90 YEARS

ON THE THEORY OF NUCLEAR REACTIONS INVOLVING COMPOSITE PARTICLES

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As known, the interaction of an individual particle with a central field can be described in terms of phases which unambiguously determine both the scattering and absorption cross-sections of particles [1]. The interaction between the central field and a system of particles interacting with one another can also be described, in the adiabatic approximation, with the help of phases. This approach is suitable for obtaining the general formulas for the cross-sections of various processes in terms of phases which characterize the interaction of individual particles of the composite system with the central field.

1. First, we consider the scattering of an individual particle in a central field. We denote the wave vector of an incident particle by $\mathbf{k}_0$ ($|\mathbf{k}_0| = k$) and take the plane wave

$$\psi_0 = e^{i\mathbf{k}_0\mathbf{r}}$$

as the pre-collision wave function of the particle. We will describe the particle-field interaction in terms of the unitary scattering matrix S. Then the post-collision wave function of the particle can be written in the form

$$\psi = S\ \psi_0. \tag{1}$$

Both the energy and momentum of a particle scattered by the central field are conserved. Therefore, the scattering matrix is diagonal in the representation, in which energy, squared momentum, and its projection are diagonal. In this case, the eigenvalues of the scattering matrix can depend on the particle energy and momentum, $S = S(E,\ l)$ (l is the orbital quantum number). In the quasiclassical approximation, the particle momentum is unambiguously related to the impact parameter $\rho = l\lambda$ (λ is the wavelength). Hence, we have

$$S = S(E, \rho),$$

where ρ may be regarded as the polar coordinate of a particle in the plane perpendicular to the initial wave vector $\mathbf{k}_0\big(\boldsymbol{\rho}\perp\mathbf{k}_0\big)$. The eigenvalues of the scattering matrix S may be expressed in terms of the scattering phases η at infinity by the relation

$$S(E,\rho) = e^{2i\eta\ (E,\rho)}. \tag{2}$$

To find the asymptotics of the wave function $\psi(r)$, we employ the Huygens principle. In the region of small angles (where the quasiclassical approximation is valid), it can be written in the form

$$\psi(\mathbf{r}) = \frac{k}{2\pi i}\int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\ \psi(\mathbf{r}')ds', \tag{3}$$

where the integration should be executed over an arbitrary closed surface. Choosing the integration surface to be a plane perpendicular to the initial wave vector $\mathbf{k}_0\,(\boldsymbol{\rho}\mathbf{k}_0 = 0)$ and bearing in mind that $\psi_0(\rho) = 1$, we obtain

$$\psi(\mathbf{r}) = \frac{k}{2\pi i}\int \frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\ S(\rho)\ d\rho. \tag{4}$$

In a similar manner, the initial wave function looks as

$$\psi_0(\mathbf{r}) = \frac{k}{2\pi i}\int \frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\ d\rho. \tag{5}$$

Subtracting (5) from (4) yields the expression that is suitable to determine the asymptotic form of the wave function:

$$\psi(\mathbf{r}) = \psi_0(\mathbf{r}) + \frac{k}{2\pi i}\int \frac{e^{ik|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\,\{S\ (\rho) - 1\}\ d\rho. \tag{6}$$

Indeed, large values of ρ are inessential in the integrand on the right-hand side of (6), because $S(\rho) \to 1$ as $\rho \to \infty$. Therefore, for r sufficiently large, we can employ the expansion

$$|\mathbf{r}-\boldsymbol{\rho}| \approx r - \frac{\mathbf{r}}{r}\ \boldsymbol{\rho},$$

in the integrand and thus obtain

$$\psi(\mathbf{r}) \to \psi_0(\mathbf{r}) + f(\mathbf{k}_0, \mathbf{k}) \, \frac{e^{ikr}}{r}, \qquad r \to \infty, \tag{7}$$

$$f(\mathbf{k}_0, \mathbf{k}) = i\frac{k}{2\pi} \int d\rho e^{-i\mathbf{k}\boldsymbol{\rho}} \{1 - S(\rho)\} \tag{8}$$

$\left(\mathbf{k} = \frac{\mathbf{r}}{r} k \text{ is the post-collision wave vector of a particle}\right)$. The asymptotic wave function (7) is given by the sum of the incident plane wave and the diverging spherical wave that determines the scattering of particles. The elastic scattering amplitude (8) is uniquely determined by the scattering matrix $S(\rho)$ or by the scattering phases $\eta(\rho)$.

