ENVELOPES OF MULTIBEAM INTERFERENCE SPECTRA IN PLANE-PARALLEL STRUCTURES: SUBSTANTIATION AND BASIC REGULARITIES

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The matrix method of consecutive summation of the multibeam interference in single-layered structures has been applied to derive analytical expressions for interference spectrum envelopes in a multilayer system. The expressions obtained are valid for arbitrary incidence angles, wave polarizations, and absorption degrees. The main regularities of those envelopes have been discussed.

1. Introduction

The method of multibeam interference spectrum envelopes is an important supplement to the widely known spectroscopy method for nondestructive monitoring of film structure parameters [1–5], including their optical inhomogeneity as well [6–10]. Exact analytical expressions for envelopes have been substantiated only in the case of single-film structures [11–14], whereas, concerning structures made up of a larger number of layers, an opinion was stated that the corresponding analytical expressions cannot be obtained even for a system of two transparent films [15].

In this work, the matrix method of consecutive summation of the multibeam interference in single films has been used to substantiate the principle for deriving analytical expressions describing the spectrum envelope of a plane electromagnetic wave with either s- or p-polarization at its normal or inclined transmission through the interfaces in a system composed of a number of transparent and absorbing films.

2. Main Results, Their Analysis, and Conclusions

Consider a system of k isotropic layers, where the j-th layer ($j = 1 \ldots k$) is characterized by the complex refractive index $\tilde{n}_j = n_j - i\chi_j$, the thickness d_j, and the phase thickness $\tilde{\delta}_j = \frac{4\pi d_j}{\lambda}\tilde{n}_j \cos\tilde{\beta}_j = \mathrm{Re}\tilde{\delta}_j + i\mathrm{Im}\tilde{\delta}_j$; here, $\tilde{\beta}_j$ is the angle of light refraction in the j-th layer which satisfies the refraction law $\tilde{n}_{j-1}\sin\tilde{\beta}_{j-1} = \tilde{n}_j \sin\tilde{\beta}_j$. The medium above the film system is semiinfinite with the index of refraction $\tilde{n}_0$, and, below the system, there is a semiinfinite substrate characterized by the refractive index $\tilde{n}_{k+1}$.

For an interface between two films, the complex amplitude reflection, $\tilde{r} = \sigma\exp(-i\phi)$, and transmission, $\tilde{t} = \tau\exp(-i\varphi)$, coefficients for both p- and s-polarizations are calculated by the Fresnel formulas

$$\tilde{r}^s_{j-1,j} = -\frac{\sin\left(\tilde{\beta}_{j-1} - \tilde{\beta}_j\right)}{\sin\left(\tilde{\beta}_{j-1} + \tilde{\beta}_j\right)}, \quad \tilde{r}^p_{j-1,j} = \frac{\mathrm{tg}\left(\tilde{\beta}_{j-1} - \tilde{\beta}_j\right)}{\mathrm{tg}\left(\tilde{\beta}_{j-1} + \tilde{\beta}_j\right)},$$

$$\tilde{t}^p_{j-1,j} = \frac{2\cos\tilde{\beta}_{j-1}\sin\tilde{\beta}_j}{\sin\left(\tilde{\beta}_{j-1} + \tilde{\beta}_j\right)\cos\left(\tilde{\beta}_{j-1} - \tilde{\beta}_j\right)},$$

$$\tilde{t}^s_{j-1,j} = \frac{2\cos\tilde{\beta}_{j-1}\sin\tilde{\beta}_j}{\sin\left(\tilde{\beta}_{j-1} + \tilde{\beta}_j\right)}.$$

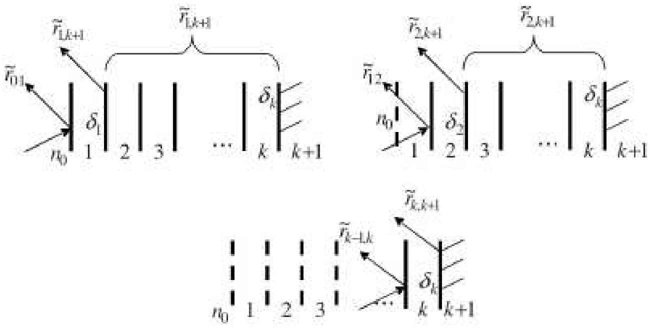

Fig. 1. Illustration of the step-by-step method for the calculation of amplitude coefficients

