The second objective is the analysis of novel experiments (in addition to the neutron decay) which could be implemented in this magnetic neutron trap. The preliminary results of our numerical simulations are presented in [6].

2. Results of Numerical Experiments and Optimization of Parameters for a Neutron Magnetic Trap

The BW magnetic neutron trap can be approximated by a central flat surface and four cylindrical and four toroidal surfaces (see Fig. 1). We will use the characteristic parameters shown in Fig. 1. The magnetic field inside the trap, produced by the permanent magnets, is given by the following approximate expressions [5]:

$$B_x = \frac{4B_r}{\pi\sqrt{2}} \sum_n \frac{(-1)^n}{4n-3} \sin\left[k_n S(x,y,z)\right] \times$$
$$\times e^{-k_n d(x,y,z)} \left(1 - e^{-k_n D}\right),$$

$$B_y = \frac{4B_r}{\pi\sqrt{2}} \sum_n \frac{(-1)^n}{4n-3} \cos\left[k_n S(x,y,z)\right] \times$$
$$\times e^{-k_n d(x,y,z)} \left(1 - e^{-k_n D}\right), \qquad (3)$$

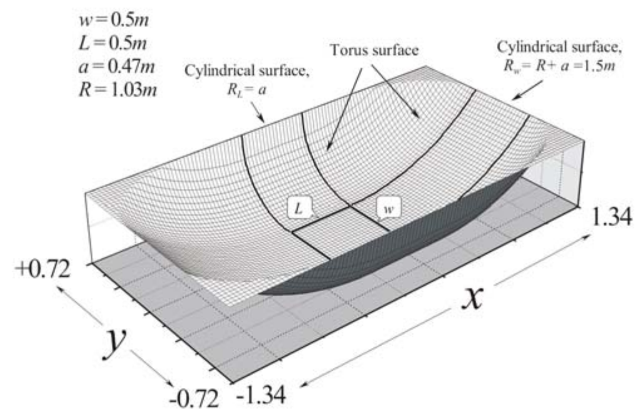

Fig. 1. Geometry of a BW magnetic neutron trap

where $k_n = \frac{2\pi}{\lambda}(4n-3), \lambda = 5.08$ cm, $D = 2.54$ cm is the size of the small magnets, and B_r is the remnant field, $S(x,y,z)$ is the shortest route, which is traced on the trap surface, from an arbitrary point on the surface to the plane XZ, and $d(x,y,z)$ is the distance from an arbitrary point $M(x,y,z)$ inside the trap to the surface.

The equation of motion for a single neutron is

$$\frac{d\mathbf{v}}{dt} = -\frac{\mu}{m}\text{grad}\left[B(x,y,z)\right] + \mathbf{g}. \qquad (4)$$

The magnetic moment of a neutron is $\mu = 9.66236 \times 10^{-27}$ JT, the neutron mass is $m = 1.67493 \times 10^{-27}$ kg, the gravitational acceleration is $g = 9.79$ m/s^2, and $B = \sqrt{B_x^2 + B_y^2}$.

The adiabatic approximation, which is used in our simulations, requires that the magnetic moment of a neutron be oriented in the positive direction of the local magnetic field. The conditions of the validity of the adiabatic approximation have been analyzed in our paper [6].

The flow of UCNs enters the trap through a special channel at the bottom of the trap. On "pumping" UCNs in, the channel is closed. Most of the neutrons don't have enough energy so that to go up to the trap borders. They are trapped and "fly" inside the trap reflecting from the thin slice of the magnetic field that covers the trap surface. The neutrons, which can escape from the trap, are adsorbed with the cleaner that is located in the plane $z = z_{\max} = 42cm$ and seals the trap on the top (see Fig. 2, the energy of escaped neutrons $E \geq mgz_{\max}$).