The differential elastic scattering cross-section is given by

$$d\sigma_e = \left|f(\mathbf{k}_0, \mathbf{k})\right|^2 d\mathfrak{o}, \tag{9}$$

where $d\mathfrak{o}$ is an element of the solid angle. Taking the relation $k^2 d\mathfrak{o} = d\varkappa$ into account ($\varkappa$ is the plane wave vector) and using the completeness of the system of plane waves $e^{i\varkappa\rho}$, we obtain the integral elastic scattering cross-section:

$$\sigma_e = \int d\rho |1 - S(\rho)|^2. \tag{10}$$

In order to derive the absorption cross-section of particles, we will determine the density of a particle flux through the plane perpendicular to the initial wave vector $\mathbf{k}_0$. Since $\psi_0(\mathbf{r}) = e^{ikz}$ and the scattering matrix $S(\rho)$ does not depend on the z-coordinate, we have

$$j = \frac{i\hbar}{2M} \left(\psi \frac{\partial\psi^*}{dz} - \psi^* \frac{\partial\psi}{dz}\right) = \frac{\hbar k}{M} \, |S(\rho)|^2.$$

The incident flux density is $j_0 = \frac{\hbar k}{M}$. A decrease of the flux density ($j < j_0$) is caused by the absorption of particles, so that $|S|^2 < 1$. We subtract the density $j(\rho)$ from j_0, integrate the difference over the whole ρ plane and thus find the number of absorbed particles. Dividing this number by the incident flux density j_0 yields the absorption cross-section of particles

$$\sigma_a = \int d\rho \left\{1 - |S(\rho)|^2\right\}. \tag{11}$$

The total cross-section of all the processes, which is related to the zero-angle elastic scattering amplitude as $\sigma_t = 4\pi\lambda \text{ Im } f(0)$, is given by the expression

$$\sigma_t = 2 \int d\rho \{1 - \text{Re } S(\rho)\}. \tag{12}$$

The total cross-section σ_t is equal to the sum of the scattering cross-section σ_e and the absorption cross-section σ_a.

2. Now let us consider the collision of a system of A interacting particles and some particle O, whose state remains unchanged in the course of the collision. (As an example, we mention the scattering of a system of interacting nucleons, i.e. a nucleus, by a central field). We suppose the interaction of each individual particle of the incident system with the scatterer to be central. The relative distance between system A and particle O is characterized by the vector $\mathbf{r} = \mathbf{R}_A - \mathbf{r}_0$, where $\mathbf{R}_A = M \sum_1^A \mathbf{r}_k / M_A$ is the center-of-inertia radius vector of the incident system, and $\mathbf{r}_0$ is the radius vector of the scatterer. (M and $\mathbf{r}_k$ are the mass and the radius vector of an individual constituent of the system, and M_A is the mass of the system).

The pre-collision wave function of the entire system Ψ_0 can be written as the product of the wave function describing the relative motion, $\psi_{i\mathfrak{o}} = e^{i\mathbf{k}_{i\mathfrak{o}}\mathbf{r}}$ ($\mathbf{k}_{i0}$ is the pre-collision wave vector of the relative motion), and the wave function Φ_i describing the intrinsic motion of the incident system. Functions Φ_j are solutions to the equation

$$H_A \Phi_j = E_j \Phi_j,$$

where H_A is the intrinsic Hamiltonian of system A, and E_j are probable values of the energy. The functions Φ_j form a complete system of orthogonal functions. The total energy of the system

$$E = \frac{\hbar^2 k_i^2}{2\mu} + E_i,$$

where $\mu = \frac{M_0 M_A}{M_0 + M_A}$ is the reduced mass of the system.

