ON THE THEORY OF CYCLOTRON RESONANCE

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The possibility for the cyclotron resonance to be observed in a metal in a magnetic field inclined to the surface is studied. The surface impedance for a metal in a magnetic field parallel to the surface is calculated for an arbitrary law of reflection of electrons from the surface.

1. Introduction

M.Ya. Azbel and the present author [1–3] worked out the theory of cyclotron resonance for the case of a metal in a magnetic field parallel to the surface, by assuming the reflection of electrons from the surface to be purely diffuse. For the parallel magnetic field and a quadratic dispersion law, all the electrons on the boundary of the Fermi distribution contribute to the resonance, since they all have the same orbital revolution frequency Ω and pass many times through the skin layer. When the dispersion law has a more general character, only electrons with an extremal value of effective mass contribute to the resonance. In this case, the relative depth of the resonance is smaller than that for a quadratic dispersion law. In an inclined magnetic field, the majority of electrons enter the skin depth only once, and then they disappear into the interior of the metal and, therefore, do not contribute to the resonance. However, Chambers [4] pointed out that, in a magnetic field inclined at a considerable angle Φ to the surface, a resonance can still be produced by the minority of electrons which have $\bar{v}_H \approx 0$ and therefore return to the skin layer many times without drifting along the direction of the constant magnetic field. The bar over $\bar{v}_H$ denotes the averaging over the path of an electron with $\varepsilon(\mathbf{p}) = \zeta$, $p_H = \text{const}$. Here, ε is the energy, $\mathbf{p}$ the wave vector, $\mathbf{v} = \partial\varepsilon/\partial\mathbf{p}$ the velocity, ζ the chemical potential of the electron, and p_H and v_H are the projections of $\mathbf{p}$ and $\mathbf{v}$ on the direction of the magnetic field.

We showed earlier [2, 3] that, to the lowest order in δ_{eff}/r, the surface impedance Z of the metal is, in general, independent of the magnetic field. The field dependence and the resonance can occur, generally speaking, only in the next orders in δ_{eff}/r. Here, δ_{eff} is the effective skin depth and r is the orbital radius of an electron in the magnetic field. Therefore, it is of importance to evaluate the magnitude of resonance effects in an inclined magnetic field in order to clarify whether such effects might be observed experimentally.

It is also of interest to consider how the reflection of electrons from the metal surface influences the high-frequency surface impedance. If the surface were strictly regular, then the reflection of electrons from it should be specular. But even a single crystal always has surface irregularities of sizes comparable with interatomic spacings, though they can be small as compared with the effective skin depth. Therefore, the reflection of electrons from the surface will be diffuse or almost diffuse. This means that the distribution function of the electrons reflected from the surface does not correlate with that of the incident electrons.

In the present paper, we discuss the cyclotron resonance in an inclined magnetic field and investigate the dependence of the impedance on the reflection coefficient of electrons at the metal-vacuum boundary in the case where a constant magnetic field is parallel to the metal surface.

2. Cyclotron Resonance in an Inclined Magnetic Field

The calculation of the total surface impedance tensor

$$Z_{\mu\nu} \equiv R_{\mu\nu} + iX_{\mu\nu} = \partial E_\mu(0)/\partial I_\nu = = (4\pi\omega/ic^2)\partial E_\mu(0)/\partial E'_\nu(0) \qquad (\mu, \nu = x, y) \tag{1}$$

reduces to a joint solution of Maxwell's equations

$$\text{rot}\mathbf{E} = -i\omega\mathbf{H}/c; \quad \text{div}\mathbf{H} = \text{div}\mathbf{E} = 0;$$

$$\text{rot}\mathbf{H} = 4\pi\mathbf{j}/c; \qquad \mathbf{j} = -\frac{2e}{h^3}\int \mathbf{v} f_1 d\mathbf{p} \tag{2}$$

and of the kinetic equation for the addition f_1 to the equilibrium Fermi distribution function $f_0(\varepsilon)$

$$i\omega f_1 + v_z\frac{\partial f_1}{\partial z} + \Omega\frac{\partial f_1}{\partial \tau} + \frac{f_1}{t_0} = e\mathbf{E}\mathbf{v}\frac{\partial f_0}{\partial \varepsilon}. \tag{3}$$

