RESONANT PHENOMENA ON THE DIFFRACTION OF A PLANE *H*-POLARIZED WAVE ON THE METAL-BAR GRATING

Translated and reprinted from Zh. Tekhn. Fiz. **38**, No. 9, pp. 1505–1520 (1968)

V.G. SOLOGUB, V.P. SHESTOPALOV

Kharkov Institute of Radioelectronics,
Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian SSR
(Kharkov, Ukraine)

The resonant phenomena appearing on the diffraction of an H-polarized plane wave on periodic gratings consisting of perfectly conducting bars of rectangular section are studied. These phenomena represent an abrupt change of the transmitted or reflected field intensity in a certain short interval of the frequencies or the incidence angles adjacent to the resonant values. It is established that, for gratings with the nonzero bar thicknesses, there always exist the conditions, under which the incident wave transmits through the grating completely even if the slits between the bars are very narrow. The natural modes of a grating are studied, and it is shown that resonant phenomena occur if the incident plane wave excites a mode close to the natural one.

1. Considerable attention has recently been given to the study of open resonant systems and the means of their excitation in connection with the uptake of millimeter and submillimeter wave ranges. A periodic grating from perfectly conducting bars of rectangular section considered in the present paper can be assigned to systems of this kind. When a plane H-polarized wave is diffracted on this grating, a number of resonant phenomena arise such as the anomalous transmission in the neighborhood of sliding points, full transmission at certain frequencies and incidence angles, *etc.* Some of the effects are similar to those for reflecting gratings (for example, the anomalous transmission and Wood anomalies). These phenomena can be explained naturally with the help of the notion, given in the present work, about the natural mode of a grating. It is shown that the system under consideration possesses three types of natural modes, which are the solutions of Maxwell's equations in the absence of sources: a) a mode with a complex frequency (negative imaginary part) and real phase propagation velocities along the grating; b) a mode with a real frequency and a complex (negative imaginary part) propagation constant along the grating; c) a mode with a real frequency and real phase velocities. It is also shown that, in each resonant case, the incident plane wave excites the mode close to one of the above-listed.

Note that, in the available literature, these grating natural modes are called 'damped resonances', 'leaky waves', and the surface wave mode, respectively. In [1, 2], the relationship between damped resonances and leaky waves with the excitation of open systems is discussed. The mode of the third type is inherent in a wide class of slow-wave structures and has been studied in detail in [3] for a reflecting grating.

R. Wood was the first who observed, back in 1902, the resonant phenomena on reflecting diffraction gratings in the optical range [4]. Since that time, a lot of papers have been published, in which these resonances named the Wood anomalies, are investigated experimentally [5] and theoretically [6–8]. In the millimeter range, the Wood anomalies on reflecting gratings with rectangular grooves were studied experimentally in the recent work [9]. The most consistent theory of the diffraction of a plane wave on a grating of this kind in the case where the groove width is small in relation to the wavelength is presented in [3]. In particular, this theory allowed one to study some features of the Wood anomalies. In this case, it was established that the Wood anomalies appear on the diffraction of a plane H-polarized wave in the spectra of the 1st, 2nd, *etc.* orders in a narrow frequency band or a narrow interval of incidence angles adjacent to the so-called 'sliding points', i.e. the points of the initiation of higher propagating harmonics.

In the present paper, the resonant effects that arise not only in the vicinity of the sliding points are studied in detail. Thus, the theory developed here allows one to reveal practically all things, including Wood's resonances that take place on the diffraction of a plane H-polarized wave on perfectly conducting gratings made from bars of rectangular section if the width of the slits between bars is small [10].

2. If a plane H-polarized wave is incident on a grating (Fig. 1) from the side $y<0$ at an angle $\psi = 0$, then the diffraction field is defined by relations (1)–(4) in [10]. If the width of slits between bars d is small as compared with the grating period l, i.e., if $\theta^2 = \left(\frac{d}{l}\right)^2 \ll 1$, we can restrict ourselves to the first-order approximation. Then, for $H_z(x, y; \kappa, \alpha) = H(\xi, \eta; \kappa, \alpha)$ (where $\xi = \frac{2x}{l}$, $\eta = \frac{2y}{l}$ if $\frac{d}{\lambda} < \frac{1}{2}$), we obtain the following expressions (see [10]):

$$H(\xi, \eta; \kappa, \alpha) = e^{i\pi\alpha\kappa\xi}\Big[e^{i\pi\kappa\gamma\eta} + e^{-i\pi\kappa\gamma(\eta+2\delta)} +$$

$$+2\frac{i\kappa\theta}{D}e^{-i\pi\kappa\gamma\delta}(i\kappa\theta S_0 \sin 2\pi\kappa\delta - \cos 2\pi\kappa\delta)\times$$

$$\times \sum_{n=-\infty}^{\infty} R_n e^{-i\pi r_n(\eta+\delta)} e^{i\pi n\xi}\Big]; \quad \eta < -\delta,$$

$$H(\xi, \eta; \kappa, \alpha) = 2\frac{i\kappa\theta}{D}e^{i\pi\kappa(\alpha\xi-\gamma\delta)}\times$$

$$\times \sum_{n=-\infty}^{\infty} R_n e^{i\pi r_n(\eta-\delta)} e^{i\pi\eta\xi}; \quad \eta > \delta,$$

$$H(\xi, \eta; \kappa, \alpha) = \frac{2\sin\alpha\kappa\theta\pi}{D\alpha\kappa\theta\pi}e^{-i\pi\kappa\gamma\delta}\Big\{\sin(\delta-\eta)\pi +$$

$$+i\kappa\theta S_0 \cos\pi\kappa(\delta-\eta) + 2i\kappa\theta\times$$

$$\times \sum_{m=1}^{\infty} e^{-im\frac{\pi}{2}} \frac{S_m \cos\frac{\pi m}{2\theta}(\xi+\theta)}{\operatorname{sh}\frac{\pi\delta}{2\theta}\tau_m}\times$$

$$\times\left[\operatorname{sh}\left[\frac{\pi(\eta+\delta)}{2\theta}\tau_m\right] - \operatorname{sh}\left[\frac{\pi(\delta-\eta)}{2\theta}\tau_m\right]\times\right.$$

$$\left.\times(\cos 2\pi\kappa\delta - i\kappa\theta S_0 \sin 2\pi\kappa\delta)\right]\Big\} + 4e^{-i\pi\kappa\delta}\alpha\kappa\theta\pi\times$$

$$\times \sum_{m=1}^{\infty} e^{-im\frac{\pi}{2}} \frac{\sin\left(\alpha\kappa\theta\pi - m\frac{\pi}{2}\right)}{\left(\alpha\kappa\theta\pi\right)^2 - \left(m\frac{\pi}{2}\right)^2} \frac{\operatorname{sh}\left[\frac{\pi(\delta-\eta)}{2\theta}\tau_m\right]}{sh\left[\frac{\pi\delta}{2\theta}\tau_m\right]}\times$$