We introduce the scattering matrix S and write the post-collision wave function of the entire system as

$$\Psi = S\Psi_0. \tag{13}$$

Since the system of functions Φ_j is complete, this function can be expanded in a series:

$$\Psi = \sum_j \psi_j(\mathbf{r}) \, \Phi_j.$$

According to (13), the expansion coefficient ψ_j is given by

$$\psi_j(\mathbf{r}) = (\Phi_j,\ S\psi_{i0}\Phi_i)\,. \tag{14}$$

We regard it as the wave function of the relative motion of the post-collision system. The wave vector of the relative motion of the post-collision system is determined by the energy conservation law:

$$k_j^2 = \frac{2\mu}{\hbar^2}\ (E - E_j)\,.$$

On the elastic scattering, the intrinsic state of the incident system is not changed. In this case, the wave function ψ_i at infinity is composed from the incident plane wave ψ_{i0} and the diverging spherical wave. On the inelastic scattering, the intrinsic state of the incident system is changed, and the wave function ψ_j at long distances is a diverging spherical wave. To demonstrate this, we employ the Huygens principle which yields

$$\psi_j(\mathbf{r}) = \frac{k_j}{2\pi i}\int \frac{e^{ik_j|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\ \psi_j(\mathbf{r}')\ ds'. \tag{3$'$}$$

Let the integration surface in (3$'$) be a plane perpendicular to the initial wave vector $\mathbf{k}_{i0}$. Then we have

$$\psi_j(\mathbf{r}) = \frac{k_j}{2\pi i}\int \frac{e^{ik_j|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\ (\Phi_j,\ S(\rho)\ \Phi_i)\ d\rho. \tag{15}$$

Since the scattering matrix tends to unity for large ρ, $S(\rho) \to 1$, large ρ are insignificant in integral (15) on the inelastic scattering ($j \neq i$), because the functions Φ_j and Φ_i are orthogonal. Therefore, at sufficiently large distances ($r \to \infty$), we can use the expansion $|\mathbf{r}-\boldsymbol{\rho}| \approx r - \frac{\mathbf{r}}{r}\ \boldsymbol{\rho}$, which implies that the asymptotics of ψ_j reduces to a diverging spherical wave

$$\psi_j(\mathbf{r}) \to f_{ij}\frac{e^{ik_jr}}{r}, \qquad r \to \infty. \tag{16}$$

The inelastic scattering amplitude is then given by

$$f_{ij} = \frac{k_j}{2\pi i}\int d\rho e^{i\mathbf{k}_j\boldsymbol{\rho}}\,(\Phi_j,\ S\Phi_i) \qquad (j \neq i), \tag{17}$$

where $\mathbf{k}_j = \frac{\mathbf{r}}{r}\ k_j$ is the post-collision wave vector of relative motion.

In the case of elastic scattering, expression (15) cannot be directly applied to the calculation of the asymptotics of ψ_i. Therefore, using the relation

$$\psi_{i0}(\mathbf{r}) = \frac{k_i}{2\pi i}\int \frac{e^{ik_i|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\ d\rho,$$

we represent $\psi_i(\mathbf{r})$ as

$$\psi_i(\mathbf{r}) = \psi_{i0}(\mathbf{r}) + \frac{k_i}{2\pi i}\int \frac{e^{ik_i|\mathbf{r}-\boldsymbol{\rho}|}}{|\mathbf{r}-\boldsymbol{\rho}|}\ (\Phi_i,\ \{S-1\}\ \Phi_i)\ d\rho. \tag{18}$$

The first term on the right-hand side of (18) is the wave function in the form of a plane wave; the asymptotics of the second term is a diverging spherical wave. (Large values of ρ are inessential in the integrand of (18) since $S \to 1$ as $\rho \to \infty$). Thus, for $r \to \infty$, we have

$$\psi_i(\mathbf{r}) \to \psi_{i0}(\mathbf{r}) + f_{ii}\frac{e^{ik_ir}}{r}. \tag{19}$$

In this case, the elastic scattering amplitude is determined by the formula

$$f_{ii} = \frac{k_i}{2\pi i}\int d\rho e^{-i\mathbf{k}_i\boldsymbol{\rho}}\,(\Phi_i,\ \{S-1\}\ \Phi_i)\,, \quad \mathbf{k}_i = \frac{\mathbf{r}}{r}\ k_i. \tag{20}$$

Expressions (17) and (20) for the elastic and inelastic scattering amplitudes can be unified in view of the orthogonality of the functions Φ_i and Φ_j. Thus, we have

$$f_{ij} = i\ \frac{k_j}{2\pi}\int d\rho e^{-\chi\rho}\,(\Phi_j,\ \{1-S\}\ \Phi_i)\,. \tag{21}$$

If the duration of the effective interaction of the system with an external field is small as compared to the characteristic time interval of intrinsic motions in the system, we can use the adiabatic approximation, according to which each particle of the system interacts with the external field independently. In this case, the scattering matrix of the composite system can be written in the form of the product of scattering matrices for individual particles,

$$S - \prod_{k=1}^{A} S_k\ \ (\rho_k - \rho_0) = e^{2i\sum\limits_{k=1}^{A}\eta_k\ \ (\rho_k-\rho_0)}, \tag{22}$$

where η_k is the phase of the k-th particle under the scattering of the incident system by the field. The validity of the adiabatic approximation means that the averaged binding energy per particle is small as compared with its kinetic energy.