Here, $R_{\mu\nu}$ and $X_{\mu\nu}$ are, respectively, the active and reactive resistance tensors, $E_\mu(z)$ are the tangential components of the electric field, I_ν are the components of the total current, ω is the frequency of the external field, $\omega = eH/mc$ is the "cyclotron" frequency, $m = (1/2\pi)\partial S(\varepsilon, p_H)/\partial\varepsilon$, $S(\varepsilon, p_H)$ is the area cut out from the surface $\varepsilon(\mathbf{p}) = \varepsilon$ by the plane p_H = const, τ is the dimensionless time of revolution of an electron in its orbit, $t_0(\mathbf{p})$ is the relaxation time, and the z-axis and the x-axis are directed, respectively, along the inward normal to the metal surface and the projection of the constant magnetic field $\mathbf{H}$ on the metal surface.

The boundary condition for the kinetic equation (3) is the condition of diffuse scattering

$$f_1 = 0 \quad \text{for} \quad z = 0, \quad v_z > 0. \tag{4}$$

As shown in [2, 3], to find the surface impedance tensor $Z_{\mu\nu}$, it is convenient to carry out the Fourier transformation with respect to z in Eq. (2) and (3). After calculations fully analogous to those made earlier in [2,3], we obtain the following equations for the Fourier transforms:

$$-k^2\mathcal{E}_\mu(k) - 2E'_\mu(0) = (4\pi i\omega/c^2) j_\mu(k); \tag{5}$$

$$j_\mu(k) = \sum_\nu \left\{ K_{\mu\nu}(k)\mathcal{E}_\nu(k) - \int_0^\infty Q_{\mu\nu}(k;k')\mathcal{E}_\nu(k')\,dk' \right\};$$

$$\mathcal{E}_\mu(k) = 2\int_0^\infty E_\mu(z)\cos kz\,dz;$$

$$j_\mu(k) = 2\int_0^\infty j_\mu(z)\cos kz\,dz,$$

$$K_{\mu\nu}(k) = -\frac{2e^2}{\pi h^3}\int_0^\infty \frac{\partial f_0}{\partial\varepsilon}\,d\varepsilon \int_{p_H^{\min}}^{p_H^{\max}} \frac{m}{\Omega}\,dp_H \int_0^{2\pi} v_\mu(\tau)\,d\tau\times$$

$$\times\int_{-\infty}^{\tau} v_\nu(\tau_1)\exp\left(\int_\tau^{\tau_1}\gamma\,d\tau_2\right)\cos\left(\frac{k}{\Omega}\int_\tau^{\tau_1} v_z\,d\tau_2\right)d\tau_1,$$

$$Q_{\mu\nu}(k;k') = -\frac{2e^2}{\pi h^3}\int_0^\infty \frac{\partial f_0}{\partial\varepsilon}d\varepsilon \int \frac{m}{\Omega^2}\,dp_H \int_0^{2\pi} v_\mu\,d\tau\times$$

$$\times\int_{-\infty}^{\tau} |v_z(\tau_1)|\cos\left(\frac{k}{\Omega}\int_\tau^{\tau_1} v_z\,d\tau_2\right)d\tau_1\times$$

$$\times\int_{\varphi(\tau_1)}^{\tau_1} v_\nu(\xi)\exp\left(\int_\tau^{\xi} v_z\,d\tau_2\right)\cos\left(\frac{k'}{\Omega}\int_\xi^{\tau_1} v_z\,d\tau_2\right)d\xi,$$

$$\gamma = i\omega/\Omega + 1/\Omega t_0. \tag{6}$$

Here, the function $\varphi(\tau)$ stands for the root of the equation

$$\int_{\varphi(\tau)}^{\tau} v_z\,d\tau_2 = 0$$

immediately preceding τ. If there is no such root, then $\varphi(\tau) = -\infty$. The quantity (kv/Ω) which appears in the argument of the co- sines in Eq. (6) is of the order of magnitude of $(r/\delta_{\text{eff}}) \gg 1$. Therefore, the integrals in (6) can be evaluated by the saddle-point method. The elementary but very lengthy calculation leads to the following result.