If we take only the first harmonic in the expressions for B_x, B_y into consideration, then the value of the

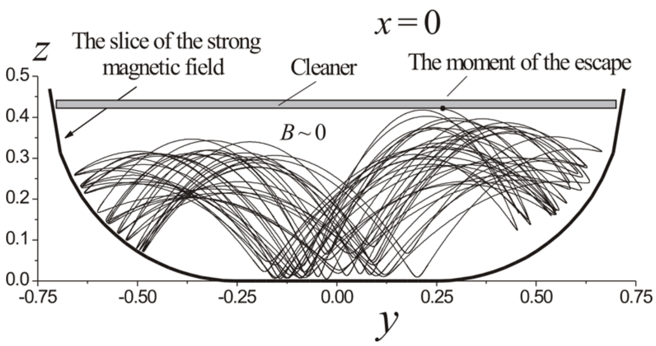

Fig. 2. Trajectory of a neutron which has escaped the trap for some while (projection to the plane $x = 0$)

magnetic field depends only on the distance to the surface, d. In what follows, we call this field a smooth field. Our first numerical calculations were done for a smooth field:

$$B = 0.82 \exp(-kd), \ k \approx 123.7 \ \mathrm{m}^{-1}. \tag{5}$$

About the distribution of neutrons at the initial time. The initial energy is defined by the maximal height which a neutron can reach $h_{\max}$, $E = mgh_{\max}$. For each neutron, its initial coordinates in the central part of the trap ($d \geq 5$ cm) were chosen randomly. After this, the initial kinetic energy was defined at a given point. The direction of the initial velocity was chosen randomly. The number of neutrons in our numerical experiments is equal to 200,000.

As Fig. 3 shows, a relatively large number of neutrons still remain in the trap. These neutrons slowly leave the trap. This undesirable phenomenon can cause an error in the measurement of the neutron lifetime. We consider two possible methods for the improvement of the parameters of the trap.

The first method: Formation of the diffuse reflection from the surface. For the case of a smooth magnetic field, the surface of the trap acts as a "mirror" for neutrons. Thus, the behavior of neutrons is analogous to that of tennis balls which rebound elastically from a smooth surface. The escape of neutrons from the trap can be accelerated, if one assumes that the surface of the trap can act not as a mirror but as a diffuse surface. The corresponding numerical experiments were carried out. We assumed that, after approaching a minimal distance to the surface, the direction of the neutron's velocity changes randomly. The following cases were considered: (i) the whole surface of the trap is diffuse, (ii) only the curved part of

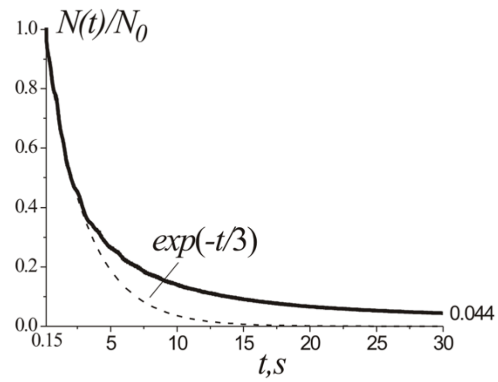

Fig. 3. Fraction of neutrons in the trap as a function of time (in the tail of the distribution), $N_0 = N(t = 0.15s)$ for a smooth magnetic field $z_{\max}$=0.42 m, $h_{\max}$=0.47 m

the trap is diffuse, and (iii) only the flat part of the trap (bottom) is a diffuse one ($h_{\max} = 0.47$ m).

The obtained results display that cases (i) and (iii) work identically well: $N(t = 20 \text{ s}) = 5$, $N(t = 30 \text{ s}) = 0$; in case (ii) $N(t = 30 \text{ s}) = 1400$. (If the whole surface reflects neutrons as a mirror, then $N(t = 30 \text{ s}) \approx 5600$.) Thus, the diffuse reflection only from the trap bottom provides a rapid escape for untrapped neutrons.

