90 YEARS

QUANTUM LINEAR GEOMETRY AND PARALLEL TRANSFER

Translated and reprinted from Comp. Rend. Acad. **188**, pp. 1470–1472 (1929)

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1. The latest development of quantum theory leads to the conclusion that the Riemann geometry with its fundamental quadratic form ds^2 remains suitable to explain the gravitation phenomena, while the quantum and electric phenomena demand introducing the geometric notions both new and strange for the Riemann geometry. Such notions were first introduced by Dirac (though implicitly) in the theory of electrons. The geometric nature of Dirac operators α_k was noted by us in [1], where we suggested to introduce the operators analogous to the Dirac matrices into geometry, by considering a linear differential form

$$ds = \sum \gamma_\nu dx_\nu, \tag{1}$$

the square of which produces an ordinary Riemann interval ds^2. This version of geometry was called a quantum linear geometry.

2. To continue, it is useful to introduce (according to Ricci and Levi-Civita) an orthogonal n-hedron of directions determined at each point of the space. By means of such an n-hedron, we can define some geometric quantity, whose components are transformed as the Dirac function Ψ on arbitrary rotations of the n-hedron. This quantity will be called a semivector (Landau).

The geometry of a quadratic form ds^2 can be studied on the basis of the infinitesimal parallel transfer of a vector, according to Levi-Civita. Similarly, the notion of infinitesimal transfer of a semivector can serve as a starting point while studying the quantum linear geometry. We now write down an increment of components of some semivector,

$$\delta\Psi = \sum_l C_l ds_l \Psi, \tag{2}$$

where C_l are operators (matrices) acting on the components of Ψ, and ds_l are the transfer components along the n-hedron directions. The equation for the quantity Ψ^+ conjugated to Ψ reads

$$\delta\Psi^+ = \Psi^+ \sum_l C_l^+ ds_l. \tag{2*}$$

We introduce the Dirac matrices α_k satisfying the relations

$$\alpha_i\alpha_k + \alpha_k\alpha_i = 2\delta_{ik},$$

and form the vector $A_i = \Psi^+\alpha_i\Psi$. In view of formulas (2) and (2*), we can calculate a variation of the vector A as

$$\delta A_i = \delta(\psi^+\alpha_i\psi)\psi = \psi^+ \sum_l (C_l^+\alpha_i + \alpha_i C_l) ds_l \psi. \tag{3}$$

This change should be some linear function of A_k, namely,

$$\delta A_i = \sum_{kl} \gamma_{ikl} A_k ds_l, \tag{4}$$

where γ_{ikl} are the Ricci coefficients. Comparing (3) and (4), we obtain

$$C_l^+\alpha_i + \alpha_i C_l = \sum_k \gamma_{ikl}\alpha_k. \tag{5}$$

If we multiply further (5) by α_i from the right and the left, add the results, and take, in addition, the identity $\gamma_{iil} = 0$ into account, we get

$$\alpha_l(C_l + C_l^+) + (C_l + C_l^+)\alpha_l = 0. \tag{6}$$

It is verified directly that the identities $\gamma_{ikl} + \gamma_{kil} = 0$ hold. Relation (6) shows that the operator C_l is of the form

$$C_l = g_l + i\Phi_l, \tag{7}$$

where g_l and Φ_l are the Hermitian operators (which are identical to the adjoint ones). Moreover, g_l satisfy the relations $g_l\alpha_i + \alpha_i g_l = 0$. Substituting (7) into (5), we obtain

$$\mathrm{i}(\alpha_i\Phi_l - \Phi_l\alpha_i) = \sum_k \gamma_{ikl}\alpha_k. \quad (8)$$

3. Formula (2) gives immediately the law of covariant differentiation of semivectors, namely,

$$\nabla_l\psi = \left(\frac{\partial}{\partial s_l} - C_l\right)\psi. \quad (9)$$

We set $g_l = 0$ and $\Phi_l = \frac{2\pi e}{hc}\varphi_l$ in (7), where φ_l are the components of the vector-potential, and introduce the coordinates x_α. Then formula (9) yields

$$\nabla_\alpha\psi = \left(\frac{\partial}{\partial x_\alpha} - \frac{2\pi\mathrm{i}}{h}\frac{e}{c}\varphi_\alpha\right)\psi. \quad (10)$$

This is exactly the expression which is present in the Dirac equation. By introducing φ_α into formula (2), we obtain

$$\delta\psi = \frac{2\pi\mathrm{i}e}{hc}\sum_\alpha \varphi_\alpha dx_\alpha\psi. \quad (11)$$

Thus, it is the transfer law for a semivector represented by a Weyl linear differential form.

