MAGNETIZED DUSTY PLASMA WITH FERROMAGNETIC GRAINS AS A NEGATIVE REFRACTIVE MEDIUM IN A NARROW SHF BAND

V.M. MAL'NEV, E.V. MARTYSH[1], V.V. PAN'KIV[1]

Addis Ababa University
(Addis Ababa 1176, Ethiopia),

[1]**Taras Shevchenko Kyiv National University**
(6, Academician Glushkov Ave., Kyiv 03680, Ukraine; e-mail: emart@univ.kiev.ua)

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The paper is devoted to the analysis of high-frequency dispersion properties of a dusty plasma with ferromagnetic grains in a strong constant external magnetic field. The dispersion of the magnetic permeability of the magnetic subsystem along with the dispersion of the dielectric permittivity of the charged subsystem are taken into account. The dispersion of the magnetic permeability tensor is important only near the ferromagnetic resonance frequency of an individual grain ω_M that is related to the SHF range. Therefore, in such a plasma, one more typical frequency ω_M appears. The magnetic subsystem strongly interacts with the eigenwaves of the conventional magnetized electron-ion plasma and considerably affects their dispersion near this frequency.

In particular, the presence of ferromagnetic grains opens a transparency window in a magnetized dusty plasma with ferromagnetic grains in the vicinity of $\omega_M, \omega \approx \omega_M < \Omega_e$ (electron plasma frequency) that does not exist in the conventional electron-ion magnetized plasma. The group and phase velocity of these new waves are opposite, and we can prescribe them a negative refraction index. Their group velocity is much smaller than the velocity of light.

We claim that the dusty plasma with ferromagnetic grains in a strong constant magnetic field can be related to the left-handed media or the negative refraction media in a narrow SHF band near the frequency of the grain ferromagnetic resonance.

1. Introduction

For the last decade, a great deal of attention was given to theoretical and experimental studies of physical properties of the so-called left-handed media (LHM) or media with a negative refraction index (NRM). It may be useful to recall that, in the ordinary right-handed media (RHM), the Poynting vector and the wave vector have the same direction. This means that the phase velocity of a wave in RHM and its group velocity have the same direction. The electromagnetic waves propagating in LHM have the oppositely directed group and phase velocities. This fact was firstly noted by Mandelshtam in [1].

Later on, Veselago studied theoretically optical properties of the media with negative dielectric permittivity ε and magnetic permeability μ simultaneously and showed that they can be described phenomenologically with the help of a negative refraction index $\eta = -\sqrt{(-\varepsilon)(-\mu)} < 0$ [2].

The serious problem on the way of experimental realizations of LHM is to find the frequency ranges where $\varepsilon(\omega)$ and $\mu(\omega)$ can be negative simultaneously. Basically, a realization of this condition is limited by the dispersion properties of the magnetic permeability. The point is that $\mu(\omega)$ practically equals to unity in natural substances at frequencies much lower than the optical one [3].

In [4], Pendry with colleagues considered the electrodynamic properties of a lattice of wires (a low-frequency plasma medium with negative permittivity at frequencies lower than the plasma frequency) and split-ring resonators (a medium with negative permeability) and showed that this system can be treated as LHM in some comparatively narrow SHF range. His prediction concerning a realization of the perfect lens with resolution beyond the diffraction limit on a parallel slab with negative refraction index [5] stimulated the intense study of the negative refraction phenomena with the aim to pass to higher frequencies and to optical ones. It was not surprising that the superlensing effect was experimentally verified for some artificially created metamaterials in a microwave frequency range [6]. Manufacturing these artificial materials for the time being requires modern technologies. A detailed survey of recent results on the usage of metal-dielectric nanostructures to develop NRM in the optic range is given in [7].

There is a point of view that materials with negative refraction do not exist in nature [8]. However, the recent publication demonstrates experimentally that materials with anomalous magnetoresistance like

$La_{2/3}Ca_{1/3}MnO_3$ possess the negative refraction in the GHz range [9].