As demonstrated below, the role of the flat part of the trap varies – in some cases it improves the trap; in others, it makes the situation worse. The diffuse reflection can be achieved by small-scale variations of a geometry of the surface of the trap bottom or by an inhomogeneous magnetic field. In this connection, we present our results for including the first five harmonics in the expression for the magnetic field, Eq. (3). Note that the higher harmonics decrease rapidly with increase in the distance from the surface. For revealing the high harmonics in the neutron dynamics, the value of B_r in Eq. (3) should be chosen in an optimal way. If the value of B_r is too small, then neutrons in some regions of the trap could penetrate through the magnetic wall. If the value of B_r is too large, then neutrons with total energy near the critical value, $E_c = mgz_{\max}$, cannot approach the surface close enough to be affected by the high harmonics.

For simplicity, we consider the case of the flat bottom of the trap. For a smooth magnetic field, its gradient is oriented normally to the surface, providing a mirror-type reflection surface. If one takes the short-wave components of the magnetic field into account, then the angle α between the normal direction and the gradient of the magnetic field will change along the surface. The following fact must be taken into account. For a given total neutron energy, the minimal distance between this neutron and the surface, $d_{\min}$, is a function of y. (According to Eq. (3), the modulus of the magnetic field changes only along the y-axis.) Consequently, the angle α is a complicated function of y: $\alpha = f\left(B_r\left(d_{\min}(y)\right)\right)$. The reflection of neutrons, at a given angle of approaching the flat surface, will take place at different angles, depending on the point contact with the magnetic wall. Up to some extent, this case is analogous to a diffuse reflecting surface. Naturally, this effect is revealed more explicitly in weaker magnetic fields, B_r =1.07 T, when the neutron can approach the surface more closely. In this case, the speed of the cleaning of a trap increases: $N(t = 30$ s$) = 2800$ – half as great as for the smooth magnetic field. Note that, for B_r=1.3 T, the characteristics of the trap do not change, if the number of harmonics in Eq. (3) exceeds 2. In this case, the neutron does not feel the short-wave components of the field, because it cannot move close enough to the wall: $N(t = 30$ s$) = 4300$.

It is possible to design a diffuse reflective bottom by covering it with granular magnetic particles. The parameters of granules (or the regular edges on the surface) should be optimized using computer models.

T h e s e c o n d c a s e: T h e c o n s t r u c t i o n o f a s y m m e t r i c t r a p s. As numerical experiments demonstrate, the existence of a "mirror" flat bottom in the trap increases the time for the cleaning of the trap. This result requires a more thorough consideration in order to provide a qualitative and quantitative understanding. Here, we present only the results of numerical simulations by letting the parameters of the trap, w and L, approach zero: $N(t = 30$ s$) = 2400$. Thus, an important factor is the general shape of the trap. Namely, it should be asymmetric and should not have a flat bottom. The first numerical simulations were done for the trap shown in Fig. 4. We will not describe here its shape in detail, because it appeared to be not the best version of an asymmetric trap, but we present only the final results. This result is better by about 30%, than that for a symmetric trap without flat bottom, but it still is not good enough for carrying out the experiments. Recall that we consider a system of neutrons with $h_{\max} = 0.47$. For smaller energies, $z_{\max} < h_{\max} < 0.47$, the escape time increases. The trap demonstrated in Fig. 4 is asymmetric and has a sharp back wall ($x < 0$) and a smoother front wall ($x > 0$).

Much better parameters were obtained from a trap that was designed from the trap shown in Fig. 4 in the following way. The part of the trap closest to the reader is reflected in the plane yz in the region $x < 0$. Thus, a "semi-symmetric" trap is realized, when the symmetry relative to the plane xz is destroyed. The detailed description of the trap (see Fig. 5) and its effectiveness are given below.