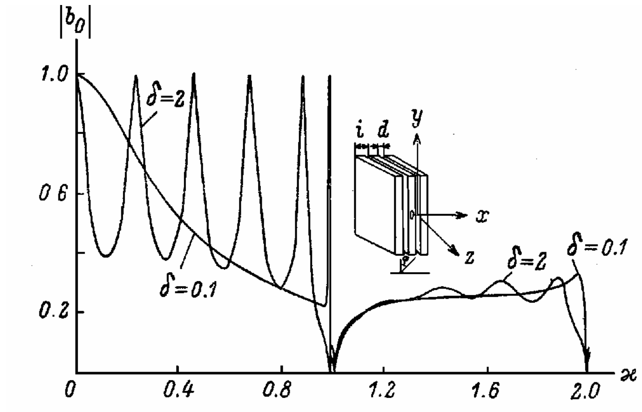

Fig. 1. Transmission coefficients $|b_0|$ versus the frequency $\kappa = \frac{\omega l}{2\pi c} = \frac{l}{\lambda}$ in the case of normal incidence ($\psi = 0$). Different kinds of frequency resonances are shown: in the vicinity of the sliding points ($\kappa = 1, 2$ for $\delta = 0.05$) and far from them ($\delta = 2$). $\theta = 0.2$, $\gamma = 0.99999$

$$\times \cos\frac{\pi m}{2\theta}(\xi+\theta); \ |\eta| < \delta; \ |\xi| < \theta,$$

$$E_x = -\frac{1}{ik}\frac{\partial H_z}{\partial y}; \quad E_y = \frac{1}{ik}\frac{\partial H_z}{\partial x};$$

$$E_z = 0; \quad H_x = 0; \quad H_y = 0, \tag{1}$$

where

$$D = \left(1 + \kappa^2\theta^2 S_0^2\right)\sin 2\pi\kappa\delta + 2i\kappa\theta S_0 \cos 2\pi\kappa\delta;$$

$$R_n = \frac{\sin(n+\alpha\kappa)\theta\pi}{r_n(n+\alpha\kappa)\theta\pi}\ \frac{\sin\alpha\kappa\theta\pi}{\alpha\kappa\theta\pi},$$

$$S_m = \sum_{s=-\infty}^{\infty} \frac{\sin(s+\alpha\kappa)\theta\pi}{r_s} \frac{\sin\left[(s+\alpha\kappa)\theta\pi - \frac{m\pi}{2}\right]}{(s+\alpha\kappa)^2\theta^2\pi^2 - \left(\frac{m\pi}{2}\right)^2};$$

$$r_n = \sqrt{\kappa^2 - (n+\alpha\kappa)^2}; \ \tau_m = \sqrt{m^2 - (2\kappa\theta)^2};$$

$$\alpha = \sin\psi; \quad \gamma = \cos\psi; \quad \kappa = \frac{kl}{2\pi} = \frac{l}{\lambda}; \quad \delta = \frac{h}{l}; \quad k = \frac{\omega}{c}, \tag{2}$$

k is the wave number, λ is the incident wavelength, and h is the bar height.

The approximate expression for the field given by (1) at $|y| > \frac{h}{2}$ $(-\infty < x < \infty)$ and at $|y| < \frac{d}{2}$ $(|x - nl| < \frac{d}{2}$; $n = 0, \pm 1, \pm 2, \dots)$ satisfies the Helmholtz equation, is continuous throughout the entire space, and is such that the condition $E_{\text{tang}} = 0$ is fulfilled on the bars. The electromagnetic field defined by (1) and (2) does not have, however, the required singularity on the bar edges, since the first-order approximation is inadequate to take the higher harmonics into account. As a consequence, the continuity condition for $E_x(x, y; \kappa, \alpha)$ in the slits for $y = \pm\frac{h}{2}$ is fulfilled with a moderate accuracy. However, the accuracy of formulas (1) and (2) improves as θ and κ decrease. It is significant that the amplitude coefficients in (1) of spatial harmonics propagating away from the grating satisfy the energy conservation law

$$\sum_{|n+\alpha\kappa|<\kappa} \left(|a_n|^2 + |b_n|^2\right) r_n = \kappa\gamma, \tag{3}$$

where a_n correspond to the reflected field, while b_n correspond to the transmitted field.

3. Going to the analysis of formula (1), we note, first of all, that the smaller the θ, the greater the amplitudes $|b_n|$ $(n = 0, \pm 1, \pm 2, \dots)$ of the spatial harmonics of the transmitted field (for $|y| \geq \frac{h}{2}$) if the following relation is not fulfilled:

$$\tan 2\pi\kappa\delta = -\frac{2\kappa\theta V(\alpha, \kappa)}{1 - \kappa^2\theta^2 |S_0(\alpha, \kappa)|^2} = -U(\alpha, \kappa), \tag{4}$$

which results in

$$|b_n| = \frac{R_n}{R} \quad . \tag{5}$$

Here,

$$S_0 = R - iV = \sum_{n=-\infty}^{\infty} \frac{\sin^2(n + \alpha\kappa)\theta\pi}{r_n (n + \alpha\kappa)^2 \theta^2\pi^2};$$

$$R \equiv R(\alpha, \kappa) = \sum_{|n+\alpha\kappa|<\kappa} \frac{\sin^2(n + \alpha\kappa)\theta\pi}{r_n (n + \alpha\kappa)^2 \theta^2\pi^2} > 0,$$

$$V \equiv V(\alpha, \kappa) = \frac{\sin^2(n + \alpha\kappa)\theta\pi}{\sqrt{(n + \alpha\kappa)^2 - \kappa^2 (n + \alpha\kappa)^2 \theta^2\pi^2}}. \tag{6}$$

For the harmonic amplitudes in the reflected field, taking into account (3), we have

$$|a_n| = |b_n|, \quad (n \neq 0); \quad |a_0| = 1 - \frac{1}{\kappa\gamma R} \frac{\sin^2 \alpha\kappa\theta\pi}{(\alpha\kappa\theta\pi)^2}.$$

Thus, relation (4) determines the values of the parameters α and κ (which will be referred to as the resonant values), for which the transmission of the incident field is maximal.

In this case, if (4) holds for

$$\kappa(1 + \alpha) \leq 1, \tag{7}$$

i.e., if only the principal wave propagates, then, as seen from (5) and (6), the transmission coefficient $|b_0|$ is equal to unity (the reflection coefficient equals zero, since (3) yields $|a_0|^2 = 1 - |b_0|^2$). In this situation, the incident wave passes through the grating completely, without reflection even if the slits between the bars are very narrow, i.e., if $\theta \ll 1$.

When analyzing relation (4), we will distinguish two cases: the function $U(\kappa, \alpha)$ is positive, and it is negative. The latter takes place if

$$\kappa\theta |S_0(\kappa, \alpha)| \geq 1. \tag{8}$$

With the help of this inequality, we determine the neighborhoods of the sliding points.

First, we consider frequency resonances (by κ). For the present, we restrict our consideration to the case $\kappa\theta |S_0| < 1$. In this case, the resonant values $\kappa = \kappa_N$ $(N = 1, 2, \dots)$ satisfying (7) have the form, to within θ^2,

$$\kappa_N = \frac{N}{2\delta}\left[1 - \frac{1}{N\pi} \arctan U\left(\alpha, \frac{N}{2\delta}\right)\right];$$

$$N = 1, 2, \dots, \left[\frac{2\delta}{1 + \alpha}\right].$$

In this case, the inequality $\delta > \frac{1}{4}$ should be necessarily valid. If the value of $\left(\frac{N\theta}{2\delta}\right)^2 \left|S_0\left(\alpha, \frac{N}{2\delta}\right)\right|^2$ can be neglected, since it is considerably less than unity, the corresponding value of κ_N is close to

$$\kappa_N = \frac{N}{2\delta}\left(1 + \frac{2\theta}{\pi\delta} \ln \sin \frac{\pi\theta}{2}\right), \tag{9}$$

whereas if $\left(\frac{N\theta}{2\delta}\right) \left|S_0\left(\alpha, \frac{N}{2\delta}\right)\right|$ tends to unity, we have $\kappa_N \to \frac{1}{2\delta}\left(N - \frac{1}{2}\right)$.