In our papers [10, 11], while studying the low-frequency waves in a magnetized dusty plasma with ferromagnetic grains (MDPFG), we obtained the permeability tensor of the magnetized dusty plasma with ferromagnetic grains. It possess a dispersion in a very narrow frequency interval (in contrast to a nonmagnetoactive dusty plasma with nonmagnetic grains, where it is a scalar and equal to unity). The dispersion was associated with small mechanical rotational vibrations of the magnetic dipoles of grains with respect to an external strong magnetic field H_0 with a frequency $\omega_0 = \sqrt{d_m H_0/J}$, ($d_m$ and J are the magnetic moment and the inertia moment of a grain, respectively). In a very narrow frequency band near ω_0, the nonzero components of the permeability tensor $\mu_{xx}(\omega) = \mu_{yy}(\omega)$ can be negative. The ferromagnetic grain subsystem affect the dispersion law of low-frequency electromagnetic waves. In particular, the dependence of a frequency ω on a wave vector k of Alfven's waves takes the form $\omega = kV_A/\sqrt{\mu(\omega)}$ (where V_A is the velocity of Alfven's waves that remains unchanged). This means that the Alfven waves with frequencies corresponding to $\mu(\omega) < 0$ cannot propagate in MDPFG. There is no such opacity band in the conventional magnetized plasma. The dielectric permittivity tensor is controlled by the charged components and is given by the standard formulas. By varying the number densities of electrons, ions, and grains and a magnetic field H_0, one can realize $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ in this system simultaneously. For typical parameters of ferromagnetic grains and typical magnetic fields, ω_0 corresponds to long radio waves. This means that MDPFG can manifest properties of LHM in a low-frequency range. Unfortunately, this claim cannot be verified under laboratory conditions.

2. Dispersion Equation Of SHF Waves in MDPFG

In this paper, we consider the high-frequency dispersion properties of a dusty plasma with ferromagnetic grains in a strong constant magnetic field. The high-frequency magnetic permeability tensor of MDPFG is associated with the ferromagnetic resonance of an individual grain. The frequency of the ferromagnetic resonance is determined by the frequency of precession of the electron spin magnetization vector around a constant external magnetic field H_0 and corresponds to the cm range for the fields of the order of 10^3 Gs.

As was mentioned above, the tensor of magnetic permeability of MDPFG is completely controlled by the magnetized grain subsystem. The high-frequency tensor can be obtained on the base of the magnetic susceptibility tensor of an individual ferromagnetic grain χ_{ij} [3]. Its nonzero components in the coordinate system with the z-axis along H_0 can be written in the form

$$\chi_{xx} = \chi_{yy} = -\frac{1}{\beta}\frac{\omega_M(\omega_M+\omega_H)}{\omega^2-(\omega_M+\omega_H)^2},$$

$$\chi_{xy} = -\chi_{yx} = i\frac{1}{\beta}\frac{\omega\omega_M}{\omega^2-(\omega_M+\omega_H)^2}. \qquad (1)$$

We introduce the following notations:

$$\omega_M = \gamma\beta M, \quad \omega_H = \gamma H_0, \gamma = g/(2mc), \qquad (2)$$

g is the gyromagnetic ratio, e and m are the electron charge and mass, respectively, c is the velocity of light, and β is the anisotropy coefficient. Here, M is the static magnetization in the constant field H_0.The permeability tensor of the grain subsystem μ_{ij} is obtained according to the relation

$$\mu_{ij} = \delta_{ij} + 4\pi v N_g \chi_{ij}/\beta, \qquad (3)$$

where v is the volume of a grain, and N_g is the grain number density. Using (1)and (3), we can write the nonzero components of the permeabillity tensor of MDPFG as

$$\mu_{xx} = \mu_{yy} = 1 - \xi\frac{\omega_M(\omega_M+\omega_H)}{\omega^2-(\omega_M+\omega_H)^2} \equiv \mu_1(\omega),$$

$$\mu_{xy} = -\mu_{yx} = i\xi\frac{\omega\omega_M}{\omega^2-(\omega_M+\omega_H)^2} \equiv i\mu_2(\omega),$$

$$mu_{zz} = 1. \qquad (4)$$

Here, we introduced the parameter $\xi = 4\pi v N_g/\beta$. As one may expect, $\xi \ll 1$ for typical dusty plasmas. This allows us to consider the magnetization of the grain subsystem as a sum of magnetizations of individual ferromagnetic grains and to neglect the interaction between the magnetic dipoles. This inequality also results in that the dispersion of (4) is important in a narrow frequency band near $\omega \approx \omega_M \gg \omega_H$