Finally, the scattering amplitude of a system of particles in a central field is as follows:

$$f_{ij} = i\ \frac{k_j}{2\pi}\int d\rho e^{-i\chi\rho}\int d\tau\Phi_j^* \times \tag{23}$$

$$\times (\mathbf{r}_1, \ldots, \mathbf{r}_A) \left\{ 1 - e^{2i \sum\limits_{k=1}^{A} \eta_k \ (\rho_k - \rho_0)} \right\} \Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A).$$

Here, $d\tau$ is the elementary volume in the space of intrinsic coordinates of the incident system. For the differential scattering cross-section, we have

$$d\sigma_{ij} = \left|f_{ij}\right|^2 dO. \tag{24}$$

Taking both the relation $k_j^2 dO = d\varkappa$ and the completeness of the system of plane waves into account, we obtain the following formulas for the integral elastic and inelastic scattering cross-sections:

$$\sigma_{ii} = \int d\rho \left| \int d\tau \left\{ 1 - e^{2i \sum\limits_{k=1}^{A} \eta_k \ (\rho_k - \rho_0)} \right\} \times \right.$$

$$\left. \times \left|\Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A)\right|^2 \right|^2, \tag{25}$$

$$\sigma_{ij} = \int d\rho \left| \int d\tau \Phi_j^* (\mathbf{r}_1, \ldots, \mathbf{r}_A) \, e^{2i \sum\limits_{k=1}^{A} \eta_k \ (\rho_k - \rho_0)} \times \right.$$

$$\left. \times \ \Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A) \right|^2 \quad (i \neq j). \tag{26}$$

In view of the completeness of the system of functions Φ_j, the total scattering cross-section $\sigma_s = \sum\limits_j \sigma_{ij}$ looks as

$$\sigma_s = \int d\rho \int d\tau \left| 1 - e^{2i \sum\limits_{k=1}^{A} \eta_k \ (\rho_k - \rho_0)} \right|^2 \times$$

$$\times \left|\Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A)\right|^2. \tag{27}$$

The momentum distribution of recoil particles is given by the formula

$$d\sigma(\varkappa) = \int d\tau \left| \int d\rho e^{-i\chi\rho} \left(1 - e^{2i \sum\limits_{k=1}^{A} \eta_k \ (\rho_k - \rho_0)} \right) \right|^2 \times$$

$$\times \left|\Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A)\right|^2. \tag{28}$$

The scattering cross-section σ_s describes the processes under which the number of particles in the incident system remains unchanged (the elastic scattering and the scattering accompanied by the partial or full disintegration of the incident system).

3. Let us find the total cross-section of a reaction induced by a composite incident particle. Without the external field, the particle flux density through a plane perpendicular to the initial wave vector $\mathbf{k}_{i0}$ is equal to $j_0 = \dfrac{\hbar k_i}{\mu}$. Let now the external field be present. In view of (14), the expression for the particle flux density through the same plane under the transition from the state i to the state j is

$$j_j = \frac{\hbar k_i}{\mu} \left|\Phi_j, \ S \ (\rho) \ \Phi_i\right|^2.$$

Then the total particle flux density is given by

$$j = \sum_j j_j = \frac{\hbar k_i}{\mu} \left(\Phi_i, \ |S|^2 \ \Phi_i\right).$$

To obtain the reaction cross-section σ_r, we divide the difference $j_0 - j$ by j_0 and integrate the quotient over the whole plane. Thus, we have

$$\sigma_r = \int d\rho \int d\tau \left\{ 1 - e^{-4 \ \mathrm{Im} \sum\limits_{1}^{A} \eta_k \ (\rho_k - \rho_0)} \right\} \times$$

$$\times \left|\Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A)\right|^2. \tag{29}$$

The reaction cross-section σ_r describes all the processes, in which the number of particles in the incident system is changed (absorption, partial absorption, stripping, capture, etc.).