Hence, for $\delta > \frac{1}{4}$, there exist a number of values $\kappa = \kappa_N$ (Fig. 2) depending on θ, δ, and the angle of incidence ψ such that the intensities of the transmitted and incident waves are equal to 1. For $\kappa = \kappa_N$, the reflected wave is absent; the value of $|H(\xi, \eta)|$ in a slit is as much as the values of the order of $\frac{\gamma}{\theta}$. However,

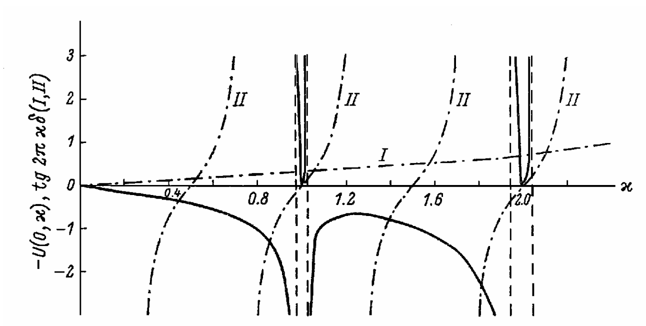

Fig. 2. Transmission coefficients $|b_0|$ versus the frequency $\kappa = \frac{\omega l}{2\pi c} = \frac{l}{\lambda}$ in the case of normal incidence ($\psi = 0$). Different kinds of frequency resonances are shown: in the vicinity of the sliding points ($\kappa = 1, 2$ for $\delta = 0.05$) and far from them ($\delta = 2$). $\theta = 0.2$, $\gamma = 0.99999$

when κ is displaced from the resonant values κ_N, the field picture changes: the reflected wave arises, and the intensity of the transmitted wave as well as the magnitude of $|H(\xi, \eta)|$ in the slits decreases sharply; the smaller is θ, the more abruptly they change (Figs. 1 and 5,*a*).

We determine the width of the grating transmission band $\Delta\kappa$ as the difference between two values of κ being closest to a resonant value and such that the intensity of the transmitted wave for these κ is twice lower than the intensity for the corresponding resonant value. These values of κ satisfy the equation

$$\tan(2\pi\kappa\delta) = - \frac{U}{1 - \frac{R^2}{V^2}U^2}\left[1 \pm \frac{R}{V}\sqrt{1 + \left(1 - \frac{R^2}{V^2}\right)U^2}\right], \tag{10}$$

which implies that, for $\left(\frac{N\theta}{2\delta}\right)^2 \left|S_0\left(\alpha, \frac{N}{2\delta}\right)\right|^2 \ll 1$, the transmission band width $\Delta\kappa_N$ in the vicinity of κ_N is close to

$$\Delta\kappa_N = \frac{2\theta}{\pi\gamma\delta} \tag{11}$$

and tends to zero as θ decreases or δ increases.

For $\kappa(1 + \alpha) > 1$, new types of propagating waves arise. In this case, there also exist the resonant values κ satisfying relation (4) (Fig. 2) and being such that the total intensity of propagating harmonics in the transmitted field reaches the maximum. However, as it follows from (5), (6), and the energy conservation law (3), this intensity is now invariably less than the incident wave intensity.

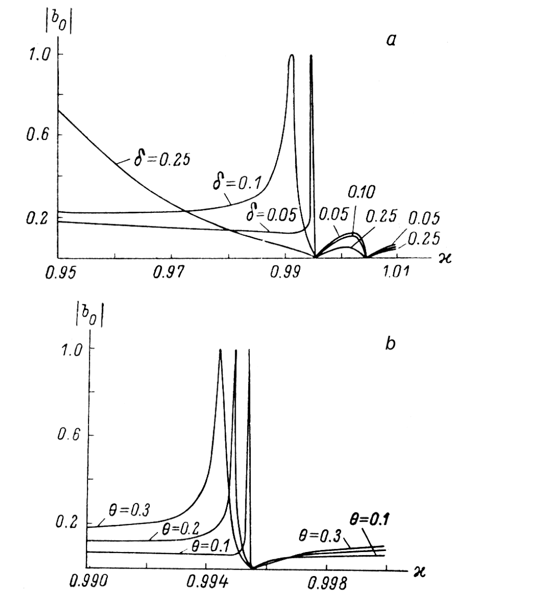

Fig. 3. Resonance dependence of $|b_0|$ on κ in the vicinity of the first sliding point at a near-normal incidence ($\gamma = 0.99999$) for $\theta = 0.2$ and various δ (*a*) and for $\delta = 0.05$ and various θ (*b*)

Passing to the study of the neighborhoods of the sliding points

$$\kappa(1 \pm \alpha) = n \quad (n = 1, 2, \ldots), \tag{12}$$

which have been determined by condition (8), we restrict ourselves, for simplicity, to the neighborhood of the sliding point $\kappa = 1$ in the case of the normal incidence ($\psi = 0$). This neighborhood, with an accuracy of $\theta^6 \ln^2 \sin\frac{\pi\theta}{2}$, is given by the inequalities

$$\sqrt{1 - 4\theta^2(1 + G\theta)^2} \le \kappa \le \sqrt{1 + 4\theta^2(1 + \theta)^2},$$

where

$$G = 2\sum_{s=2}^{\infty} \frac{\sin^2 s\pi\theta}{\sqrt{s^2 - 1}s^2\pi^2\theta^2}.$$

Suppose there exist κ such that $\tan 2\pi\kappa\delta > 0$ within the interval

$$\sqrt{1 - 4\theta^2(1 + G\theta)^2} \le \kappa \le 1.$$

Then the intensity of the transmitted wave changes sharply from small values up to the unity and then decreases abruptly at the sliding point $\kappa = 1$ down to zero if κ is varied in this interval (Fig. 3,*a*,*b*). The

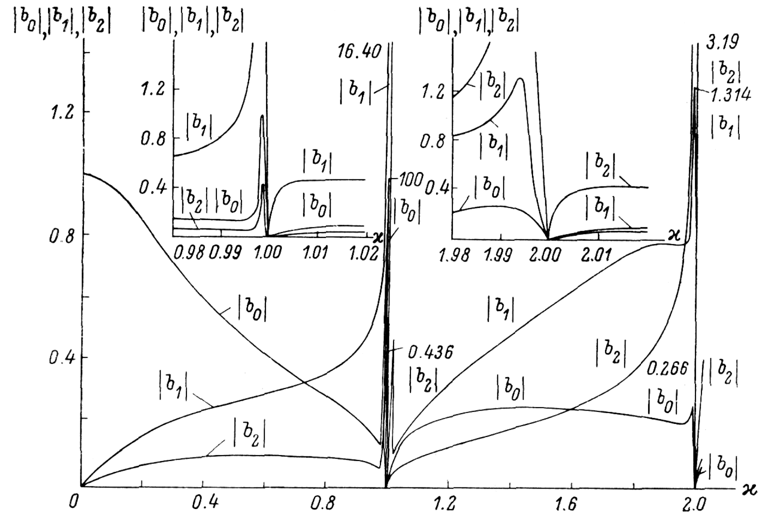

Fig. 4. Amplitudes of the first spatial harmonics in the reflected field $|b_0|$, $|b_1|$, and $|b_2|$ as a function of κ in the case of a normal incidence. As seen, these amplitudes are abruptly changed in the vicinity of the sliding points (abnormal transmission); this concerns especially the amplitude of a harmonic that propagates by passing through a sliding point ($|b_1|$ near $\kappa = 1$; $|b_2|$ near $\kappa = 2$)

resonant value $\kappa = \kappa_0$, with an accuracy of θ^4, is equal to

$$\kappa_0 = \sqrt{1 - \varsigma_0^2}, \tag{13}$$

where

$$\zeta_0 = \frac{2\theta\left(\sqrt{1+\tan^2 2\pi\delta} - 1 + G\theta\tan 2\pi\delta\right)}{\tan 2\pi\delta + 2G\theta}.$$

In this case, the transmission bandwidth in the vicinity of $\kappa = \kappa_0$ is close to

$$\Delta\kappa_0 = \zeta_0^3. \tag{14}$$

Relations (13) and (14) are derived on the assumption that $\tan 2\pi\delta > 0$.