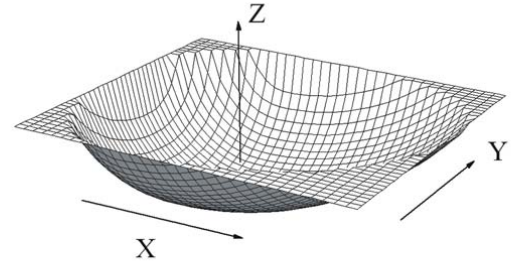

Fig. 4. One of the versions of an asymmetric trap

R e s u l t s f o r a n a s y m m e t r i c t r a p. First, we assume that the neutron absorber is placed at $z_{\max} = 0.42$ m, which is significantly below the "maximal" height, $h_{\max} = 0.47$ m ($\Delta = h_{\max} - z_{\max} = 0.05$ m, Fig. 6, curve a). Curve b in Fig. 6 presents the result for neutrons with a smaller difference $\Delta = h_{\max} - z_{\max} \leq 0.02$ m. Thus, the asymmetric trap, even under these conditions, appeared to be the most effective in comparison with the symmetric trap.

S u p p l e m e n t. The effective cleaning of a trap from neutrons with energies close to the critical one ($E_{\rm cr} = z_{\max}$ mg) can be achieved in at least two ways: (i) by optimizing the trap's shape and (ii) by creating diffuse conditions for the diffuse reflection of neutrons. However, for the optimal shape of a trap, the additional requirement of diffuse reflective walls is not evident. The numerical modeling demonstrated that the most effective diffuse reflection takes place from the flat bottom of the trap, not from the curvilinear walls. For the chosen form of the trap (Fig. 5), which is probably close to the optimal one, the diffuse reflection does not change the cleaning time.

It could be that the most effective solution of the problem corresponds to the intermediate region, in which both the form of the trap and the diffuse reflection should be incorporated. The solution of this problem requires a thorough theoretical and numerical analysis.

3. Analysis of New Possible Experiments in a Neutron Magnetic Trap

3.1. Neutron magnetic resonance (NMR) of a single neutron

First, we consider the opportunity of using a magnetic trap to detect the neutron magnetic resonance. Assume

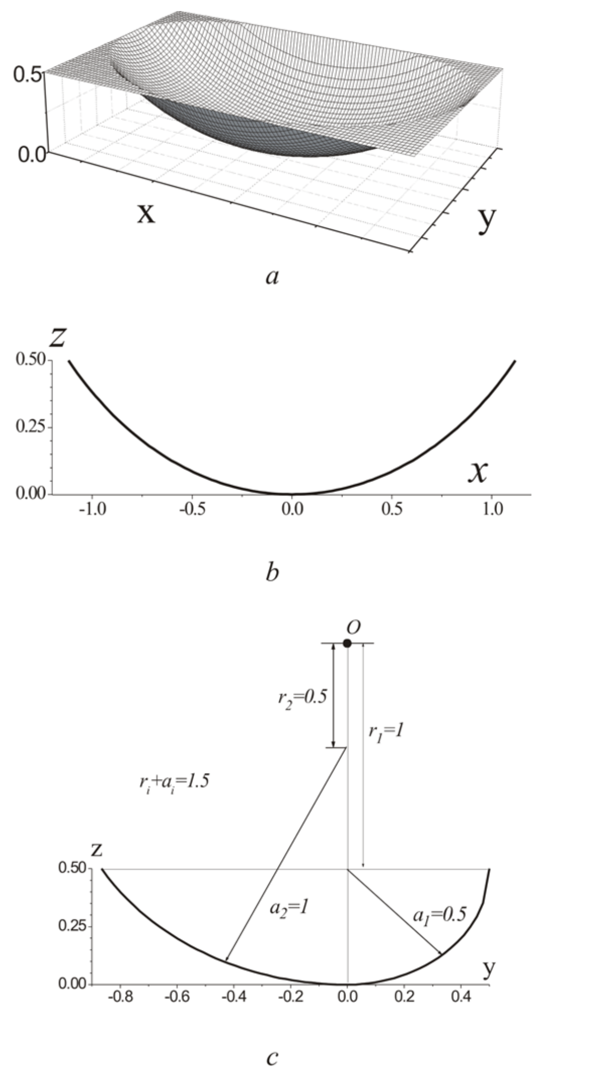

Fig. 5. (*a*) Shape of a trap which is asymmetric relative to the $y = 0$ plane; (*b*) *zx*-plane; (*c*) *zy*-plane