4. Making use of expression (23) for the elastic scattering amplitude yields the following formula for the total cross-section of all processes:

$$\sigma_t = 2 \int d\rho \int d\tau \left\{ 1 - \mathrm{Re} \ e^{2i \ \mathrm{Im} \sum\limits_{1}^{A} \eta_k \ (\rho_k - \rho_0)} \right\} \times$$

$$\times \left|\Phi_i \ (\mathbf{r}_1, \ldots, \mathbf{r}_A)\right|^2. \tag{30}$$

As could be expected, the total cross-section σ_t is equal to the sum of the scattering cross-section σ_s and the reaction cross-section σ_r.

It is worth noting that the total cross-section of the interaction of a composite system with any field is not equal to the sum of individual cross-sections of particles' interaction with this field. This is related to the presence of intrinsic scattering effects which violate the additivity of the cross-sections. Indeed, the difference $1 - \prod\limits_1^A S_k$ contained in (21) is identically equal to the sum

$$1 - \prod_1^A S_k = \sum_1^A (1 - S_k) - \sum_{k \neq j} (1 - S_k)(1 - S_j) +$$

$$+\sum_{k\neq j\neq i}(1-S_k)(1-S_j)(1-S_i)+\ldots+$$

$$+(-1)^{A+1}(1-S_1)(1-S_2)\ldots(1-S_A).$$

It is not difficult to understand the physical meaning of each term on the right-hand side of the last equation. The difference $1-S_k$ determines the amplitude of the individual particle scattering by the field. Hence, the first term on the right-hand side corresponds to the independent scattering of individual constituents of the composite system by the field, the second term describes effects associated with the additional intrinsic scattering, the third term is responsible for the effects produced by the additional intrinsic double scattering, and so on. Disregarding the intrinsic scattering effects, i.e., substituting the sum $\sum\limits_1^A(1-S_k)$ for $1-\prod\limits_1^A S_k$ yields

$$\sigma_t\simeq\sum_{k=1}^A\sigma_t^{(k)}.$$

If the interaction is weak, $|\eta_k|\ll 1$, then the elastic scattering amplitude (23) is the same as that in the Born approximation. Indeed, in view of the relation

$$e^{2i\sum\eta_k}-1\simeq 2i\sum\eta_k,$$

and the definition of scattering phase

$$\eta(\rho)=-\frac{1}{2\hbar v}\int\limits_{-\infty}^{\infty}V(\mathbf{r})\,dz,$$

where $V(r)$ is the interaction potential, we obtain the scattering amplitude in the form

$$f_{ij}=-\frac{\mu}{2\pi\hbar^2}\int d\mathbf{r}e^{-i\varkappa\mathbf{r}}\times$$

$$\times\int d\tau\Phi_j^*(\mathbf{r}_1,\,\ldots,\mathbf{r}_A)\sum_{k=1}^A V_k(\mathbf{r}_k-\mathbf{r}_0)\;\Phi_i(\mathbf{r}_1,\,\ldots,\mathbf{r}_A).\tag{31}$$

Here, $\varkappa$ stands for the difference of the pre-collision and post-collision wave vectors ($\varkappa\to\mathbf{k}_0-\mathbf{k}$).

5. Let us find the cross-section of a stripping reaction, in which one of the particles of the incident system is captured by the scatterer. Let the frame of reference be such that the particle-scatterer with number 0 is at rest before the collision, and let a particle with number 1 belonging to the incident system be captured. We introduce the relative coordinates

$$\mathbf{r}'=\mathbf{r}_1-\mathbf{r}_0,\qquad\mathbf{r}''=\mathbf{R}_{A-1}-\mathbf{r}_0,$$

where $\mathbf{R}_{A-1}$ is the center-of-inertia radius vector of the system after the capture. We denote the intrinsic wave function of this system by $\varphi_f(\mathbf{r}_2,\,\ldots,\mathbf{r}_A)$ and the center-of-inertia wave vector by $\mathbf{k}_f$.