To the zero approximation in $(\Omega/kv)^{1/2} \sim (\delta_{\text{eff}}/r)^{1/2} \ll 1$, the surface impedance is independent of the magnetic field and is equal to its value for $H = 0$. In the first and second approximations, the dependence on H does not have a resonant behavior. A weak resonant dependence appears only due to higher terms in the expansion of the impedance in powers of $(\delta_{\text{eff}}/r)^{1/2}$. It turns out also that only the Z_{yy} component of the surface impedance tensor shows a resonance. Therefore, the resonant absorption of power can occur only for electromagnetic waves incident on the metal with a definite polarization (see [3]). The electric vector of the incident wave must be parallel to the y-axis (perpendicular to both $\mathbf{H}$ and the normal to the metal surface). The relative magnitude of the resonant addition is

$$\frac{\Delta Z_{\text{res}}}{Z(0)} \sim A\left(\frac{\delta_{\text{eff}}}{r}\right)^{3/2}\ln\left[\frac{1}{1-\exp(-2\pi i\omega/\Omega_0 - 2\pi\nu_0/\Omega_0)}\right]. \tag{7}$$

That is, at the resonance (at $\omega \simeq q\Omega_0$, where q is an integer), this contribution has only a logarithmic singularity with respect to $(\nu_0/\omega) \ll 1$. Here, Ω_0, r, and $\nu_0 = \overline{(1/t_0)}$ are, respectively, the Larmor frequency, the orbit radius, and the collision rate for electrons with $\bar{v}_H = 0$ averaged along the orbit, A is a complex-valued constant independent of H,

$$\delta_{\text{eff}} = |c^2 Z(0)/4\pi\omega| \sim (\delta^2 l)^{1/3}; \quad \delta = (mc^2/2\pi ne^2)^{1/2},$$

and l is the mean free path.

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The resonant effect is small because it is produced only by a small number of electrons which return many times into the skin depth. Since the resonant contribution is small, we do not present the exact formulae for it.

The result obtained is correct if the angle Φ between the magnetic field and the metal surface is not close to zero or $\pi/2$, i.e., if $\sin 2\Phi \gg \delta_{\rm eff}/r$.

The case of a perpendicular magnetic field ($\cos\Phi \ll \ll \delta_{\rm eff}/r$) is special, because, in this case, the electrons which give the main contribution to the current density have orbits lying entirely within the skin layer. We have then the diamagnetic resonance ($\omega = \Omega$) analogous to that in semiconductors, rather than the cyclotron resonance ($\omega = q\Omega$). As was shown in [5], in the zero approximation in ($\delta_{\rm eff}/l$) [in a perpendicular field, the expansion parameter is ($\delta_{\rm eff}/l$) instead of ($\delta_{\rm eff}/r$)], the effective mean free path $l^* = l/[1+i(\omega-\omega)t_0]$ disappears at all from the formula for the surface impedance which is independent of H in this approximation. A resonance (i.e., an extremum with respect to H) appears only due to the second-approximation term proportional to $(\delta_{\rm eff}/l^*)^2 \ln(l^*/\delta_{\rm eff})$.

3. Cyclotron Resonance in a Parallel Magnetic Field on the Arbitrary Reflection of Electrons from the Metal Surface

To obtain the dependence of the surface impedance on the reflection coefficient of electrons at the metal surface, we have to change the boundary condition (4). We suppose that a fraction ρ of the electrons is reflected specularly (i.e., without any change in their distribution function), while the rest are distributed after the reflection by the equilibrium function $f_0(\varepsilon)$. It is clear that

$$f(0; v_x, v_y, v_z) = \rho f(0; v_x, v_y, -v_z) + (1-\rho) f_0(\varepsilon)$$

for $v_z > 0$. For the addition $f_1 = f - f_0(\varepsilon)$, the last condition becomes

$$f_1(0; v_x, v_y, v_z) = \rho f_1(0; v_x, v_y, -v_z), \quad v_z > 0. \tag{8}$$

As was shown in [2], the Fourier transform of the current density can be determined if we know the relation between $\partial\psi_-(0;\mathbf{v})/\partial z$ and $\psi_-(0;\mathbf{v})$, where $\psi_\pm(z;\mathbf{v}) = f_1(z;\mathbf{v}) \pm f_1(z;-\mathbf{v})$. Equation (3) yields

$$\partial\psi_-(z;\mathbf{v})/\partial z = -\{i\omega + 1/t_0(\mathbf{p}) + \Omega\partial/\partial\tau\}\psi_+(z;\mathbf{v})/v_z \tag{9}$$