that the neutron spins are polarized initially opposite to the direction of the external magnetic field, $S_z = -1/2$. One applies a $\pi/2$ pulse or a pulse of the fast adiabatic half-reversal to the neutrons confined in the trap. If the frequency of the pulse matches the frequency of the neutron magnetic resonance, the spins transfer to the transversal plane. Due to decoherence, the neutron spin wave functions collapse to stationary states: some neutron spins collapse to the initial state, $S_z = -1/2$, and some collapse to the state with $S_z = 1/2$. Because of the conditions in the magnetic trap, the neutrons with the spin $S_z = 1/2$ leave the trap. They leave the trap during the time of collisions with the "trap walls" t_c which is much shorter than the neutron lifetime. It is

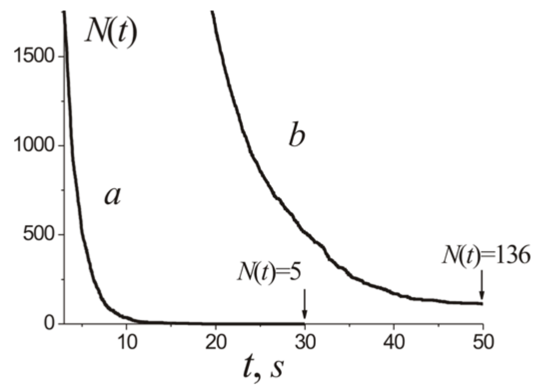

Fig. 6. *a* – $z_{\max}$ =0.42 m, $h_{\max}$=0.47 m; *b* – $z_{\max}$=0.42 m, $h_{\max}$=0.44 m. The asymmetric trap very effectively throws out the neutrons with energies of 0.47 mg. When the energy decreases, the time of cleaning increases (b)

assumed that the detector of the trap does not register the beta-decay of these neutrons. As a result, the number of the registered decays decreases. The described method would provide an ultra-sensitive detection of the neutron magnetic resonance. Probably, even a single neutron spin transition can be detected through the beta-decay of a single neutron.

3.2. Decoherence time of a neutron spin

Next, we consider the possible application of the ultra-sensitive NMR for a measurement of the decoherence time, t_d, of weakly interacting neutron spins (or even a single spin). We describe here an idealized situation (see below). We propose the following protocol. One applies two coherent $\pi/2$ NMR pulses with a time delay "t". After the first pulse, all neutron spins transfer from the state $S_z = -1/2$ to the transversal plane. If $t \gg t_c$, all neutrons will collide with the "trap walls" before the second pulse. Thus, half of the spins will leave the trap before the second pulse. This scenario is not of our choice. We will take the delay time, t, to be much smaller than t_c. Then, the fate of neutrons will depend on the value of the spin decoherence time, t_d which we assume to be much smaller than $t_c \, (t_d \ll t_c)$. In the simplest case, $t_d \gg t$, the decoherence does not affect the neutron spins. After the second NMR pulse, the spins transfer to the state $S_z = 1/2$ and disappear after colliding with the "walls" of the trap.