We multiply the total wave function of the whole system $\Psi=S\Psi_0$ by $e^{-i\mathbf{k}_f\mathbf{r}''}\varphi_f^*(\mathbf{r}_2\,\ldots,\mathbf{r}_A)$. By integrating this product over the variable $\mathbf{r}''$ and the intrinsic coordinates of the escaping system, we obtain the wave function of the captured particle as

$$\psi_f(\mathbf{r}')=\left(e^{i\mathbf{k}_f\mathbf{r}''}\varphi_f(\mathbf{r}_2,\ldots,\mathbf{r}_A),\prod_1^A S_k(\rho_k-\rho_0)\times\right.$$

$$\left.\times e^{i\mathbf{k}_{i0}\mathbf{r}}\Phi_i(\mathbf{r}_1,\ldots,\mathbf{r}_A)\right).$$

We note that $\mathbf{r}=\dfrac{M_{A-1}}{M_A}\,\mathbf{r}''+\dfrac{M}{M_A}\,\mathbf{r}'$ and introduce a new integration variable $\boldsymbol{\xi}=\mathbf{r}'-\mathbf{r}''$. Then we have

$$\psi_f(\mathbf{r}')=e^{i(\mathbf{k}_{i0}-\mathbf{k}_f)\,\mathbf{r}'}S_1(\rho')\times$$

$$\times\left(e^{-i\mathbf{k}_f\boldsymbol{\xi}}\varphi_f,\;\prod_2^A S_k e^{-i\frac{M_{A-1}}{M_A}\,\mathbf{k}_{i0}\boldsymbol{\xi}}\;\Phi_i\right).\tag{32}$$

We make use of (32) to calculate the flux density of captured particles through the plane perpendicular to the initial relative-motion wave vector $\mathbf{k}_{i0}$:

$$J=\frac{\hbar}{M}(\mathbf{k}_{i0}-\mathbf{k}_f)_z\left|S_1(\rho')\right|^2\times$$

$$\times\left|\left(e^{-i\mathbf{k}_f\boldsymbol{\xi}}\varphi_f,\;\prod_2^A S_k e^{-i\frac{M_{A-1}}{M_A}\,\mathbf{k}_{i0}\boldsymbol{\xi}}\;\Phi_i\right)\right|^2.$$

If the captured particle does not interact with the scatterer ($S_1=1$), this density is given by

$$J_0=\frac{\hbar}{M}(\mathbf{k}_{i0}-\mathbf{k}_f)_z\times$$

$$\times\left|\left(e^{-i\mathbf{k}_f\boldsymbol{\xi}}\varphi_f,\;\prod_2^A S_k e^{-i\frac{M_{A-1}}{M_A}\,\mathbf{k}_{i0}\boldsymbol{\xi}}\;\Phi_i\right)\right|^2.$$

A decrease of the flux density in the case of interacting particles ($I<I_0$) is caused by the absorption of particles by the scattering system. Integrating the difference I_0-I over the whole plane $\left|\rho'\mathbf{k}_{i0}\right|=0$ and dividing the result by the incident flux density $j_0=\dfrac{\hbar k_i}{M_A}$

yields the stripping cross-section referred to the unit density of final states. If the escaping system is in the state f, the differential cross-section of the stripping reaction is determined by the formula

$$d\sigma(\mathbf{k}_f) = \frac{M_A}{M} \left(1 - \frac{k_{fz}}{k_i}\right) \int d\rho' \left\{1 - \left|S_1)\rho'\right|^2\right\} \times$$

$$\times \left| \int d\mathbf{r}'' d\tau_{A-1} e^{-i\mathbf{k}_f \mathbf{r}''} \varphi_f^* \left(\mathbf{r}_2, \cdots, \mathbf{r}_A\right) \prod_2^A S_k \left(\rho_k - \rho_0\right) \times\right.$$

$$\left. \times e^{i \frac{M_{A-1}}{M_A} \mathbf{k_{i0}} \mathbf{r}''} \Phi_i \left(\mathbf{r}_1, \ldots, \mathbf{r}_A\right) \right|^2 \frac{d\mathbf{k}_f}{(2\pi)^3}. \qquad (33)$$