(in this case, we considered the fact that $\varepsilon(\mathbf{p})$ is an even function). Therefore, it is sufficient only to find the relation between $\psi_+(0;\mathbf{v})$ and $\psi_-(0;\mathbf{v})$. By using the boundary condition (8), we get

$$\psi_+(0;\mathbf{w},v_z) = \rho\psi_+(0;\mathbf{w},-v_z) +$$
$$+ \mathrm{sgn} v_z\{\rho\psi_-(0;\mathbf{w};-v_z) - \psi_A(0;\mathbf{w},v_z),\} \tag{10}$$

where $\mathbf{w}$ is a two-dimensional vector with components (v_x, and v_y), and sgn$x$ denotes the sign of x. Replacing v_z by $-v_z$ in Eq. (10) and eliminating $\psi_+(0;\mathbf{w},-v_z)$, we obtain

$$\psi_+(0;v_x,v_y,v_z) = -\frac{\mathrm{sgn} v_z}{1-\rho^2}\{(1+\rho^2)\psi_-(0;v_x,v_y,v_z) -$$
$$-2\rho\psi_-(0;v_x,v_y,-v_z)\}. \tag{11}$$

As $\rho \to 0$, we get the boundary condition used in [2]. When $\rho \to 1$, we find

$$\psi_-(0;\mathbf{w};v_z) = \psi_-(0;\mathbf{w};-v_z).$$

The Fourier transform of the current density can be determined with the help of Eq. (11). For simplicity, we write down the formula for $\mathbf{j}(k)$ in the case of the isotropic quadratic dispersion law $\varepsilon(\mathbf{p}) = p^2/2m$, where m is the effective mass, by assuming the relaxation time t_0 independent of $\mathbf{p}$ (the residual resistance), and taking the variable and constant magnetic fields to be perpendicular to each other:

$$j_\mu(k) = K(k)\mathcal{E}_\mu(k) - \int_0^\infty Q(k;k')\mathcal{E}_\mu(k')\,dk';$$

$$K(k) = \frac{3\sigma\gamma}{\pi}\int_0^{\pi/2}\cos^2\vartheta\sin\vartheta\,d\vartheta\int_0^\infty e^{-\gamma x}\,dx \times$$
$$\times\int_0^\pi \cos[kr\sin\vartheta(\cos\tau - \cos(\tau - x))]\,d\tau;$$

$$Q(k;k') = \frac{3\sigma\gamma r}{\pi^2}\int_0^{\pi/2}\cos^2\vartheta\sin^2\vartheta\,d\vartheta\int_0^\infty e^{-\gamma x}\,dx I(\vartheta;x);$$

$$I(\vartheta;x) = \int_0^\pi d\lambda\sin\lambda\big\{\cos[kr\sin\vartheta(\cos\lambda - \cos(\lambda + x))] -$$
$$-\rho\cos[kr\sin\vartheta(\cos\lambda - \cos(\lambda - x))]\big\}/(e^{\gamma\lambda} - \rho e^{-\gamma\lambda}) \times$$
$$\times\int_0^\lambda \cosh\gamma\eta\cos[k'r\sin\vartheta(\cos\lambda - \cos\eta)]\,d\eta;$$

$$\sigma = \frac{ne^2 l}{mv};\ \gamma = \frac{r}{l};\ r = \frac{mvc}{eH};\ l = \frac{vt_0}{1+i\omega t_0};\ v = \left(\frac{2\zeta}{m}\right)^{1/2};$$

$$\mathbf{v} = v(\cos\vartheta, \sin\vartheta\cos\tau, \sin\vartheta\sin\tau);\quad n = 8\pi p^3/3h^3. \quad (12)$$