We note that $\zeta_0 \approx \theta\tan 2\pi\delta$ for $\tan 2\pi\delta \ll 1$ and $\theta \ll 1$.

Among the amplitudes of spatial harmonics in the transmitted field ($\eta \geq \delta$), the harmonics with $n = -1$ and $n = 1$ show the highest amplitudes for $\kappa = \kappa_0$ (Fig. 4):

$$|b_{-1}| = |b_1| = \frac{\kappa_0}{\zeta_0}\frac{\sin\pi\theta}{\pi\theta}.$$

In the reflected field ($\eta \leq -\delta$), the highest amplitudes have the harmonics with $n = -1$ and $n = 1$ as well:

$$|a_{-1}| = |a_1| = \frac{\kappa_0}{\zeta_0}\frac{\sin\pi\delta}{\pi\delta}.$$

By taking into consideration that the principal reflected wave is absent for $\kappa = \kappa_0$, we get the following field picture in this resonant situation: far from the grating, the field under the grating is determined by the incident wave only, the field above the grating is determined by the transmitted wave, whose intensity is equal to that of the incident wave; in the vicinity of the grating, these fields are determined to a large extent by the harmonics with $n = -1$ and $n = 1$. These harmonics are the surface waves of high amplitudes that propagate along the grating in opposite directions with the velocity close to the velocity of light, and they exponentially decrease with a small damping factor as the observation point moves away from the grating. Consequently, the contribution of these waves is less than that of the incident (or transmitted) wave only if

$$|\eta| > \delta + \frac{1}{\pi\zeta_0}\ln\frac{1}{\zeta_0}.$$

In this case, the field inside the slits between bars consists of the propagating and damped guided waves of high amplitudes; in the slits, the magnitude of $|H(\xi,\eta)|$ reaches the values of the order of ζ_0^{-1} (Fig. 5,*b*).

The field structure changes abruptly as κ shifts from the value κ_0; at the sliding point $\kappa = 1$, the transmitted field vanishes[1]. In the slits, the field consists of the guided waves exponentially decreasing in the direction

[1]Within the limits of the given approximation.

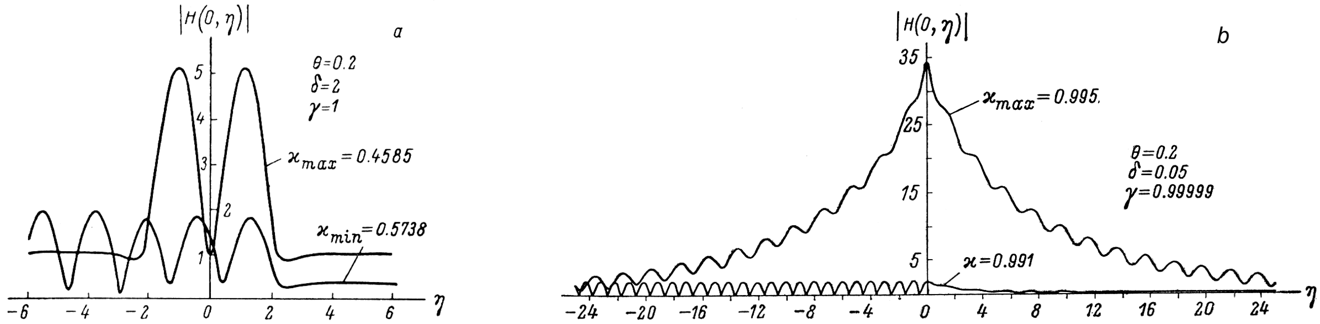

Fig. 5. Distribution of the absolute value of the magnetic field over η ($\eta = \frac{2y}{l}$) for $\xi = 0$ ($\xi = \frac{2x}{l}$) for different types of resonances far from the sliding point $\kappa = 1$. a – far from the sliding point $\kappa = 1$ (normal incidence). The field distributions in the resonant case $\kappa_{\max} = 0.4585$, the region $-2 \le \eta \le 2$ is occupied by a slit; in the absence of resonances, $\kappa_{\min} = 0.5738$; b – in the vicinity of the sliding point $\kappa = 1$ (almost normal incidence). The field distributions in the resonant case $\kappa_{\max} = 0.995$, the region $-0.5 \le \eta \le 0.5$ is occupied by a slit; in the absence of resonances, $\kappa_{\min} = 0.991$

of positive η, whose amplitude is proportional to θ^2. Beneath the grating, the field is a superposition of the incident, reflected, and two sliding waves propagating along the grating in opposite directions:

$$H(\xi,\eta) = e^{i\pi\eta} + e^{-i\pi(\eta+2\delta)} -$$

$$-\frac{\pi\theta}{\sin\pi\theta} e^{-i\pi\delta}\left(e^{i\pi\xi} + e^{-i\pi\xi}\right), \quad \eta \le -\delta.$$

On the further increase in κ, the field begins to 'seep' through the grating, new propagating modes arise, and their amplitude changes abruptly as κ varies in the range

$$1 < \kappa < \sqrt{1 + 4\theta^2(\theta + \theta^2)}.$$

At $\kappa = \kappa_0' = \sqrt{1 + \zeta_0'^2}$, where

$$\zeta_0' = \frac{2\theta\sqrt{\tan 2\pi\delta}\left(\theta\sqrt{\tan 2\pi\delta} + \sqrt{2G\theta + \tan 2\pi\delta}\right)}{2G\theta + \tan 2\pi\delta},$$

it reaches its maximal value (for $\kappa > 1$)

$$|a_n| = |b_n| = \frac{\sin\pi\theta}{\pi\theta}\frac{\kappa_0'}{\zeta_0' + 2\kappa_0'\frac{\sin^2\pi\theta}{(\pi\theta)^2}}, \quad (n = \pm 1).$$

We note that, in this case, the amplitude of the principal reflected wave differs little from 1, while the one of the transmitted wave is close to 0 (Fig. 4):

$$|a_0| = \frac{2\kappa_0'}{\zeta_0' + 2\kappa_0'\frac{\sin^2\pi\theta}{(\pi\theta)^2}}\frac{\sin^2\pi\theta}{(\pi\theta)^2};$$

$$|b_0| = \frac{\zeta_0'}{\zeta_0' + 2\kappa_0'\frac{\sin^2\pi\theta}{(\pi\theta)^2}}.$$

Consider now the resonances by an angle of incidence. It follows from (10) for $\kappa(1+\alpha) < 1$ that the transmitted wave intensity is higher than a half of the incident wave intensity for those ψ, for which the following inequality holds:

$$R^2(\alpha,\kappa) \ge \frac{1 + U^2(\alpha,\kappa)}{U^2(\alpha,\kappa)} V^2(\alpha,\kappa), \quad (\alpha = \sin\psi > 0).$$

For $\kappa < \frac{1}{2}$, these angles of incidence belong, with an accuracy of θ^4, to the interval

$$\frac{\pi}{2} - \arcsin\frac{\theta}{\sqrt{3 - 2\sqrt{2} + (\sqrt{2}-1)\kappa^2\theta^2\left[\frac{1}{\sqrt{1-2\kappa^2}} + P(1,\kappa)\right]}} <$$

$$< \psi < \frac{\pi}{2} - \arcsin\frac{\theta}{\sqrt{3 + 2\sqrt{2} + (\sqrt{2}+1)\kappa^2\theta^2\left[\frac{1}{\sqrt{1-2\kappa^2}} + P(1,\kappa)\right]}},$$

where

$$P(\alpha,\kappa) = \sum_{\substack{n \ne 0 \\ n \ne -1}} \frac{\sin^2(n+\alpha\kappa)\pi\theta}{\pi^2\theta^2(n+\alpha\kappa)^2\sqrt{(n+\alpha\kappa)^2 - \kappa^2}}.$$

We determine the angle of full transmission ψ_0 as the angle of incidence, at which the incident wave passes through the grating without reflection. It is seen from (1) that, in this case, $\kappa(1+\alpha) < 1$.