In the region of validity of the adiabatic approximation, the distribution over k_{fz} has a sharp maximum at $k_{fz} = \dfrac{M_{A-1}}{M_A} k_i$. Indeed, the adiabatic approximation is applicable provided the binding energy per particle is small. But this means that the wave function must weakly depend on the coordinates. The Fourier component of a weakly varying function is maximum for small values of the wave vector. Therefore, the distribution which is proportional to the square modulus of $\int dz e^{i\left(\frac{M_{A-1}}{M_A} k_i - k_{fz}\right) z} \Phi_i(z, \cdots)$ is characterized by a maximum at $k_{fz} = \dfrac{M_{A-1}}{M_A} k_i$. In this case, the factor $\dfrac{M_A}{M} \left(1 - \dfrac{k_{fz}}{k_i}\right)$ in (33) can be approximately replaced by unity. After this replacement, it is not difficult to find the total cross-section of the stripping reaction by using the condition that the system of functions $e^{i\mathbf{k}_f \boldsymbol{\xi}} \varphi_f$ is complete:

$$\sum_f \left(e^{i\mathbf{k}_f \boldsymbol{\xi}} \varphi_f(\zeta),\ e^{i\mathbf{k}_f \boldsymbol{\xi}'} \varphi_f(\zeta')\right) = (2\pi)^3 \delta\ (\xi - \xi')\ \delta_{\zeta\zeta'}.$$

Thus, the formula for the total cross-section of the stripping reaction, in which one of the particles of the incident system is captured, reads

$$\sigma_{str} = \int d\rho' \{1 - e^{-4\ \mathrm{Im}\ \eta_1(\rho')}\} \times$$

$$\times \int d\tau e^{-4\mathrm{Im} \sum_2^A \eta_k(\rho_k - \rho_0)} \left| \Phi_i \left(\mathbf{r}_1, \ldots, \mathbf{r}_A\right) \right|^2. \qquad (34)$$

Since $|S_k|^2 \leq 1$, the cross-section σ_{str} is always smaller that the total cross-section σ_r of the reaction.

6. It is obvious that the results thus obtained are valid both for the scattering of a system of interacting nucleons by an individual particle and for the scattering of a nucleon by a system of interacting nucleons. Formulas (25) and (26) describe the elastic and inelastic scatterings of nucleons by nuclei, while formula (29) determines the reaction cross-section. In particular, formulas (33) and (34) determine the cross-section of the pick-up reaction where the incident nucleon captures a nucleon from the nucleus-scatterer.

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1. A.I. Akhiezer and I.Ya. Pomeranchuk, *Some Problems of Nuclear Theory* (Gostekhizdat, Moscow, 1950) (in Russian).

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O.G. Sitenko was the outstanding physicist-theoretician. He graduated from Kharkiv State University in 1949. There he worked from 1949 till 1961. In 1961–1968, O.G. Sitenko was Head of a department at the Institute of Physics of the Acad. of Sci. of UkrSSR. From 1968, he guided a department at the Institute for Theoretical Physics of the Nat. Acad. of Sci. of Ukraine, and, from 1988, O.G. Sitenko was Director of this institute. At the same time, starting from 1963, he was Professor and Head of a chair at Taras Shevchenko Kyiv National University. O.G. Sitenko was Editor-in-Chief of Ukrainian Physical Journal (from 1988); Academician of the Nat. Acad. of Sci. of Ukraine (from 1982); laureate of the State Prize of Ukraine (1992) and Sinel'nikov's (1976) and Bogolyubov's (1994) prizes; Honored Worker in Science and Technique of Ukraine (1996).

O.G. Sitenko is well known due to his works in the fields of nuclear physics and plasma physics. He foresaw (together with O.I. Akhiezer) the diffraction splitting of a deuteron, developed (1958) the theory of diffraction nuclear processes at high energies with participation of complex nuclear particles (the Glauber–Sitenko theory); developed (1960) the theory of quasielastic scattering of high-energy electrons by nuclei; introduced, for the first time, the tensor of dielectric permeability for plasma in a magnetic field with regard for spatial dispersion(1955); developed the theory of electromagnetic fluctuations in plasma, the nonlinear theory of fluctuations and turbulent processes in plasma; and foresaw the Raman scattering of electromagnetic waves in plasma. His main scientific results are presented in the monographs:

1. A.I. Akhiezer, I.A. Akhiezer, R.V. Polovin, A.G. Sitenko, and K.N. Stepanov, *Collective Oscillations in Plasma* (Lockheed Company, Sunnyvale, 1965).
2. A.G. Sitenko, *Fluctuations and Non-Linear Wave Interactions in Plasmas* (Pergamon Press, Oxford, 1982).
3. A. Sitenko and V. Malnev, *Plasma Physics Theory* (Chapman and Hall, London, 1995).
4. A. Sitenko and V. Tartakovskii, *Theory of Nucleus: Nuclear Structure and Nuclear Interaction* (Kluwer, Dordrecht, 1997).