Taking the multibeam interference in the layered structure into account, as well as the following formula for matrix product [16],

$$\begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix} = \begin{bmatrix} 1 & \tilde{r}_{01} \\ \tilde{r}_{01} & 1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} 1 & \tilde{r}_{12} \\ \tilde{r}_{12} \exp\left(-i\tilde{\delta}_1\right) & \exp\left(-i\tilde{\delta}_1\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} 1 & \tilde{r}_{23} \\ \tilde{r}_{23} \exp\left(-i\tilde{\delta}_2\right) & \exp\left(-i\tilde{\delta}_2\right) \end{bmatrix} \times \cdots$$

$$\times \begin{bmatrix} 1 & \tilde{r}_{k,k+1} \\ \tilde{r}_{k,k+1} \exp\left(-i\tilde{\delta}_k\right) & \exp\left(-i\tilde{\delta}_k\right) \end{bmatrix}, \qquad (1)$$

we calculate the amplitude reflection coefficient for a system composed of k films as

$$\tilde{r}_1 = \frac{\tilde{M}_{21}}{\tilde{M}_{11}}.$$

For the j-th film, the amplitude coefficients equal

$$\tilde{r}_j = \frac{\tilde{r}_{j-1,j} + \tilde{r}_{j+1}\Omega_j \exp(-i\delta_j)}{1 + \tilde{r}_{j-1,j}\tilde{r}_{j+1}\Omega_j \exp(-i\delta_j)} \qquad (2)$$

and

$$\tilde{t}_j = \frac{\tilde{t}_{j-1,j}\tilde{t}_{j+1}\sqrt{\Omega_j}\exp\left(-i\frac{\delta_j}{2}\right)}{1 + \tilde{r}_{j-1,j}\tilde{r}_{j+1}\Omega_j \exp(-i\delta_j)}, \qquad (3)$$

where $\Omega_j = \exp\left(\mathrm{Im}\tilde{\delta}_j\right)$, and $\delta_j = \mathrm{Re}\tilde{\delta}_j$. Therefore, by introducing a matrix system that is similar to expression (1),

$$\begin{bmatrix} \tilde{M}_{j,11} & \tilde{M}_{j,12} \\ \tilde{M}_{j,21} & \tilde{M}_{j,22} \end{bmatrix} = \begin{bmatrix} 1 & \tilde{r}_{j-1,j} \\ \tilde{r}_{j-1,j} & 1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} 1 & \tilde{r}_{j,j+1} \\ \tilde{r}_{j,j+1} \exp\left(-i\tilde{\delta}_j\right) & \exp\left(-i\tilde{\delta}_j\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} 1 & \tilde{r}_{j+1,j+2} \\ \tilde{r}_{j+1,j+2} \exp\left(-i\tilde{\delta}_{j+1}\right) & \exp\left(-i\tilde{\delta}_{j+1}\right) \end{bmatrix} \times \cdots$$

$$\times \begin{bmatrix} 1 & \tilde{r}_{k,k+1} \\ \tilde{r}_{k,k+1} \exp\left(-i\tilde{\delta}_k\right) & \exp\left(-i\tilde{\delta}_k\right) \end{bmatrix},$$

the recurrent expressions (2) and (3) can also be rewritten as

$$\tilde{r}_j = \frac{\tilde{M}_{21,j}}{\tilde{M}_{11,j}} \qquad (4)$$

and

$$\tilde{t}_j = \frac{\tilde{t}_{j-1,j} \prod_{m=j}^{k} (\tilde{t}_{m,m+1} \exp(-i\frac{\tilde{\delta}_m}{2}))}{\tilde{M}_{j,11}}, \qquad (5)$$

respectively.

The single-film simulation is consistently applied to each layer, starting from the upper one and making use of formulas (2) and (3). However, in the case of the j-th layer, we use formulas ([4]) and (5), according to the algorithm schematically depicted in Fig. 1. Therefore, let us first express the energy coefficient of light reflection by the upper layer as

$$R_1 = \frac{R_{\max 1} - a_1^2 \sin^2 \Delta_1^+}{1 - a_1^2 \sin^2 \Delta_1^-} = \frac{R_{\min 1} + b_1^2 \cos^2 \Delta_1^+}{1 + b_1^2 \cos^2 \Delta_1^-}, \qquad (6)$$

where $\Delta_1^\pm = (\delta_1 \pm \phi_{01} - \phi_{1,k+1})/2$, $a_1^2 = 4\sigma_{01}\sigma_{1,k+1}\Omega_1/ /(1 + \sigma_{01}\sigma_{1,k+1}\Omega_1)^2$ and $b_1^2 = 4\sigma_{01}\sigma_{1,k+1}\Omega_1/(1- -\sigma_{01}\sigma_{1,k+1}\Omega_1)^2$. In such a representation, the functions