First, we consider the case where the right-hand part of relation (4) is negative ($\kappa\theta\,|S_0(\alpha,\kappa)| < 1$). It follows from the properties of the function $U(\alpha,\kappa)$ that the angle of full transmission exists only if $-\tan 2\pi\kappa\delta \ge U(0,\kappa) > 0$. For $\kappa < \frac{1}{2}$ (in this case, $V(\alpha,\kappa)$ is a slowly varying function of ψ and κ; hence, we can consider that

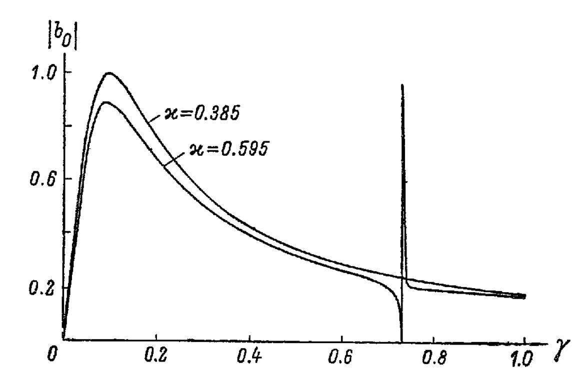

Fig. 6. $|b_0|$ as a function of the angle of incidence ψ ($\gamma = \cos\psi$) for various κ. Different types of resonances by the angle of incidence are shown: a full transmission at $\kappa = 0.385$ and an abnormal transmission in the vicinity of the sliding point $\psi_{\text{sl.}} = \arcsin\frac{1-\kappa}{\kappa}$ for $\kappa = 0.595$, $\theta = 0.0917$; $\delta = 7$

$V(\alpha,\kappa) = -2\ln\sin\frac{\pi\theta}{2}$ making no significant error), the angle ψ_0 is equal to

$$\psi_0 = \frac{\pi}{2} - \arcsin\theta\Big[1 - 4\kappa\theta\,\ln\,\sin\frac{\pi\theta}{2}\Big(\text{cotan}2\pi\kappa\delta + $$

$$+\kappa\theta\,\ln\,\sin\frac{\pi\theta}{2}\Big)\Big]^{-1/2}; \qquad (15)$$

whereas, for $1 > \kappa > \frac{1}{2}$, the angle of full transmission, for which $1 - \kappa\theta\,|S_0(\alpha,\kappa)| \ll 1$, is close to

$$\psi_0 = \arcsin\Big\{\frac{1-\kappa}{\kappa} - \frac{\theta^2\left[1 + \sqrt{\tan 2\pi\kappa\delta} - \kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right]}{2\left[\tan 2\pi\kappa\delta + 2\kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right]}\Big\}. \qquad (16)$$

The intensity of the transmitted wave for the angle of incidence given by (15) or (16) is the same as that for the incident wave; it decreases rapidly as ψ moves away from these values (Fig. 6).

If $\tan 2\pi\kappa\delta > 0$, then the resonant value of ψ belongs to the neighborhoods, which have been determined by (7), of the sliding points given by (12). We restrict our consideration to a neighborhood of the sliding point $\psi_{\text{sl.}} = \arcsin\frac{1-\kappa}{\kappa}$ for $1 > \kappa > \frac{1}{2}$, which can be determined with an accuracy of θ^4 by the inequalities

$$\arcsin\left\{\frac{1-\kappa}{\kappa} - \frac{\theta^2}{2}\left[1 + \kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right]\right\} < \psi <$$

$$< \arcsin\left[\frac{1-\kappa}{\kappa} + \frac{\theta^2}{2}\left(1 + \frac{\kappa\theta}{\sqrt{2\kappa^2-1}}\right)^2\right].$$

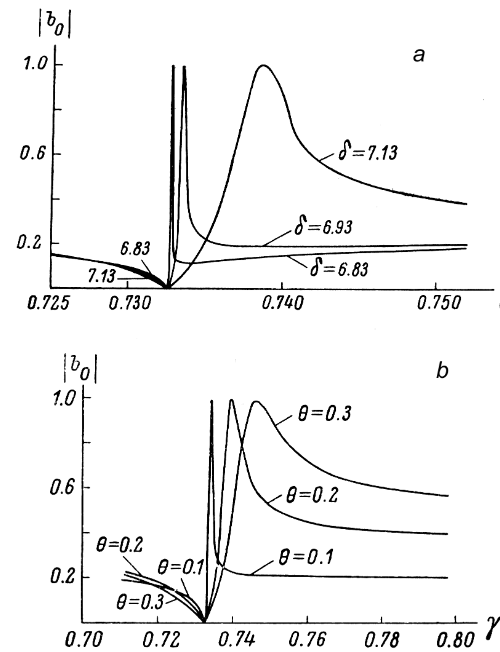

Fig. 7. Resonant curve $|b_0|$ as a function of the angle of incidence ψ ($\gamma = \cos\psi$) in the vicinity of the sliding point $\psi_{\text{sl.}} = \arcsin\frac{1-\kappa}{\kappa}$; for $\theta = 0.1$, $\kappa = 0.595$, and different δ (a) and for $\kappa = 0.595$, $\delta = 0.7$, and different θ (b)

The transmitted wave intensity changes abruptly as ψ varies in the range

$$\arcsin\left\{\frac{1-\kappa}{\kappa} - \frac{\theta^2}{2}\left[1 + \kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right]\right\} < \psi \le$$

$$\le \arcsin\frac{1-\kappa}{\kappa},$$

reaching its maximal value equal to 1 at the angle of incidence

$$\psi_0 = \arcsin\left(\frac{1-\kappa}{\kappa} - \eta_0^2\right), \qquad (17)$$

where

$$\eta_0^2 = \frac{\theta}{\sqrt{2}}\frac{\sqrt{1+\tan^2 2\pi\kappa\delta} - 1 + \kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\tan 2\pi\kappa\delta}{2\kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right) + \tan 2\pi\kappa\delta},$$

and reducing to zero at the sliding point $\psi = \psi_{\text{sl.}}$ (Figs. 6 and 7).

The transmission bandwidth by the angle of incidence $\Delta\psi$ in the vicinity of ψ_0 is close, in this case, to

$$\Delta\psi = \frac{4\sqrt{2}\kappa^2}{2\kappa-1}\eta_0^3 \qquad (18)$$

and tends to zero as θ and $\tan 2\pi\kappa\theta$ decrease as $(\theta \tan 2\pi\kappa\delta)^3$.

