90 YEARS

ON GRAVITATIONAL FIELDS OF THE THIRD TYPE

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A special place in Einstein's gravity theory belongs to fields of the third type, according to my earlier classification [1, Sec. 18]. Whereas gravitational fields of the first type include practically all the physically interpretable solutions of the field equations (the Schwarzschild solution, static field, etc.), and fields with cylindrical waves can be indicated among the known solutions of the second type [2], gravitational fields of the third type do not admit yet a physical interpretation. Moreover, even the determination of formal solutions belonging to this class of field is a rather difficult technical problem. However, if we start from the assumption that Einstein's theory of gravitation is logically closed and requires no additional definition, which would possibly exclude fields of the third type, such fields are worthy of special attention. The existence of such fields was predicted before specific examples were presented. The first example of such a field was apparently the metric obtained by the author in 1955 (unpublished),

$$ds^2 = e^{-x^2}[e^{-2x^4}(dx^1)^2 + (dx^2)^2] -$$

$$-2dx^3dx^4 + x^2(x^3 + e^{x^2})(dx^4)^2$$

, which satisfies the field equations $R_{\alpha\beta} = 0$ and admits a non-Abelian two-element group of motions. It was later observed [1, Sec. 30] that there exists a two-parameter family of gravitational fields of the third kind having the same group of motions in vacuum ($R_{\alpha\beta} = 0$),

$$ds^2 = e[a(dx^1)^2 + (dx^2)^2] + 2dx^3dx^4 + \lambda(dx^4)^2,$$

$$a = e(x^2)^2, \quad \lambda = 2\left[x^3 + \frac{e}{4}(x^2)\right]^2 \ln(px^2) - e(x^2)^2 + q,$$

$$p,\, q = \text{const}, \quad e = \pm 1,$$

and the assertion was made that this is a space of maximum mobility of the third type in vacuum. However, an arithmetic error has crept in the laborious calculations related to the formulation of this assertion, so that these gravitational fields admit, in fact, also a three-element group of motions. This was pointed out by Collinson and French [3] who made a remark concerning the possibility of a three-element group without indicating the metrics. The fields of greatest physical interest among the gravitational fields are, as a rule, those admitting various symmetries, and particularly groups of motions. Therefore, the determination of fields with maximum mobility is certainly worthy of special attention. An example of such a space was found by V.R. Kaigorodov. In a special coordinate system, the metric takes the form

$$ds^2 = -x^3(dx^1)^2 + 2dx^1dx^2 - \frac{(3x^2)^2}{(2x^3)^3}[(dx^3)^2 + (dx^4)^2];$$

it satisfies the field equations $R_{\alpha\beta} = 0$ and admits of a three-member group of motions with the group structure

$$[X_1,\, X_2] = 0, \quad [X_2,\, X_3] = -2X_2, \quad [X_3,\, X_1] = -X_1$$

and the operators of the group of motions

$$X_1 = p_1, \quad X_2 = p_4, \quad X_3 = x^1p_1 - x^2p_2 - 2x^3p_3 - 2x^4p_4.$$

The analysis of the λ matrix [1, Sec. 18] $(R_{AB} - \lambda g_{AB})$, where $R_{AB} \to R_{\alpha\beta\gamma\delta}$, $g_{AB} \to g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}$; $A, B, ... = 1, ..., 6 \equiv (14,\ 24,\ 34,\ 23,\ 31,\ 12)$, shows that it has the characteristic $[(3^\circ, 3)]$, i.e., this metric defines actually a gravitational field of the third kind.

The question whether other fields of the third type with the group G_3 are admissible is presently under study.

It is remarkable that the field equations $R_{\alpha\beta} = \kappa g_{\alpha\beta}$ of the third type admit, at $\kappa > 0$, the group of motions G_4 [1, Sec. 30]. When $\kappa \to 0$, the mobility is lowered, although this fact is still difficult to be interpreted physically.

1. A.Z. Petrov, *New Methods in General Relativity Theory* (Nauka, Moscow, 1966) (in Russian).
2. A. Einstein and N. Rosen, On gravitational waves, J. Franklin Inst. **223**, 43 (1937).
3. C.D. Collinson and D.C. French, Null tetrad approach to motions in emply space-time, J. Math. Phys. **8**, 701 (1967).

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O.Z. Petrov is the famous physicist-theoretician. He graduated from Kazan' State University in 1937 and worked there from 1943 till 1969. From 1970, he was Head of a department at the Institute for Theoretical Physics of the Acad. of Sci. of UkrSSR; Academician of the Acad. of Sci. of UkrSSR (1969); Lenin's prize winner (1972).

O.Z. Petrov carried out the studies in the field of mathematical physics, general relativity theory, and philosophical questions of physics; performed a cycle of works on the group invariance methods in the theory of gravitation; proposed new methods to study the gravitation radiation and the energy of a gravitation field and to model the gravitation field; proved the existence of three types of gravitation (Petrov's types); constructed the classification of gravitation fields by the groups of motion (the conformal, affine, and projective groups). He developed the general theory of modeling of gravitation fields and advanced the idea to describe the Einstein's theory of gravitation in terms of a plane space. The scientific results of Oleksii Zynoviiovych are presented in the monographs: A.Z. Petrov, Einstein's Spaces (GIFML, Moscow, 1961) (in Russian); A.Z. Petrov, New Methods in General Relativity (Nauka, Moscow, 1966) (in Russian). These monographs occupy a particular place in the world scientific literature on general relativity theory. They obtained rapidly the common recognition and were translated into many foreign languages.