The dispersion equation of monochromatic waves that can propagate in a medium with arbitrary permittivity $\hat{\varepsilon}(\omega,\vec{k})$ and permeability $\hat{\mu}(\omega,\vec{k})$ tensors can be obtained from the Maxwell equations. For plane electromagnetic waves $\exp(i\vec{k}\cdot\vec{r} - i\omega t)$, they can be written as follows:

$$\epsilon_{ijl}k_jE_l(\omega,\vec{k}) = \frac{\omega}{c}\mu_{is}(\omega,\vec{k})H_s(\omega,\vec{k}),$$

$$\epsilon_{ijl}k_jH_l(\omega,\vec{k}) = -\frac{\omega}{c}\varepsilon_{is}(\omega,\vec{k})E_s(\omega,\vec{k}),$$

$$k_i\varepsilon_{ij}(\omega,\vec{k})E_j(\omega,\vec{k}) = 0,$$

$$k_i\mu_{ij}(\omega,\vec{k})H_j(\omega,\vec{k}) = 0, \qquad (5)$$

where ϵ_{ijl} is the completely antisymmetric tensor, $E_l(\omega,\vec{k})$ and $H_s(\omega,\vec{k})$ are the Fourier components of the electric and magnetic fields, respectively. Eliminating the components of the magnetic field $H_s(\omega,\vec{k})$ from system (5), we obtain the system of linear equations for $E_j(\omega,\vec{k})$. To obtain $\omega(\vec{k})$ for the eigenwaves, one has to nullify the determinant of the following system of linear equations:

$$\Lambda_{ij}(\omega,\vec{k})E_j(\omega,\vec{k}) = 0. \qquad (6)$$

The components of the tensor $\hat{\Lambda}(\omega,\vec{k})$ are given by the formulas

$$\Lambda_{xx} = \varepsilon_1\tilde{\mu}^2 - \eta^2\mu_1\cos^2(\theta),$$

$$\Lambda_{xy} = -\Lambda_{yx} = -i[\varepsilon_2\tilde{\mu}^2 - \eta^2\mu_2\cos^2(\theta)],$$

$$\Lambda_{xz} = \Lambda_{zx} = \eta^2\mu_1\sin(\theta)\cos(\theta),$$

$$\Lambda_{yy} = (\varepsilon_1 - \eta^2\sin^2(\theta))\tilde{\mu}^2 - \eta^2\mu_1\cos^2(\theta),$$

$$\Lambda_{yz} = -\Lambda_{zy} = i\eta^2\mu_2\sin(\theta)\cos(\theta),$$

$$\Lambda_{zz} = \varepsilon_3\tilde{\mu}^2 - \eta^2\mu_1\sin^2(\theta). \qquad (7)$$

Here, we introduced the notation $\tilde{\mu}^2 = \mu_1^2 - \mu_2^2$. The expressions for μ_1 and μ_2 are given by formulas (4).

The dielectric permeability tensor ε_{ij} is controlled by the charged components and is chosen according to the model of cold electron-ion plasma [12]:

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \sum_\alpha \frac{\Omega_\alpha^2}{\omega^2 - \omega_H\alpha^2} \equiv \varepsilon_1,$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = -i\sum_\alpha \frac{\omega_H\alpha}{\omega}\frac{\Omega_\alpha^2}{\omega^2 - \omega_H\alpha^2} \equiv -i\varepsilon_2,$$

$$\varepsilon_{zz} = 1 - \sum_\alpha \frac{\Omega_\alpha^2}{\omega^2} \equiv \varepsilon_3. \qquad (8)$$