For $\psi > \psi_{\text{sl.}}$, a new propagating mode arises, whose amplitude sharply changes within the interval

$$\psi_{sl.} < \psi < \arcsin\left[\frac{1-\kappa}{\kappa} + \frac{\theta^2}{2}\left(1 + \frac{\kappa\theta}{\sqrt{2\kappa^2-1}}\right)^2\right],$$

by reaching its maximal value (4) at $\psi = \psi_0' = \arcsin\left(\frac{1-\kappa}{\kappa} + \eta_0'^2\right)$, where

$$\eta_0' = \frac{\theta\sqrt{\tan 2\pi\kappa\delta}}{\sqrt{2}} \times \frac{\left[\frac{\kappa\theta\sqrt{\tan 2\pi\kappa\delta}}{\sqrt{2\kappa-1}} + \sqrt{2\kappa\theta P(1,-\kappa) + \tan 2\pi\kappa\delta}\right]}{2\kappa\theta P(1,-\kappa) + \tan 2\pi\kappa\delta}.$$

We note that the field structure at the resonant values of the angle of incidence is similar to that at the resonant values of κ; hence, we will not dwell on it.

Thus, if only a principal wave $(\kappa(1+\alpha) < 1)$ propagates, then, for each angle of incidence ψ, there exists a series of values of κ, for which the incident wave passes through the grating completely even in the case of narrow slits and thick bars. Similarly, for any fixed κ satisfying inequality (6), there exists an angle of full transmission ψ_0. The change of the diffracted field, when κ or ψ shifts from the resonant value, is especially abrupt if it lies in the vicinity of the sliding point $\kappa(1+\alpha) = 1$.

Similar phenomena occur in the case where several types of propagating waves exist $(\kappa(1+\alpha) \geq 1)$; however, the intensity of the transmitted field is invariably less now than the incident wave intensity.

In order to understand the origin of the resonance phenomena considered, let us turn our attention to the study of the natural modes of a grating.

4. The natural mode of a grating is the solution of the homogeneous Maxwell equations which is continuous in the domain complementary to the bars, is given by

$$H_z^c(x,y) = \begin{cases} \sum\limits_{n=-\infty}^{\infty} a_n e^{-i\pi\sqrt{\kappa^2-(n+\alpha\kappa)^2}(\eta+\delta)} e^{i\pi(n+\alpha\kappa)\xi}, \quad \eta < -\delta, \\ e^{i2\pi N\kappa\alpha} \sum\limits_{m=0}^{\infty} \left[c_m e^{i\pi\sqrt{\kappa^2-\left(\frac{m}{2\theta}\right)^2}\eta} + d_m e^{-i\pi\sqrt{\kappa^2-\left(\frac{m}{2\theta}\right)^2}\eta}\right] \cos\frac{\pi m}{2\theta}(\xi - 2N + \theta); \quad |\eta| < \delta, \quad |\xi - 2N| < \theta, \\ \sum\limits_{n=-\infty}^{\infty} b_n e^{i\pi\sqrt{\kappa^2-(n+\alpha\kappa)^2}(\eta-\delta)} e^{i\pi(n+\alpha\kappa)\xi}, \quad \eta > \delta, \ (N = 0, \pm 1, \pm 2, ...) \end{cases}$$

$$E_x^c = -\frac{1}{i\pi\kappa}\frac{\partial H_z^c}{\partial\eta}; \quad E_y^c = \frac{1}{i\pi\kappa}\frac{\partial H_z^c}{\partial\xi}; \quad E_z^c = 0;$$

$$H_y^c = 0; \quad H_x^c = 0; \quad \eta = \frac{2y}{l}; \quad \xi = \frac{2x}{l}$$

and satisfies the condition $E^c_{\text{tang}} = 0$ (on the bars).

In these relations, we have chosen the branch $\sqrt{\kappa^2 - s^2}$ (in the complex plane s being cut along vertical lines that go upwards from κ and downwards from $-\kappa$) such that $\operatorname{Im}\sqrt{\kappa^2 - s^2} \to +\infty$ as $s \to \pm\infty$. Such a choice of the branch ensures the continuity of $\sqrt{\kappa^2 - (n+\alpha\kappa)^2}$ and $\sqrt{\kappa^2 - \left(\frac{m}{2\theta}\right)^2}$ for κ and α located in the right half-plane ($\operatorname{Re}\kappa > 0$, $\operatorname{Re}\alpha > 0$).

The natural mode is characterized by two parameters κ and α which are dependent, generally speaking, on geometrical parameters of the grating and determine, respectively, the eigenfrequency and the eigenvalue of the propagation constant along the grating.

If $\operatorname{Im}\kappa \geq 0$ and ψ is real-valued ($\alpha = \sin\psi$), then the natural mode is absent for $\kappa \neq 0$; the only solution of the homogeneous Maxwell equations satisfying the above-listed conditions is the trivial solution. Hence, for a given value of ψ, the eigenfrequencies possess negative imaginary parts, while, for fixed κ, the eigenvalue of the propagation constant is determined by the complex value of the angle ψ.

If the slits between the bars are narrow, the values of κ and α associated with the natural mode, as it follows from (1), can be estimated from the equation[2]

$$\tan 2\pi\kappa\delta = -\frac{2i\kappa\theta S_0}{1 + \kappa^2\theta^2 S_0^2}, \tag{19}$$

where $S_0 = S_0(\alpha, \kappa)$ is determined by (5).

If the angle ψ is real-valued and fixed, then the roots $\kappa = \kappa_N^c$ ($N = 1, 2, \dots$) of this equation not belonging to neighborhoods of the sliding points (12) are equal, with

[2]With these values of κ and α, the fields given by (1) tend to infinity.

an accuracy of θ^2, to

$$\kappa_N^c = \frac{N}{2\delta}\left[1 - \frac{1}{N\pi}\times\right.$$

$$\times\arctan\frac{\sqrt{(1-\sigma_N^2-t_N^2)+4\sigma_N^2}-1+\sigma_N^2+t_N^2}{2\sigma_N}-$$

$$\left.-\frac{i}{2\pi N}\ln\frac{\sqrt{\left(1+\sigma_N^2+t_N^2\right)^2-4t_N^2}}{1+\sigma_N^2+t_N^2-2t_N}\right] \quad (N=1,2,...) \quad (20)$$

$$\sigma_N = \frac{U_N V_N^2}{V_N^2+R_N^2 U_N^2};$$

$$t_N = \frac{V_N R_N U_N}{V_N^2+R_N^2U_N^2}\left(\frac{2\delta U_N}{N\theta V_N}-1\right),$$

where U_N, V_N, R_N are the values of the corresponding functions at $\kappa = \frac{N}{2\delta}$. At the same time, the root $\kappa = \kappa_0^c$ located in the vicinity of the sliding point $\kappa_{\text{sl.}}(1+\alpha)=1$ has the following form for $\alpha \gg \theta^2$:

$$\kappa_0^c = \frac{1-\varepsilon^c}{1+\alpha},$$

$$\varepsilon^c = \frac{\theta^2\tan\dfrac{2\pi\delta}{1+\alpha}}{(1+\alpha)(2-\alpha)\left\{\dfrac{\theta}{\gamma}\tan\dfrac{2\pi\delta}{1+\alpha}+i\left[1+\sqrt{1+\tan^2\dfrac{2\pi\delta}{1+\alpha}}\right]-\dfrac{\theta P\left(\alpha,\frac{1}{1+\alpha}\right)}{1+\alpha}\tan\dfrac{2\pi\delta}{1+\alpha}\right\}^2}.$$