Under conditions of the anomalous skin effect, $kr \sim (r/\delta_{\rm eff}) \gg 1$. By using the saddle-point method, we get the asymptotic formulas

$$K(k) = \frac{3\pi\sigma}{4l}\frac{1+\exp(-2\pi\gamma)}{1-\exp(-2\pi\gamma)}\frac{1}{k},$$

$$Q(k;k') = \frac{3\sigma}{4l}\frac{\exp(-2\pi\gamma)}{1-\exp(-2\pi\gamma)}\frac{1-\rho}{1-\rho\exp(-2\pi\gamma)}\frac{1}{\sqrt{kk'}}\times$$

$$\times\left[\pi\delta(k-k') + \frac{\cosh^2\pi\gamma}{k+k'}\right] +$$

$$+\frac{3(1+\rho)\sigma}{4\pi l}\left\{\frac{\cosh\pi\gamma\exp(-\pi\gamma)}{1-\rho\exp(-2\pi\gamma)} +\right.$$

$$\left.+\left[1-\rho+\frac{\sqrt{2}\gamma}{\pi\mathrm{B}(3/4,3/2)}\frac{\ln(k/k')}{\sqrt{kr}-\sqrt{k'r}}\right]^{-1}\right\}\times$$

$$\times\frac{\ln(k/k')}{k^2-k'^2}, \quad (13)$$

where $\mathrm{B}(p,q)$ is the Eulerian integral of the first kind. The last relation in (13) can be regarded as an interpolation formula, since it gives the correct result both for

$1-\rho \gg |\gamma\sqrt{\delta_{\rm eff}/r}|$ and for $1-\rho \ll |\gamma\sqrt{\delta_{\rm eff}/r}|$.

The study shows that, for all values of $\rho \neq 1$ [more precisely, for $1-\rho \gg |\gamma|(\delta_{\rm eff}/r)^{1/2}$], the surface impedance near the resonance ($\omega \simeq q\Omega$) and in the strong magnetic fields ($|\gamma| \ll 1$) is independent of the reflection coefficient ρ. In this case, the formulae for Z obtained earlier [1–3] remain valid.

The case of specular reflection is particular, because, in this case, the main contribution to the current density comes from electrons which collide repeatedly with the metal surface. It turns out that, in the zero approximation in $(\delta_{\rm eff}/r)^{1/2}$, the impedance manifests a non-resonant dependence on the magnetic field, and the resonance appears only in the next approximation $(\delta_{\rm eff}/r)$.

In strong fields ($|\gamma| \ll 1$), the dependence of the surface impedance on the magnetic field is quite different for $\rho = 1$ and for $\rho \neq 1$:

$$Z(H) \sim Z(0)\gamma^{1/5}(\delta/l)^{2/15} \sim H^{-1/5} \quad (\rho = 1);$$

$$Z(H) \sim Z(0)\gamma^{1/3} \sim H^{-1/3} \quad (1-\rho \gg |\gamma|(\delta_{\rm eff}/r)^{1/2}). \quad (14)$$

The exact formulae for the impedance with $\rho = 1$ will not be given here, since this case is of purely theoretical interest.

In conclusion, I take the opportunity to thank I.M. Lifshitz and M.Ya. Azbel for the discussion of the results of this work.

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The published article by E.A. Kaner belongs to a cycle of his works dedicated to the development of the theory of cyclotron resonance discovered by him along with M.Ya. Azbel in 1956 [1]. The discovery of cyclotron resonance is of general physical significance. It was included in the State Register of the USSR (now FSU).

E.A. Kaner is the first who discovered the effect of anomalous penetration of radio waves into metals (together with M.Ya. Azbel and V.F.Gantmakher [2]). He also published a great number of pioneer works relevant to other areas of theoretical physics, solid-state physics, and plasma physics. The total number of his works is more than 180. The major works are reprinted in the collection of his works [3]. In 1980, the state award of Ukraine was conferred upon him for the cycle of works on the magneto-acoustic spectroscopy of metals.

E.A. Kaner was a distinguished researcher who was efficient in combining the scientific and research activities with a good deal of organization talent. He was the founder of the solid-state theory department that is currently functioning as an independent entity of the IRE of the NASU. E.A. Kaner was the co-editor of the international journal “Solid State Communications” and a member of the editorial boards of the journals “Low Temperature Physics” and “Ukrainian Journal of Physics”. In addition, he was a member of numerous Scientific Boards of the Academy of Sciences of the USSR and the Academy of Sciences of Ukrainian SSR.

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