Summation is taken over all kinds of charged particles α (electrons, ions, grains), and the notations $\Omega_\alpha = \sqrt{4\pi e_\alpha^2 n_\alpha/m_\alpha}$, $\omega_{H\alpha} = |e|H_0/m_\alpha c$ are introduced for the plasma and cyclotron frequencies of the α-kind charged particles. After the expansion of the determinant of system (5) with account of (4), (7), and (8), we obtain the following biquadratic equation for the refractive index $\eta = ck/\omega$:

$$A\eta^4 + B\eta^2 + C = 0, \qquad (9)$$

where

$$A = \tilde{\mu}^2[\varepsilon_1\sin^2(\theta) + \varepsilon_3\cos^2(\theta)][\cos^2(\theta) + \mu_1\sin^2(\theta)],$$

$$B = \tilde{\mu}^2[\mu_1(\varepsilon_2^2 - \varepsilon_1^2)\sin^2(\theta) - \varepsilon_1\varepsilon_3(2\mu_1\cos^2(\theta)+$$

$$\tilde{\mu}^2\sin^2(\theta)) + 2\varepsilon_2\varepsilon_3\mu_2\cos^2(\theta)],$$

$$C = \tilde{\mu}^4\varepsilon_3[\varepsilon_1^2 - \varepsilon_2^2]. \qquad (10)$$

Here, θ is the angle between the z-axis and the direction of the wave vector $\vec{k}$ which lies in the $x-z$ plane. This can be done due to the cylindrical symmetry of the problem. At $\mu_2 = 0$, coefficients (10) coincide with the corresponding coefficients obtained in [11]; at $\mu_2 = 0, \mu_1 = 1$, they are reduced to those in the case of the conventional electron-ion magnetized plasma [12].

From solutions of (9),

$$\eta_\pm^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad (11)$$

it follows that two waves with different refractive indices can propagate in MDPFG (as in the conventional magnetized electron-ion plasma) in the same direction. The first wave with the refractive index η_+ is called the conventional wave, and the second one with the index η_- is the unconventional wave.

We now make some simplifications of the tensor components (4) and (8), by basing on the physical peculiarities of our problem. In strong fields H_0 closely to the saturation, the static magnetization in a ferromagnetic grain $M \gg H_0$, and $\omega_M \gg \omega_H$. This allows us to neglect ω_H in (4) and rewrite these formulas as

$$\mu_{xx} = \mu_{yy} = 1 - \xi\frac{\omega_M^2}{\omega^2 - \omega_M^2} \equiv \mu_1(\omega),$$

$$\mu_{xy} = -\mu_{yx} = i\xi\frac{\omega_M\omega}{\omega^2 - \omega_M^2} \equiv i\mu_2(\omega). \qquad (12)$$

The simplifications of the permeability tensor (8) are dictated by the fact that, in the vicinity of the frequencies $\omega \approx \omega_M$ which are of interest for us and are

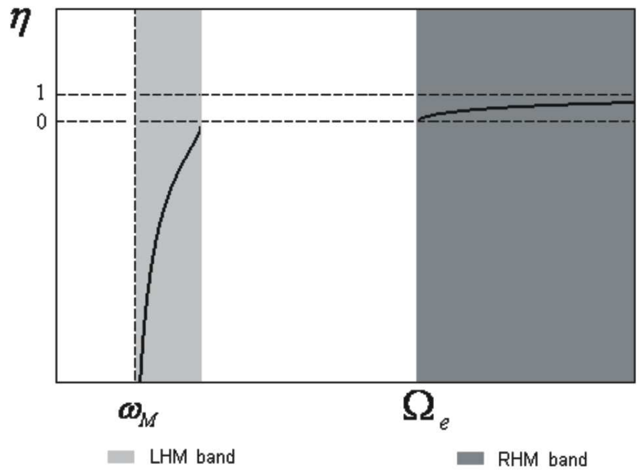

Fig. 1. The typical behavior of the refractive index

related to the high-frequency band, we can neglect the ion motion and the electron cyclotron frequency:

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \frac{\Omega_e^2}{\omega^2} \equiv \varepsilon_1,$$
$$\varepsilon_{xy} = -\varepsilon_{yx} = -i\frac{\omega_{He}}{\omega}\frac{\Omega_e^2}{\omega^2} \equiv -i\varepsilon_2,$$
$$\varepsilon_{zz} = 1 - \frac{\Omega_e^2}{\omega^2} \equiv \varepsilon_3. \tag{13}$$

Here, $\Omega_e = 4\pi e^2 n_e/m_e$ and ω_{He} are the electron Langmuir and cyclotron frequencies, respectively. In this approximation, we have

$$\varepsilon_1 = \varepsilon_3, \quad \varepsilon_1 \gg \varepsilon_2. \tag{14}$$

But even with these simplifications, the analysis of the dispersion equation (11) for a wave propagating at an arbitrary angle θ is very complicated. Below, we consider two particular cases.