Note that, for $\alpha = 0$, we have

$$\kappa_0^c = 1-\varepsilon_0^c;$$

$$\varepsilon_0^c = \frac{2\theta^2\tan 2\pi\delta}{\left[\theta\tan 2\pi\delta + i\left(1+\sqrt{1+\tan^2 2\pi\delta}-\theta G\tan 2\pi\delta\right)\right]^2}. \quad (21)$$

For the roots whose real part is less than $(1+\alpha)^{-1}$, in the case where $\left(\frac{N\theta}{2\theta}\right)^2\left|S_0\left(\alpha,\frac{N}{2\delta}\right)\right| \ll 1$, (20) yields

$$\kappa_N^c = \frac{N}{2\delta}\left(1+\frac{2\theta}{\pi\delta}\ln\sin\frac{\pi\theta}{2}\right)-i\frac{\theta}{\pi\gamma\delta};$$

$$\left(N=1,2,\,...\,,\left[\frac{2\delta}{1+\alpha}\right]\right). \quad (22)$$

It follows from the formulas derived that the real part of κ_N^c $(N=1,2,\dots)$ is close to the resonant values κ_N (for example, (8) and (13)), while the imaginary part is negative and tends to zero as θ decreases.

We recall that, in the case of diffraction, a number of resonant curves $\kappa = \kappa_N(\psi)$ $(N=0,1,2,\dots)$ exist in the plane κ,ψ as ψ varies in the range $\left(0,\frac{\pi}{2}\right)$. Similarly, in the case of the natural mode in the three-dimensional space $\operatorname{Re}\kappa$, $\operatorname{Im}\kappa$, ψ, there exist a number of curves being the intersection of three mutually perpendicular cylinders

$$\begin{array}{l}\operatorname{Re}\kappa = \operatorname{Re}\kappa_N^c(\psi)\\ \operatorname{Im}\kappa = \operatorname{Im}\kappa_N^c(\psi)\end{array}\qquad 0\le\psi\le\frac{\pi}{2}$$

which determine the eigenvalues of the parameters κ and α. These curves are close to the corresponding resonant curves; the curve corresponding to the neighborhood of the sliding point $\kappa_{\text{sl.}}(1+\alpha)=1$ differs the least from the resonant curve.

We determine the Q-factor of the natural mode in the way similar to that for an open resonator [11],

$$Q = -\frac{\operatorname{Re}\kappa^c}{2\operatorname{Im}\kappa^c},$$

where κ^c is one of the eigenvalues of the parameter κ. At this determination, the Q-factor is little different from the quantity

$$Q' = \frac{\kappa_N}{\Delta\kappa_N},$$

where κ_N is a resonant value, and $\Delta\kappa_N$ is the transmission bandwidth in the vicinity of this value. Note that the Q-factor is a function of the angle ψ and of grating's parameters θ and δ. For the natural mode characterized by values (22), it equals

$$Q_N = \frac{\pi\gamma N}{4\theta}\left(1+\frac{2\theta}{\pi\delta}\ln\sin\frac{\pi\theta}{2}\right), \quad (N=1,2,\,...\,,[2\delta]),$$

while, for the mode specified by (21) in the case where $\tan 2\pi\delta \ll 1$, it is close to

$$Q_0 = \frac{1 - \frac{1}{2}\theta^2 \tan^2 2\pi\delta}{\theta^3 \tan^3 2\pi\delta}.$$

It is seen from these formulas that the Q-factor of the natural mode increases as θ decreases, by reaching the highest value when the eigenvalue of the parameter κ is close to one of the sliding points (12).

The eigenfield for each value of κ^c consists of oscillations of two kinds. At some distance away from the grating, oscillations of the first kind are waves of the form

$$A_n e^{i\pi\left[(\kappa^c\alpha+n)\xi+\sqrt{(\kappa^c)^2-(n+\alpha\kappa^c)^2}|\eta|\right]}, \tag{23}$$

where $\mathrm{Im}\,\kappa^c < 0$, $\mathrm{Im}\, r_n^c < 0$, and

$$r_n^c = \sqrt{\kappa^{c2} - \left(n + \alpha\kappa^c\right)^2}.$$

Their number is finite and coincides with that of integers falling in the interval $(\mathrm{Re}\left\{\kappa^c\left(1+\alpha\right)\right\}, \mathrm{Re}\left\{\kappa^c\left(1-\alpha\right)\right\})$. A wave of this kind grows exponentially in the direction normal to the grating and along positive values of ξ. However, one can specify a sector of directions, along which this wave is decaying exponentially. This sector is bounded by two rays

$$-\mathrm{Im}\,\kappa^c\alpha\xi - \mathrm{Im}\,r_n^c\left|\eta\right| = \mathrm{const}, \tag{24}$$

and its opening is less than π and equals $2\arctan\frac{\mathrm{Im}\,\kappa^c}{\mathrm{Im}\,r_n^c}$.

Rays (24) specify the directions, along which the wave given by (23) propagates.

Oscillations of the second kind have also the form of waves (23). However, they are infinite in number, and we have $\mathrm{Im}\,r_n^c > 0$ for them. Waves of this kind decay exponentially in the direction normal to the grating and grow along positive values of ξ. The opening of the sector of directions, along which they decay exponentially, is greater than π and equals $2\left(\pi + \arctan\frac{\mathrm{Im}\,\kappa^c}{\mathrm{Im}\,r_n^c}\right)$. The propagation direction of the waves is also determined by the directions of the rays given by (24).

Note that the oscillations of these kinds belong to a class of the so-called damped resonances [1] or the eigenoscillations of open systems characterized by a complex-valued frequency with a negative imaginary part, which indicates the decay of these waves with time and the energy leakage in the directions of their exponential growth. Consequently, they cannot exist without inflow of energy from the outside.

Let now κ be real-valued and fixed. First, let us consider a surface natural mode arising if $\kappa < \frac{1}{2}$ and $\alpha > 1$ (the angle ψ has the form $\frac{\pi}{2} - i\varphi$, $\varphi > 0$). Let also $\alpha\kappa = n_0 + \nu\kappa$, where n_0 is the integer nearest to $\alpha\kappa$ (hence, $|\nu\kappa| \le \frac{1}{2}$). Separate the term $n = -n_0$ in S_0. The remained sum for $\theta \ll 1$ is a slowly varying function of ν and κ. So, without any substantial error, the sum may be considered equal to $-2i\ln\sin\frac{\pi\theta}{2}$. Relation (19) yields the eigenvalue of the propagation constant $\alpha\kappa$ along the grating, $\alpha_N^c\kappa = N + \nu^c\kappa$ $(N = 0, 1, 2, \ldots)$, where ν^c can be determined from the formula

$$\sqrt{1 - \left(\nu^c\right)^2} =$$

$$= \frac{i\theta}{\cot\! \mathrm{an}\, 2\pi\kappa\delta + \sqrt{1 + \mathrm{cotan}^2\, 2\pi\kappa\delta} + 2\kappa\theta\ln\sin\frac{\pi\theta}{2}},$$

and the following inequality must be satisfied:

$$\mathrm{cotan}\, 2\pi\kappa\delta \ge \frac{1 - 4\kappa^2\theta^2\left[\left(1 - 4\kappa^2\right)^{-1/2} - \ln\sin\frac{\pi\theta}{2}\right]^2}{4\kappa\theta\left[(1 - 4\kappa^2)^{-1/2} - \ln\sin\frac{\pi\theta}{2}\right]}.$$

In this case, the natural mode does not contain waves which grow while moving away from the grating and involves only the surface waves propagating along the grating with phase velocities

$$v_n^{\mathrm{ph}} = \frac{\kappa c}{\alpha_N^c\kappa + n} \quad (n = 0, \pm1, \pm2, \ldots)$$

which are less than the velocity of light. The greatest phase velocity corresponds to the wave with $n = -N$ and is determined by the relation $v_{-N}^{\mathrm{ph}} = \frac{c}{\nu^c}$.

