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# ON THE INTERACTION OF A CHARGED PARTICLE BEAM WITH ELECTRON PLASMA

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If the infinite plasma is passed by a nonmodulated parallel beam of electrons, whose initial velocity exceeds the average thermal velocity of plasma electrons, then longitudinal electric waves are excited in plasma with exponentially increasing amplitude. In this case, the state of the beam becomes unstable, and the fluctuations of density and velocity that exist in the beam also propagate in the form of longitudinal waves with increasing amplitude. In other words, the area with increased or decreased density naturally arise in the beam and propagate in the form of waves with increasing amplitude.

These assertions are correct if the role of the collisions of plasma electrons with positive ions and with beam electrons is not taken into account and the deviations of different quantities from their equilibrium values are considered small.

A linearized system of equations describing the interaction of plasma with a beam of electrons without regard for collisions of particles takes the form [1, 2]

$$\frac{df}{dt} + u\frac{df}{dx} + \frac{e}{m}E\frac{df_0}{du} = 0, \tag{1}$$

$$\frac{\partial E}{\partial x} = 4\pi e\int_{-\infty}^{\infty} f du + 4\pi\rho, \tag{2}$$

$$\frac{\partial v}{\partial t} + v_0\frac{\partial v}{\partial x} = \frac{e}{m}E, \tag{3}$$

$$\frac{\partial \rho}{\partial t} + \rho_0\frac{\partial v}{\partial x} + v_0\frac{\partial \rho}{\partial x} = 0, \tag{4}$$

where $f$ is the deviation of the distribution function of plasma electrons from the equilibrium function $f_0$; $u$ is the projection of the velocity of plasma electrons on the $x$-axis, in which direction the beam electrons move; $E$ is the electric field; $\rho$ and $v$ are,respectively, deviations of the beam charge density and the velocity of beam particles from the equilibrium values $\rho_0$ and $v_0$; $e$ and $m$ have usual meaning. The deviations of all quantities from their equilibrium values are assumed to be small in comparison with the equilibrium values.

A solution of the system of equations (1)–(4) will be sought in the form of plane waves like $\mathrm{const}\cdot e^{i(\omega t - kx)}$. If $|ka| \ll 1$, where $a = \left(\theta/4\pi n_0 e^2\right)^{1/2}$ ($\theta$ is the absolute temperature, and $n_0$ is the plasma electron density), then the relation between $\omega$ and $k$ (the so-called dispersion equation) reads

$$\left(\omega^2 - v_T^2 k^2\right)\left[1 - \frac{\Omega^2}{(\omega - v_0 k)^2}\right] = \omega_0^2, \tag{5}$$

where $\omega_0^2 = 4\pi e^2 n_0/m$, $\Omega^2 = 4\pi e\rho_0/m$, $v_T = (3\theta/m)^{1/2}$.

If the beam velocity $v_0$ exceeds the average thermal velocity of plasma electrons $v_T$, then relation (5) considered as an equation for $k$ has complex roots at a given frequency $\omega$. This means that both the field $E$ and the deviation $\rho$ of the beam density from the equilibrium value look like waves, whose amplitudes increase exponentially with $x$. This implies that, as the beam is passing through plasma, the generation and amplification of high frequency oscillations in plasma are possible.

The maximum value of the modulus of the imaginary part of $k$, being a function of the frequency $\omega$, is attained at $\omega = \omega_0/\sqrt{1 - (v_T/v_0)^2}$ and is equal to

$$\Gamma_{\max} = \frac{3^{1/2}}{2^{4/3}}\frac{\omega_0}{v_T}\left(\frac{v_T}{v_0}\right)^{2/3}\left(\frac{v_0^2}{v_T^2} - 1\right)^{1/6}\left(\frac{\Omega}{\omega_0}\right)^{2/3}. \tag{6}$$

This quantity has, in turn, a maximum at $v_0 = \sqrt{2}v_T$, which corresponds to the maximally amplified frequency equal to $\sqrt{2}\omega_0$.

Let us dwell on the problem of the generation of microradiowaves with the help of electron plasma. Usually, the conclusion is drawn [3] about the impossibility to generate ultrahigh frequencies with the help of plasma in view of the facts that the period of plasma oscillations $T \sim n_0^{1/2}$ ($n_0$ is the plasma electron density) and the time between two collisions $\tau \sim n_0^{-1}$. Therefore, with increase in $n_0$ which is necessary for the generation of microradiowaves, the effect of the electrons – positive ions collisions withdrawing electrons from the oscillation process strongly increases.