3. Negative Refractive Index in MDPFG

We now consider waves which propagate along the external magnetic field ($\theta = 0$). In this case, the refractive index (11) takes a very simple form:

$$\left(\frac{ck}{\omega}\right)^2 = (\varepsilon_1 \mp \varepsilon_2)(\mu_1 \pm \mu_2). \tag{15}$$

It follows from (6) that these waves are transverse $E_z = 0$ and circularly polarized $E_y/E_x = \mp i$ ones. In the SHF range, the pole $\omega = \omega_M$ corresponds to the resonance associated with the grain subsystem. The frequency and the direction of rotation of the vector $\vec{E}$ coincide with those of the Larmor rotation of electrons and ions and the rotation of the magnetization vector of an individual grain.

Comparing (15) with the corresponding equation for the conventional magnetoactive plasma [12], we see the extra factor $(\mu_1 \pm \mu_2)$ that is related to the permeability tensor of the grain subsystem. The propagation of undamped electromagnetic waves is possible if

$$\eta^2(\omega) > 0. \tag{16}$$

This condition in the conventional magnetized electron-ion plasma reduces to

$$\varepsilon_1 \mp \varepsilon_2 > 0. \tag{17}$$

In the case of MDPFG, the transparency condition (16) includes the term depending on the dispersion properties of the ferromagnetic grain subsystem,

$$\frac{k^2c^2}{\omega^2} = [1 - \frac{\Omega_e^2}{\omega^2}(1 + \frac{\omega_{He}}{\omega})][1 - \xi\frac{\omega_M}{\omega - \omega_M}] > 0. \tag{18}$$

The magnetoactive electron-ion plasma is opaque for the electromagnetic waves with wave frequencies ω lower than the plasma frequency Ω_e. However, the presence of the ferromagnetic magnetized grains makes it transparent at frequencies

$$1 - \xi\frac{\omega_M}{\omega - \omega_M} < 0,$$

$$\omega = \omega_M(1 + \alpha\xi), \quad 0 < \alpha < 1, \tag{19}$$

which corresponds to the narrow frequency band ($\xi \ll 1$) in the vicinity of ω_M. The appearance of this transparency window is a property of MDPFG.

We now consider the refractive index of MDPFG as a function of ω in the frequency band (19)

$$\eta(\omega) = \pm\sqrt{[1 - \frac{\Omega_e^2}{\omega^2}(1 + \frac{\omega_{He}}{\omega})][1 - \frac{1}{\alpha}]}. \tag{20}$$

Here, the sign "+" corresponds to the case where both terms in brackets under the square root are positive and is related to the usual positive refraction, whereas the sign "−", when both these terms are negative simultaneously, is related to the negative refraction. In Fig. 1, we show the typical behavior of the refractive index in LHM ($\eta < 0$) and RHM ($\eta > 0$) transparency bands. The graph of the refractive index versus the reduced frequency (ω/ω_M) calculated according to (20) for typical parameters of MDPFG is shown in Fig. 2.

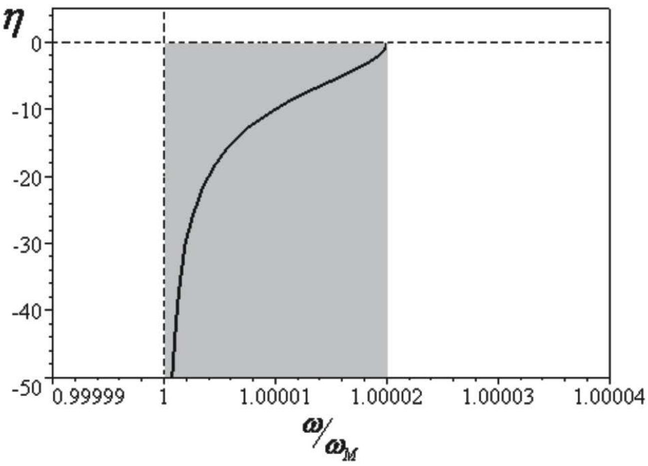

Fig. 2. Refractive index in the LHM transparency band in the case where $\theta = 0$, $\Omega_e = 100$ GHz, $\omega_M = 10$ GHz, and $\xi = 2 \times 10^{-5}$