Oscillations of this kind can exist without inflow of energy from the outside like the eigenoscillations of a closed system. The field structure of such surface natural mode can be generated with the help of charges moving with a certain velocity near the grating.

If $1 > \kappa > \frac{1}{2}$, then, at

$$\sqrt{1 + \mathrm{cotan}^2 2\pi\kappa\delta} + \mathrm{cotan} 2\pi\kappa\delta \gg \theta\ln\sin\frac{\pi\theta}{2},$$

the root α^c of relation (19) located in the vicinity of the sliding point $\alpha_{\mathrm{sl.}} = \frac{1-\kappa}{\kappa}$ can be written as

$$\alpha^c = \frac{1-\kappa}{\kappa} - \frac{1}{2}\left(\eta^c\right)^2;$$

$$\eta^c =$$

$$= \frac{i\theta}{\frac{2\kappa\theta}{\sqrt{2\kappa-1}}+i\left[\sqrt{1+\text{cotan}^2 2\pi\kappa\delta}+\text{cotan}\,2\pi\kappa\delta-\kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right]}.$$

This implies that the real part of α^c is close to the resonant value (16) or (17), while its imaginary part is negative and equals

$$\text{Im}\,\alpha^c = -\frac{\kappa\theta^3\left[\sqrt{1+\text{cotan}^2\,2\pi\kappa\delta}+\text{cotan}\,2\pi\kappa\delta-\kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right]}{\sqrt{2\kappa-1}\left\{\frac{\kappa^2\theta^2}{2\kappa-1}+\left[\sqrt{1+\text{cotan}^2\,2\pi\kappa\delta}\right]+\text{cotan}\,2\pi\kappa\delta-\kappa\theta P\left(\frac{1-\kappa}{\kappa},\kappa\right)\right\}}.$$

If $\tan 2\pi\kappa\delta > 0$, then $-\text{Im}\,\alpha^c$ is little different from the transmission bandwidth in α. For $\tan 2\pi\kappa\delta \ll 1$, it is close to

$$-\text{Im}\,\alpha^c \approx \frac{\kappa\left(\theta\tan 2\pi\kappa\delta\right)^3}{8\sqrt{2\kappa-1}}.$$

In this case, the eigenfield at some distance from the grating is a superposition of an infinite number of waves of kind (23). However, we now have $\text{Im}\,\kappa = 0$, $\text{Im}\,\alpha < 0$, and $\text{Im}\,r_n > 0$ for all n, so that the waves are exponentially decaying in the direction normal to the grating and are increasing along the positive values of ξ. The propagation direction for these waves is determined by the rays

$$\kappa\text{Im}\,\alpha^c\xi + \text{Im}\,r_n\,|\eta| = \text{const} \qquad (25)$$

bounding a sector with the opening

$$2\left(\pi + \arctan\frac{\kappa\text{Im}\,\alpha^c}{\text{Im}\,r_n^c}\right),$$

within which they are exponentially decaying.

Waves of this kind belong to the class of leaky waves, though, in contrast to waves of this class in open waveguide systems [12], they decay exponentially in the direction normal to the guided-wave system. The energy leaks in the directions of the exponential growth of these waves which belong to a sector characterized by the opening $-2\arctan\frac{\kappa\text{Im}\,\alpha^c}{\text{Im}\,r_n^c}$ and bounded by rays (25).

Thus, we have three types of natural modes for the open periodic structure under consideration.

The first type (damped resonances) is characterized by complex values of κ belonging to the unphysical plane of this parameter and resulting in the damping of oscillations of this kind with time.

The natural mode of the second type (leaky waves) is characterized by the complex-valued propagation constants along the structure α, which indicates the field energy leakage into the free space in the directions of exponential growth of oscillations of this kind. Therefore, modes of the first and the second types cannot exist without inflow of energy from the outside, although they are the solutions of the Maxwell equations in the absence of sources. In this sense, they are similar to oscillations in an open resonator.

The natural mode of the third type (the mode of surface waves) is characterized by the real-valued κ and α ($\kappa < \frac{1}{2}$, $\alpha > 1$) and represents a superposition of waves propagating along the grating with the velocity, which is less than the velocity of light, and decaying exponentially as the distance from the grating increases. The eigenoscillations of this kind are inherent in a wide class of slow-wave structures. These oscillations can exist without inflow of energy from the outside like the eigenoscillations of a closed resonator.

5. From the above-presented consideration, it is not difficult to establish the relationship between the resonant phenomena appearing on the diffraction of a plane H-polarized wave and the natural modes of a grating. The analysis of the results obtained in Sections 3 and 4 shows that, in each resonant case, the diffracted field has a structure similar to the field of one of the natural modes. Moreover, the resonances by κ correspond to the mode of damped resonances, while the resonances by an angle of incidence correspond to the surface-wave mode or the leaky-wave mode. The diffracted field most closely resembles the eigenfield if the resonant values of κ and α belong to the vicinity of one of the sliding points (12). In this case, the intensity of the transmitted field changes especially abruptly as κ or α shifts from the corresponding resonant value; the lower the ratio $\frac{d}{l}$, the sharper this change. For reflecting gratings, these resonances in diffraction spectra correspond to the well-known Wood anomalies. However, as it is seen from the results of this work, a metal-bar grating possesses, in addition to the resonances associated with the Wood anomalies, a number of other resonances. Of particular interest are the resonant phenomena occurring in the long-wave region, since they could be used in the study of the processes accompanying the motion of charged particles near a diffraction grating [13].

It is significant that a metal-bar grating can be considered as a resonant system, in which high-Q oscillations can be excited with the help of an H-polarized plane wave. At the appropriate choice of κ and α, the forced oscillations possess a number of interesting properties. Among them are the presence of frequencies and angles of incidence such that the incident wave passes through a grating without reflection even if the slits are narrow and the bars are thick; abnormal transmission in the vicinities of sliding points, *etc.*

If the bar thickness is zero, the resonant phenomena considered above do not exist.

The authors would like to thank Ya.S. Komissarov and V.A. Pavlyuk for their help with the numerical calculations.

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Received January 24, 1967

SHESTOPALOV VIKTOR PETROVYCH
(23.01.1923–28.11.1999)

Ukrainian physicist, Academician of the NASU (1979). Born on January 23, 1923 at Slavyansk (Donetsk region). Graduated from the Kharkiv University (1949). Head of a department at the Kharkiv Pedagogical Institute (1950–1958), the Kharkiv University (1958–1963, 1969–1971), and the Kharkiv Institute of Radioelectronics (1963–1969), Deputy Director (1971–1973) and Director (1973–1993) of the Institute of Radiophysics and Electronics of the NASU. Died November 28, 1999.

His researches dealt with the theory of continuous spheres, optical spectroscopy, diffraction theory, physics and engineering of millimeter and submillimeter waves. In 1962, together with V.A. Marchenko and Z.S. Agranovych, he developed the method of rigorous solution of diffraction problems in radiophysics and electronics. In 1971, he created coherent radiation sources of a new type, diffraction radiation generators, and laid the foundation of diffraction electronics.

State Prize of Ukraine (1972), Honored Worker in Science of Ukraine (1983), State Prize of the USSR (1987).