$$R_{\substack{\max \\ \min} 1} = \left(\frac{\sigma_{01} \pm \sigma_{1,k+1}\Omega_1}{1 \pm \sigma_{01}\sigma_{1,k+1}\Omega_1}\right)^2 \qquad (7)$$

describe the envelopes of the maxima, $R_{\max 1}$, and the minima, $R_{\min 1}$, of spectra (2). The parameters of other films and the factors of the multibeam interference in them are included in the expressions for $\sigma_{1,k+1}$ and $\phi_{1,k+1}$.

The next step consists in calculating the energy coefficient of light reflection by the second ($j = 2$) film:

$$R_2 = \frac{R_{\max 2} - a_2^2 \sin^2 \Delta_2^+}{1 - a_2^2 \sin^2 \Delta_2^-} = \frac{R_{\min 2} + b_2^2 \cos^2 \Delta_2^+}{1 + b_2^2 \cos^2 \Delta_2^-}, \quad (8)$$

where $\Delta_2^\pm = (\delta_2 \pm \phi_{12} - \phi_{2,k+1})/2$, $a_2^2 = 4\sigma_{12}\sigma_{2,k+1}\Omega_2/(1 + \sigma_{12}\sigma_{2,k+1}\Omega_2)^2$ and $b_2^2 = 4\sigma_{12}\sigma_{2,k+1}\Omega_2/(1 - \sigma_{12}\sigma_{2,k+1}\Omega_2)^2$. Now, the envelopes of spectra (8) are described by the functions

$$R_{\substack{\max\\ \min} 2} = \left(\frac{\sigma_{12} \pm \sigma_{2,k+1}\Omega_2}{1 \pm \sigma_{12}\sigma_{2,k+1}\Omega_2}\right)^2. \quad (9)$$

Hence, continuing this algorithm, we obtain the following expression for the energy reflection coefficient for the j-th layer in the single-film approximation:

$$R_j = \frac{R_{\max j} - a_j^2 \sin^2 \Delta_j^+}{1 - a_j^2 \sin^2 \Delta_j^-} = \frac{R_{\min j} + b_j^2 \cos^2 \Delta_j^+}{1 + b_j^2 \cos^2 \Delta_j^-}, \quad (10)$$

where $\Delta_j^\pm = (\delta_j \pm \phi_{j-1,j} - \phi_{j,k+1})/2$, $a_j^2 = 4\sigma_{j-1,j} \times \sigma_{j,k+1}\Omega_j/(1 + \sigma_{j-1,j}\sigma_{j,k+1}\Omega_j)^2$ and $b_j^2 = 4\sigma_{j-1,j}\sigma_{j,k+1}\Omega_j/(1 - \sigma_j\sigma_{j,k+1}\Omega_j)^2$. Their envelopes are described by the analytical expressions

$$R_{\substack{\max\\ \min} j} = \left(\frac{\sigma_{j-1,j} \pm \sigma_{j,k+1}\Omega_j}{1 \pm \sigma_{j-1,j}\sigma_{j,k+1}\Omega_j}\right)^2 \quad (11)$$

or

$$R_{\substack{\max\\ \min} j} = \left(\frac{\sigma_{j-1,j} \pm \left|\frac{\tilde{M}_{21,j+1}}{\tilde{M}_{11,j+1}}\right|\Omega_j}{1 \pm \sigma_{j-1,j}\left|\frac{\tilde{M}_{21,j+1}}{\tilde{M}_{11,j+1}}\right|\Omega_j}\right)^2. \quad (12)$$

It should be noted that the absolute values of the amplitude reflection coefficients in Eqs. (6)–(11) are connected with the energy coefficients of light reflection by the relations $R_j = \sigma_{j-1,k+1}^2$.