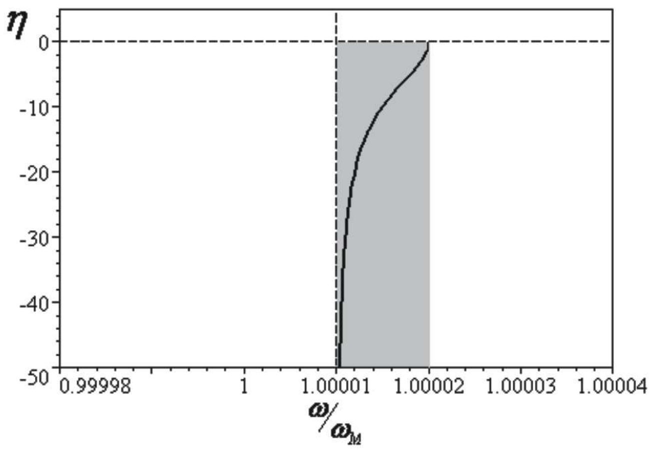

Fig. 3. Refractive index in the LHM transparency band in the case where $\theta = \pi/2$, $\Omega_e = 100$ GHz, $\omega_M = 10$ GHz, and $\xi = 2 \times 10^{-5}$

Let us consider the high-frequency electromagnetic waves in the frequency band $\omega \approx \omega_M$ provided that $\Omega_e \gg \omega_M \gg \omega_{He}$, where MDPFG has a negative refractive index. The wave vector $k(\omega)$ of these waves can written in the form

$$k = -\frac{\Omega_e}{c}\sqrt{\frac{1}{\alpha} - 1}. \quad (21)$$

The phase and group velocities of a wave are obtained according to the known formulas $v_{\text{ph}} = \omega/k$ and $v_g = d\omega/dk$ that in our case has the next view:

$$v_{\text{ph}} = -c\frac{\omega_M}{\Omega_e}\sqrt{\frac{\alpha}{1-\alpha}}, \quad (22)$$

$$v_g = 2c\frac{\omega_M}{\Omega_e}\xi\sqrt{\alpha^3(1-\alpha)}. \quad (23)$$

These formulas show that v_{ph} and v_g are oppositely directed. This situation is typical of LHM or NRM.

The energy flow of the electromagnetic wave propagates with group velocity v_g, and its direction coincides with that of the Poynting vector. In the frequency range where MDPFG has a negative refractive index, the group velocity of electromagnetic waves turn out to be considerably smaller than the velocity of light,

$$v_g = c\frac{\omega_M}{2\Omega_e}\xi, \quad \xi \ll 1. \quad (24)$$

Here, we use $\alpha = 1/2$, which means that we are exactly at the middle of the frequency band $\Delta\omega = (1/2)\xi\omega_M$ corresponding to a negative refraction index. At the same α, $|v_{\text{ph}}| \gg v_g$ because $\xi \ll 1$. Thus, these waves are related to slow electromagnetic waves.

Let us consider the electromagnetic waves propagating transversally to the external magnetic field H_0 ($\theta = \pi/2$). In this case, we have two roots of the dispersion equation (9):

$$\left(\frac{ck}{\omega}\right)^2 = \varepsilon_1 - \frac{\varepsilon_2^2}{\varepsilon_1}, \quad \left(\frac{ck}{\omega}\right)^2 = \varepsilon_3\left(\mu_1 - \frac{\mu_2^2}{\mu_1}\right). \quad (25)$$

The first root does not depend on the grain subsystem and coincides with that for the conventional electron-ion plasma. The second root describes the electromagnetic waves with dispersion depending on the magnetic subsystem $\mu_1(\omega)$ and $\mu_2(\omega)$. The refractive index of these waves takes the form

$$\eta = \pm\frac{\omega}{c}\sqrt{\left(1 - \frac{\Omega_e^2}{\omega^2}\right)\frac{1 - 2\xi\Phi(\omega)}{1 - \xi\Phi(\omega)}}, \quad (26)$$

where

$$\Phi(\omega) = \frac{\omega_M^2}{\omega^2 - \omega_M^2} \approx \frac{\omega_M}{2\Delta\omega} = \frac{1}{2\alpha\xi}. \quad (27)$$

Here, we use definition (19), $\Delta\omega = \alpha\xi\omega_M$. Different signs of the square root in (26) correspond to the cases of positive and negative refraction indices. The graphs of the refractive index versus a reduced frequency (ω/ω_M) calculated according to (26) for typical parameters of MDPFG are shown in Fig. 3. As seen, the transparency band in this case is twice as narrow, as that in the case of the longitudinal propagation of electromagnetic waves.