We would like to emphasize at this point that these considerations are not applicable, strictly speaking, to the case where plasma oscillations are excited by a beam of charged electrons, since the generated frequency is determined, in this case, by both the plasma density and the ratio of velocities $v_0$ and $v_T$.

In conclusion, we note that, on the passage of a nonmodulated beam of charged particles through a waveguide filled with a dielectric or through a chain of coupled endovibrators, the increasing waves of the electric field strength and the beam charge density also arise under certain conditions and have the same character as that in the case of plasma. In all these cases, the dispersion equation similar to relation (5) is valid. In the case of a waveguide, the role of the thermal speed $v_T$ is played by the phase velocity of propagation of electromagnetic waves in an infinite dielectric, and the condition for the beam instability and the creation of charge "bunches" propagating in the beam coincides with the condition for the Cherenkov radiation arising on the movement of a single charge with velocity $v_0$ in a dielectric.

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Alexander Il'ich Akhiezer was the outstanding physicist-theorist, Full Member of the National Academy of Sciences (NAS) of Ukraine (1964), winner of the State Prize of Ukraine (1986, 2002), winner of Mandel'shtam prize of the USSR Academy of Sciences (1950) and Pomeranchuk prize (1998), winner of Sinel'nikov (1978), Bogolyubov (1995), and Davydov prizes of the NAS of Ukraine. A.I. Akhiezer worked at Ukrainian Physical-Technical Institute (UPhTI) (now National Science Center "Kharkov Institute of Physics and Technology" of NAS of Ukraine) from 1934 till 2000; from 1938 till 1988, he was Head of the Department of Theoretical Physics (after L.D. Landau), A.I. Akhiezer guided a chair at Kharkov State University from 1940 till 1975. He is one of the founders and leaders of the Kharkov school of theoretical physics. A.I. Akhiezer developed the scattering theory of light by light and by a nuclear field at high energies, he carried out the pioneering researches in the theory of neutron scattering by crystals, in the theory of resonance nuclear reactions and the theory of neutron slowing-down (with I.Ya. Pomeranchuk). A.I. Akhiezer predicted the effect of electron-beam instability in plasma (with Ya.B. Fainberg, 1948). A.I. Akhiezer with co-workers developed the theory of linear accelerators of charged particles, the theory of radiative corrections to a number of electromagnetic processes. He constructed the theory of collective processes in plasma (with K.N. Stepanov *et al.*) and the kinetic theory of fluctuations in plasma (with A.G. Sitenko *et al.*). A.I. Akhiezer proposed the method for determining the electromagnetic form factors of protons in the scattering of polarized electrons by protons (with M.P. Rekalo, 1968). Together with A.G. Sitenko, he predicted the effect of diffraction splitting of a deuteron. He constructed the theory of bremsstrahlung on the scattering of relativistic electrons and positrons in crystals (with V.F. Boldyshev and N.F. Shul'ga). A.I. Akhiezer is a co-author of the discovery of the magnetoacoustic resonance (with V.G. Bar'yakhtar, S.V. Peletminsky, 1956).

**FAINBERG YAKOV BORISOVICH**
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Yakov Borisovich Fainberg was the outstanding physicist, Full Member of the National Academy of Sciences of Ukraine (1979), winner of the State prize of Ukraine (1996). He was born in Zolotonosha (now the Cherkassy region, Ukraine). Ya.B. Fainberg worked at the Kharkov Institute of Physics and Technology (now NSC "KIPT" of NAS of Ukraine) from 1946 till 2005. During 1972–1989, he was Head of a department at KIPT. Ya.B. Fainberg was Professor at Kharkov State University (till 1972). His name is inseparable from the development of the physics and technology of accelerators of charged particles, plasma physics, nonrelativistic and relativistic plasma electronics, and controlled thermonuclear fusion. Ya.B. Fainberg together with A.I. Akhiezer theoretically predicted (1948) the plasma-beam instability and, together with co-workers in experimental studies, discovered and investigated this instability in the magnetic field and without it. He carried out the first theoretical studies on the dynamics of charged particles accelerated in a traveling wave field (1947) and proposed and developed the alternating phase focusing method (1953). Ya.B. Fainberg discovered a new type of gas discharges, the so-called plasma-beam discharge, and proposed and studied, with co-workers, a new method of the collisionless plasma heating, i.e., the beam heating. He was a founder of new collective acceleration methods (together with V.I. Veksler and G.I. Budker), by proposing the most promising one (1956). Ya.B. Fainberg is a co-author of the discovery (with E.K. Zavoisky *et al.*) of "the phenomena, previously unknown, of the anomalous increase of resistance and the turbulent heating of plasma". Ya.B. Fainberg is a founder of the school of plasma electronics and new acceleration methods.