Following the same procedure, we calculate the energy transmission coefficients:

$$T_1 = \frac{T_{\max 1}}{1 - a_1^2 \sin^2 \Delta_{1,k+1}^-} = \frac{T_{\min 1}}{1 + b_1^2 \cos^2 \Delta_{1,k+1}^-},$$

$$T_2 = \frac{T_{\max 2}}{1 - a_2^2 \sin^2 \Delta_{2,k+1}^-} = \frac{T_{\min 2}}{1 + b_2^2 \cos^2 \Delta_{2,k+1}^-}, \quad \ldots,$$

$$T_j = \frac{T_{\max j}}{1 - a_j^2 \sin^2 \Delta_{j,k+1}^-} = \frac{T_{\min j}}{1 - b_j^2 \cos^2 \Delta_{j,k+1}^-}, \quad (13)$$

where $T_{j-1,j} = \tilde{t}_{j-1,j}\tilde{t}_{j-1,j}^*$. The envelopes of their extrema are described by the functions

$$T_{\substack{\max\\ \min} 1} = \frac{n_{k+1}}{n_0}\frac{\cos\beta_{k+1}}{\cos\alpha}\frac{T_{01}T_{1,k+1}\Omega_1}{(1 \pm \sigma_{01}\sigma_{1,k+1}\Omega_1)^2},$$

$$T_{\substack{\max\\ \min} 2} = \frac{n_{k+1}}{n_1}\frac{\cos\beta_{k+1}}{\cos\beta_1}\frac{T_{12}T_{2,k+1}\Omega_2}{(1 \pm \sigma_{12}\sigma_{2,k+1}\Omega_2)^2}, \ldots$$

$$T_{\substack{\max\\ \min} j} = \frac{n_{k+1}}{n_{j-1}}\frac{\cos\beta_{k+1}}{\cos\beta_{j-1}}\frac{T_{j-1,j}T_{j,k+1}\Omega_j}{(1 \pm \sigma_{j-1,j}\sigma_{j,k+1}\Omega_j)^2} \quad (14)$$

or, in the different form,

$$T_{\substack{\max\\ \min} j} = \frac{n_{k+1}}{n_{j-1}}\frac{\cos\beta_{k+1}}{\cos\beta_{j-1}} \times$$

$$\times \frac{T_{j-1,j}\Omega_j \prod_{m=j}^{k} T_{m,m+1}}{\left|\tilde{M}_{11,j+1}\right|^2 \left(1 \pm \sigma_{j-1,j}\left|\frac{\tilde{M}_{21,j+1}}{\tilde{M}_{11,j+1}}\right|\Omega_j\right)^2}, \quad (15)$$

where $T_{j,k+1} = \left|\tilde{t}_{j+1}\right|^2$.

The most important conclusions are as follows.

1. The analytical expressions for envelopes (11), (12), (14), and (15) are general; they are valid for both transparent and absorbing structures at the normal and inclined incidence of light of both polarizations (s and p).

2. The envelopes of interference spectra given by a single film do not oscillate, i.e. they do not reveal extremum points. In the spectra of a multifilm system, the extremum manifestation degree is determined by the regularities of the multibeam interference in every particular layer, as it takes place in the case of the interaction between oscillatory systems with different characteristic frequencies.

In the case of a multilayered system with oscillating envelopes of their spectra, it is expedient to introduce the envelopes of the envelopes, $R_{\substack{h\\ l} \max,j}$ and $R_{\substack{h\\ l} \min,j}$, following the algorithm that is illustrated in Fig. 2. The envelopes of the maxima (subscript h) are described by the functions

$$R_{h\max,j} = \left(\frac{\sigma_{j-1,j} + \Omega_j\sqrt{R_{\max,j+1}}}{1 + \Omega_j\sigma_{j-1,j}\sqrt{R_{\max,j+1}}}\right)^2,$$

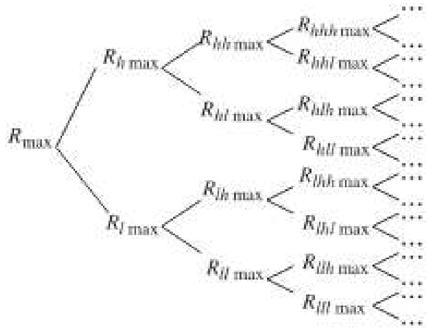

Fig. 2. Graph of a branch algorithm for the construction of the envelopes of the extremum points of oscillating envelopes

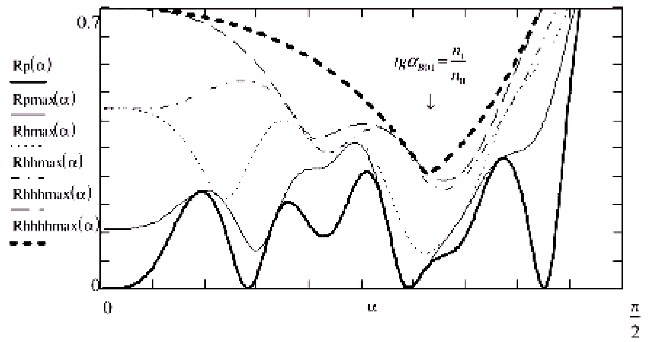

Fig. 3. Contour of the oblique light reflectance for a five-layer structure and its envelopes