In the conventional electron-ion plasma $\xi = 0$, and the wave dispersion is independent on the external magnetic field H_0:

$$\omega^2 = \Omega_e^2 + k^2c^2. \tag{28}$$

The wave propagates provided that $\varepsilon_3 = 1 - \Omega_e^2/\omega^2 > 0$ or $\omega > \Omega_e$.

Let us consider MDPFG in the frequency domain $\omega \approx \omega_M + \Delta\omega$ at $\Omega_e \gg \omega_M$, where both multipliers under the square root (26) are negative, and the refractive index is negative as well. The wave vector of this wave reads

$$k = -\sqrt{2}\frac{\Omega_e}{c}\sqrt{\frac{1-\alpha}{2\alpha-1}}. \tag{29}$$

We note that $1/2 < \alpha < 1$ in this case, and the frequency band corresponding to the negative refraction is two times narrower than that in the previous case of waves propagating along the magnetic field. The phase and group velocities of wave (29) have opposite directions:

$$v_{\rm ph} = -c\frac{\omega_M}{\Omega_e}\sqrt{\frac{2\alpha-1}{2(1-\alpha)}}, \tag{30}$$

$$v_g = \sqrt{2}c\frac{\omega_M}{\Omega_e}\xi\sqrt{(2\alpha-1)^3(1-\alpha)}. \tag{31}$$

The last formulas show that the electromagnetic wave propagating transversally to the magnetic field H_0 is slow and propagates with the velocity v_g much smaller than the velocity of light ($\xi\omega_M/\Omega_e \ll 1$). We note that formulas (22) and (30) have singularity points at $\alpha = 1$ and $\alpha = 1/2$. This takes place when we pass from the "normal" propagation of an electromagnetic wave with positive refractive index to the propagation with negative refraction index and vice versa at the corresponding variation of the frequency. From (23), it follows that, at $\alpha = 0$ and $\alpha = 1$, the group velocity of a wave propagating along H_0 turns out to be zero. The same happens with the group velocity of the wave propagating transversally to H_0 at $\alpha = 0$. The $\alpha = 0$ case corresponds to the exact resonance, when the frequency of an electromagnetic wave coincides with the frequency of the grain ferromagnetic resonance, where the dissipation processes neglected in this model becomes important. Apparently, the above described phenomena exist if the following inequality holds true: $\xi\omega_M^2\tau^2 > 1$ (τ is the relaxation time of the high-frequency magnetization).

It would be useful to evaluate numerically the parameters of our theory for typical MDPFG. Let a size of ferromagnetic grains be of the order of $a \sim 10^{-4}$ cm (this allows us to consider them as one-domain particles), and let their number density $N_g \sim 10^6$ cm^{-3}. Setting $3\beta \approx 8$, we obtain $\xi = \frac{16\pi^2}{3\beta}a^3N_g \approx 2\times10^{-5} \ll 1$. The frequency of the ferromagnetic resonance at the magnetic field $H_0 \simeq 5 \times 10^2$ Gs is $\omega_M \sim 10^{10}$ Hz. Therefore, the frequency band $\Delta\omega = \xi\omega_M$, where the dispersion of the magnetic permeability is important, is of the order of 10^5 Hz. It is worth noting that the results obtained hold true at temperatures of the grain subsystem below the Curie temperature ($T_c \sim 10^3$ K).