4. A complete theory must give equations for C_l and γ_{ikl} which are analogous to the Maxwell and Einstein equations.

One point which differs the ideas presented here from those by Einstein and Levi-Civita should be noted: this is the introduction of matrix operators into the equations for purely geometric quantities. Due to the above, one can imagine an electromagnetic field in the Euclidean space quite well, which is impossible in other theories.

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Presented May 22, 1929

FOCK VLADIMIR ALEKSANDROVICH (22.12.1898–27.12.1974)

Vladimir Aleksandrovich Fock (or Fok) was a Soviet physicist, who did foundational work on quantum mechanics. He was born in St. Petersburg, Russia. In 1922 he graduated from Petrograd University, where he continued postgraduate studies. There he became a professor in 1932. In 1919–1923 and 1928–1941 he collaborated with the State Institute of Optics, in 1924–1936 with the Leningrad Institute of Physics and Technology, in 1934–1941 and 1944–1953 with the Lebedev Physical Institute. His primary scientific contribution lies in the development of quantum physics, although he also contributed significantly to the fields of mechanics, theoretical optics, theory of gravitation, physics of continuous medium. In 1926 he generalized the Klein-Gordon equation. He gave his name to Fock space, the Fock representation and Fock state, and developed the Hartree-Fock method in 1930. He made many other scientific contributions during the rest of his life.

IWANENKO DMITRII DMITRIEVICH (20.07.1904–30.12.1994)

D.D. Iwanenko was a prominent physicist of the XX century. He was born at the Machukhy village near Poltava. He received education at Poltava Pedagogical Institute, Kharkiv University, Petrograd (Leningrad) University (1920–1927). In 1927–1929, he worked as a researcher at the Physico-Mathematical Institute of AS USSR, participated in the organization of the theoretical department of Ukrainian Physico-Technical Institute and was the first head of it, and guided the chair in the Kharkiv Mechanical Engineering Institute (1929–1931). In 1931, he returned to the Leningrad Physico-Technical Institute. In 1935, he was convicted because of political reasons for 3 years; in one year, he was dismissed and sent to Tomsk due to the intercession of S.I. Vavilov, Ya.S. Frenkel and A.F. Ioffe (he was rehabilitated only in 1989). In 1937, D.D. Iwanenko was a research worker of the Siberian Physico-Technical Institute and Professor of the Tomsk University. 1940–1941 – Professor, Head of the chair of theoretical physics at the Kyiv University. In 1943–1994, he worked as Professor of the Moscow State University; 1945: the head of a special group for the estimation of a state of the nuclear program in Germany.

D.D. Iwanenko was a co-author (together with L.D. Landau) [1] of the theory of spinors as skew-symmetric tensors and the theory of spinor fields in the Riemann space (together with V.A. Fock) [2], advanced the idea of discrete space (together with V.A. Ambarzumian [3]; co-author (together with V.A. Ambarzumian) [4] of the hypothesis about the birth and the vanishing of massive particles, which makes the basis of the whole modern quantum theory of fields and elementary particles. In 1932-1933, the hypothesis was confirmed by P. Blackett and G. Occhialini. He was the creator of the proton-neutron model of nuclei [5]. Together with I.Ya. Pomeranchuk, he prognosticated the synchrotron radiation [6]; he also began the first (in the country) biophysical studies with the use of radioactive isotopes. The monograph “Classical Field Theory” [7] by D.D. Iwanenko and A.A. Sokolov became very popular.

D.D. Iwanenko was one of the founders of the International Gravitation Committee, Soviet Gravitation Committee, and Conferences on Gravitation in the Soviet Union, as well as journals “Physikalische Zeitschrift der Sowjetunion” and “General Relativity and Gravitation”.

Such famous scientists as P.A.M. Dirac, H. Yukawa, N. Bohr, J. Wheeler, I. Prigogine, S.C.C. Ting, and M. Gell-Mann retained their expressions on the blackboard in his room in the Moscow State University.

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7. D.D. Iwanenko, A.A. Sokolov, *Classical Field Theory* (Gostekhizdat, Moscow-Leningrad, 1949) (in Russian).