$$R_{hh\,\max,j} = \left(\frac{\sigma_{j-1,j} + \Omega_j\sqrt{R_{h\,\max,j+1}}}{1+\Omega_j\sigma_{j-1,j}\sqrt{R_{h\,\max,j+1}}}\right)^2,$$

$$R_{hhh\,\max,j} = \left(\frac{\sigma_{j-1,j} + \Omega_j\sqrt{R_{hh\,\max,j+1}}}{1+\Omega_j\sigma_{j-1,j}\sqrt{R_{hh\,\max,j+1}}}\right)^2,\ldots,$$

and the envelopes of the minima (subscript l) by the functions

$$R_{l\,\max,j} = \left(\frac{\sigma_{j-1,j} + \Omega_j\sqrt{R_{\min,j+1}}}{1+\Omega_j\sigma_{j-1,j}\sqrt{R_{\min,j+1}}}\right)^2,$$

$$R_{ll\,\max,j} = \left(\frac{\sigma_{j-1,j} + \Omega_j\sqrt{R_{l\,\min,j+1}}}{1+\Omega_j\sigma_{j-1,j}\sqrt{R_{l\,\min,j+1}}}\right)^2,$$

$$R_{lll\,\max,j} = \left(\frac{\sigma_{j-1,j} + \Omega_j\sqrt{R_{ll\,\min,j+1}}}{1+\Omega_j\sigma_{j-1,j}\sqrt{R_{ll\,\min,j+1}}}\right)^2,\ldots.$$

The analytical expressions for the oscillating envelopes of transmittance spectra are substantiated analogously.

3. As one can see from Fig. 3, the contour of the light reflection by a multifilm structure at the inclined incidence does not always enable one to determine unambiguously the value of the Brewster angle for a single interface; first of all, for the interface between the external medium and the upper layer. The method, where the correlation between the minimum of the inclined incidence contour for plane-parallel structures and the Brewster angle α_B for a single interface is sought for, should be considered with some caution. For instance, while the reflectance contour minimum was directly associated in work [17] with the α_B-value, the authors of work [18] connect it with an angle, at which the reflectance contours of the structure and the pure substrate intersect. At the same time, the authors of the latter work associate the minimum of the contour of the light reflection by a film with the so-called Brewster angle for a film. As is evident from Fig. 3, the Brewster angle α_{B01} for the upper interface ($\tan\alpha_{B01} = \frac{n_1}{n_0}$), the value of which coincides with the minimum of a nonoscillating envelope, does not necessarily correspond to the minimum of the multiple-beam reflectance contour.

In fact, as was demonstrated in work [19], the minimum of the reflectance contour in the case of the light reflection by a film is formed in the range, where the envelope of interference minima is minimal – irrespective of whether the incidence is normal or oblique, the absorption is present or absent, the polarization is s or p; therefore, this minimum was called pseudo-Brewster. Hence, a complete blooming of the film can be attained only provided that the pseudo-Brewster angle α_{psB} corresponds to the condition $R_{\min} \to 0$. As follows from Eq. (6), the blooming of a system composed of a number of films can be achieved, if the following conditions are fulfilled:

$$R_{\min 1} = 0,$$

$$d_1 = \frac{\lambda}{4\pi\sqrt{n_1^2 - n_0^2\sin^2\alpha_{psB}}}(\phi_{1,k+1}\left|_{\alpha_{psB}} - \phi_{0,1}\right|_{\alpha_{psB}} +$$

$$+\pi(2m+1)), \quad (m = 1,2,3,\ldots), \qquad (16)$$

where the interference properties of the layered substrate under the upper film have been taken into account. In contrast to the conditions known before [20], conditions (16) are general.

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ОБВІДНІ СПЕКТРІВ БАГАТОПРОМЕНЕВОЇ ІНТЕРФЕРЕНЦІЇ ПЛОСКОПАРАЛЕЛЬНИХ СТРУКТУР: ОБҐРУНТУВАННЯ ТА ОСНОВНІ ЗАКОНОМІРНОСТІ

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Р е з ю м е

На основі матричного методу послідовного підсумовування багатопроменевої інтерференції в одношарових структурах обґрунтовано принцип одержання аналітичних виразів обвідних системи багатьох плівок за довільного кута падіння, поляризації хвилі та рівня поглинання та узагальнено основні їх закономірності.