In general, we have the following outcome. During the delay time, N_1 spins ($0 \le N_1 \le N$, where N is the total number of spins in the trap) collapse to the states $S_z = 1/2$, or $S_z = -1/2$, and $(N - N_1)$ spins remain in the transversal plane. After the second $\pi/2$ NMR pulse,

$(N - N_1)$ spins, which did not collapse, will transfer from the transversal plane to the state $S_z = 1/2$. After the time t_c, these neutrons leave the trap. Now, what happens to N_1 neutrons which have collapsed during the delay time? Under the action of the second NMR pulse, these spins will transfer from the states $S_z = 1/2$ and $S_z = -1/2$ back to the transversal plane and will experience the decoherence again. As a result of this decoherence, $N_1/2$ spins collapse to the state $S_z = 1/2$ and leave the trap. The other $N_1/2$ spins, which collapse to the state $S_z = -1/2$, remain in the trap. Thus, one can detect the decay of $N_1/2$ neutrons and find the value N_1 as a function of the delay time "t". This dependence will provide the needed information about the "internal decoherence" of neutron spins in the magnetic trap.

Note that we have described an idealized experiment assuming that all the neutrons experience a uniform magnetic field. In a real situation, the magnetic field is not uniform, so the $\pi/2$ NMR pulse cannot be applied to all spins. For the same reason, the spin phase in the transversal plane will not be uniform. As a result, the number of decays will depend on the magnetic field distribution in the trap. This problem requires a thorough theoretical analysis and numerical simulations.

3.3. A precise measurement of the neutron gyromagnetic ratio

Another fundamental application of the ultra-sensitive NMR could be the precise measurement of the neutron gyromagnetic ratio, γ, and the computation of a corresponding value of the neutron magnetic moment. The magnetic interaction between neutron spins in such experiments must be minimized. An advantage of a neutron trap is the possible opportunity to deal with a few or even a single neutron spin. Currently, the value of γ is known with an accuracy of 7 significant figures [7]:

$$\gamma = 1.83247188 \pm 0.00000044 \times 10^8 \text{ s}^{-1}\text{T}^{-1}.$$

In order to verify or improve this value, one must measure the resonance magnetic field in the magnetic trap and the frequency of the electromagnetic pulse with an accuracy of not less than 7 significant figures.

In conclusion, we note that the proposed method is quite analogous to neutron electric dipole experiments with UCNs (see, e.g., [8]). We note also that B. Yerozolimsky *et al.* [9] proposed the experiment with a material neutron trap which is expected to measure the neutron lifetime with an uncertainty of less than 1 s.

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1. J.D. Bowman and S.I. Penttila, JRNIST **110**, 361 (2005).
2. V.V. Vladimirskii, Sov. Phys. JETP **12**, 740 (1961).
3. W. Paul *et al.*, Z. Phys. C **45**, 25 (1989).
4. V.F. Ezhov *et al.*, JRNIST **110**, 345 (2005).
5. J.D. Bowman and P.L. Walstrom, private communication (2006).
6. V.N. Gorshkov and G.P. Berman, *Numerical simulations on cleaning the neutron trap for measuring the neutron lifetime,* arXiv:nucl-ex/0611005v1 (2006).
7. *CODATA Recommended Values of the Fundamental Physical Constants* (1998); P.J. Mohr and B.N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).
8. C.A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006).
9. B. Yerozolimsky *et al.*, JRNIST **110**, 351 (2005).

ОПТИМІЗАЦІЯ ТА НОВІ ЗАСТОСУВАННЯ МАГНІТНОЇ ПАСТКИ ДЛЯ УЛЬТРАХОЛОДНИХ НЕЙТРОНІВ

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Р е з ю м е

Розглянуто проблеми, що мають відношення до фізики та динаміки ультра-холодних нейтронів (УХН) в магнітній пастці. Наведено результати комп'ютерного моделювання нейтронної пастки на основі постійних магнітів, конструкція якої запропонована Девідом Боуменом та Пітером Волстремом (Лос Аламосська національна лабораторія США). Вказано методи мінімізації часу "очистки" пастки (часу виходу з пастки нейтронів, енергія яких перевищує критичну). Розглянуто можливості використання такої пастки УХН для надчутливого вимірювання нейтронного магнітного резонансу, часу декогеренції нейтронного спіну та більш точного визначення гіромагнітного відношення для нейтронів.