4. Conclusion

In this paper, we have analyzed the peculiarity of eletromagnetic waves in the dusty plasma with ferromagnetic grains in a strong constant magnetic field H_0. In such a plasma, one has to consider the dispersion of the magnetic permeability tensor connected with the grain subsystem. The precessing magnetization vector of ferromagnetic grains interacts with the high-frequency self-consistent plasma field. As a result, we obtain variations of the high-frequency magnetization of the grain subsystem, which results in the dispersion of the magnetic permeability of the whole system. This dispersion is important in the vicinity of the ferromagnetic resonance frequency ω_M and can considerably affect the spectrum of high-frequency electromagnetic waves in MDPFG as compared with those waves in the conventional magnetized electron-ion plasma.

On the base of the well-known permeability tensor for a ferromagnetic obtained by Landau and Lifshits [3], we have constructed the permeability tensor of MDPFG $\hat{\mu}(\omega)$. Using it, the dielectric permittivity tensor of the cold magnetized electron-ion plasma, and the Maxwell's equations, we have obtained the biquadratic equation for the refractive index of conventional and unconventional electromagnetic waves. It is obvious that the resonance character of $\hat{\mu}(\omega)$ results in that the magnetized component affects the spectrum of weakly damped waves propagating in MDPFG only in the narrow frequency band $\Delta\omega \leq \xi\omega_M$. The point is that ξ a^3N_g (a is the grain size and N_g their number density). For typical dusty plasmas, it is of the order of $10^{-5} \ll 1$. If we take $\omega_M \sim 10^{10}$ Hz (the cm range), then $\Delta\omega \sim 10^4$ Hz. The analysis of high-frequency waves $\omega \approx \omega_M$ (10 GHz) has revealed some very interesting properties of MDPFG.

It is well known that high-frequency electomagnetic waves cannot propagate in the electron-ion plasma, if their frequency ω is smaller than the Langmuir frequency Ω_e. The addition of magnetized ferromagnetic grains to the electron-ion plasma in a strong external magnetic field (10^3 Gs) creates the transparency window in the frequency band $\Delta\omega \approx 10^4$ Hz provided that $\omega_M \ll \Omega_e$. The group velocity of these waves is much smaller than the velocity of light $v_g \approx c\xi\omega_M/\Omega_e$. The refractive index of these waves is negative.

This allows us to claim that the magnetized dusty plasma with ferromagnetic grains manifests the properties of the negative refractive or left-handed medium. The above described dusty plasma with ferromagnetic grains in strong magnetic fields can be considered, in our opinion, as one more NRM and be used as a comparatively simple system for studying the physical properties of such media.

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МАГНІТОАКТИВНА ПИЛОВА ПЛАЗМА З ФЕРОМАГНІТНИМИ ГРАНУЛАМИ ЯК СЕРЕДОВИЩЕ З НЕГАТИВНОЮ РЕФРАКЦІЄЮ У ВУЗЬКІЙ НВЧ-СМУЗІ

В.М. Мальнев, Є.В. Мартиш, В.В. Паньків

Р е з ю м е

Робота присвячена дослідженню високочастотних дисперсійних властивостей запорошеної магнітоактивної плазми з феромагнітними гранулами в сильному зовнішньому магнітному полі. Враховуються дисперсія магнітної проникності намагніченої підсистеми разом з дисперсією діелектричної сприйнятливості зарядженої підсистеми. Дисперсія магнітної проникності є суттєвою лише поблизу частоти ω_M феромагнітного резонансу окремої гранули, що належить НВЧ-діапазону. Тому в плазмі з'являється ще одна характеристична частота ω_M. Магнітна підсистема сильно взаємодіє з власними коливаннями звичайної електрон-іонної плазми та суттєво впливає на їх дисперсію поблизу даної частоти. Зокрема, наявність підсистеми гранул відкриває вікно прозорості поблизу частоти ω_M, $\omega \approx \omega_M < \Omega_e$ (Ω_e – електронна плазмова частота), яке не існує у звичайній електрон-іонній магнітоактивній плазмі. Групова та фазова швидкості цих хвиль протилежно направлені, і ми приписуємо їм від'ємний показник заломлення. Групова швидкість цих хвиль значно менша за швидкість світла. Ми стверджуємо, що запорошену магнітоактивну плазму з феромагнітними гранулами в сильному зовнішньому магнітному полі можна віднести до лівих середовищ, або середовищ з негативною рефракцією у вузькій НВЧ-смузі поблизу частоти феромагнітного резонансу гранули.