INFLUENCE OF MULTIPLET STATE MIXING IN THE ^{27}Al NUCLEUS ON NEUTRON SCATTERING CROSS-SECTIONS

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A comparative analysis of experimental data on the total and neutron scattering cross-sections by ^{27}Al nuclei in the energy range 0.2–2.5 MeV has been carried out, and the applicability of the optical-statistical approach and the excited-core model for their description has been studied. The results of theoretical analysis of experimental data were used to study the contributions of the direct mechanism and the mechanism of scattering through a compound nucleus to the elastic and inelastic scatterings of neutrons by ^{27}Al nuclei in the energy range concerned.

1. Introduction

For today, the properties of aluminum as a basis of many constructional materials have been studied rather completely. In particular, a small cross-section of neutron capture by an aluminum nucleus made this metal one of the most important components in atomic engineering. However, a number of nuclear properties of aluminum have not been studied sufficiently well until now. Despite the fact that ^{27}Al nuclei have been the object of researches dealing with the cross-sections of elastic and inelastic scatterings at a lot of nuclear laboratories, the body of reliable data concerning the cross-sections of fast neutron scattering is not large enough.

The main reason for such a situation includes the experimental difficulties that one faces while measuring the scattering cross-sections, as well as the difficulties of the theoretical interpretation of the results obtained. For the aluminum nucleus, most comprehensive researches in the reactor-spectrum energy range were carried out for the total and elastic scattering cross-sections. In the range of neutron energies below 9 MeV, the elastic scattering measurements have been executed for a few energy values only. However, it was only in work [1] that the set of parameters for the spherical optical model (SOM) was obtained by analyzing those measurement data under an assumption that the compound scattering was taken into account in the framework of the Hauser–Feshbach theory [2] only at energies less than 7 MeV. However, the analysis of experimental data on elastic scattering of neutrons at energies above 8 MeV [3–5] demonstrated that the scattering through a compound nucleus at energies up to 11 MeV must be taken into consideration. Unreliability of calculations carried out for compound cross-sections makes the determination of a consistent set of parameters for the optical potential in this energy range more complicated. A consistent set of parameters for the optical potential was obtained – on the basis of data on neutron scattering by aluminum nuclei – only in work [6] but for higher energies (11–17 MeV). A reliable set of SOM parameters was obtained by the authors of work [7] as well, by analyzing the differential cross-sections of elastic neutron scattering by aluminum nuclei in the energy range 18–26 MeV.

According to the literature data, the cross-sections of inelastic scattering with the excitation of the lowest levels of aluminum nuclei have been studied in a wide energy range. However, in works [6, 8], the applicability of the excited-core model (ECM) [9] to calculate the cross-sections of elastic and inelastic scatterings was studied only at energies of 7.62, 11, 14, and 17 MeV, although this issue is of interest in the whole investigated energy range.

At present time, there are only fragmentary researches of the optical potential parameters and the applicability of the ECM to the description of the cross-

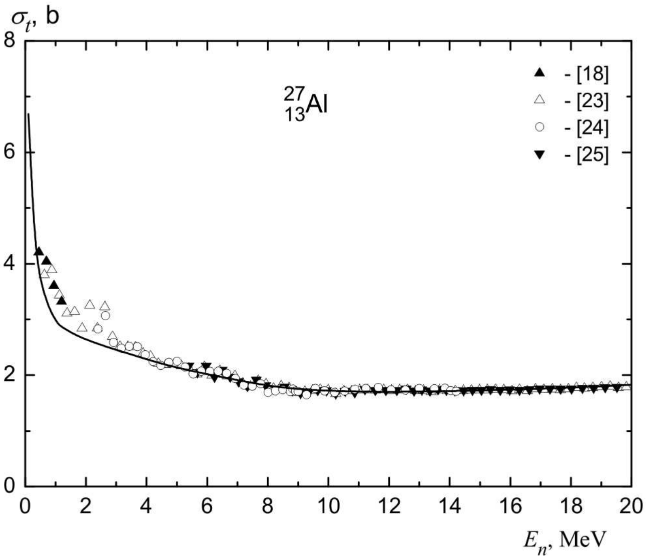

Fig. 1. Energy dependence of the total cross-section of interaction between fast neutrons and ^{27}Al nuclei. Symbols denote experimental data of works [10–13], curve corresponds to the results of cross-section calculations in the framework of the SOM with parameters (1) and (2)

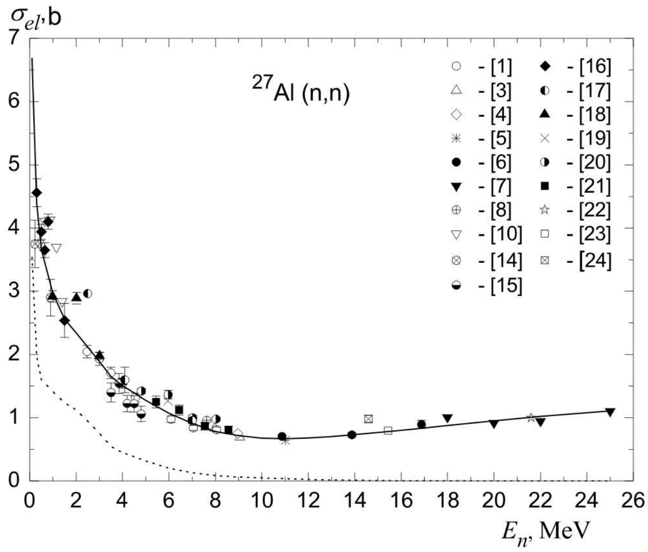

Fig. 2. Energy dependence of the cross-section of fast neutron elastic scattering by ^{27}Al nuclei. Symbols denote experimental data of works [1, 3–8, 10, 14–24], curves correspond to the results of cross-section calculations in the framework of the SOM and the SM (solid curve) and in the framework of the SM (dotted curve) with parameters (1) and (2)

sections of direct inelastic scattering for aluminum nuclei. Therefore, a conclusion can be drawn that carrying out such researches in a wide energy range, where experimental data are available, is challenging. This work aims at solving these problems.

2. Experimental Cross-Section Database for Interaction Between Fast Neutrons and Aluminum Nuclei

2.1. Total cross-sections of interaction between fast neutrons and ^{27}Al nuclei

The total cross-sections of interaction between fast neutrons and aluminum nuclei, as well as the nuclei of the majority of constructional materials, have been studied for more than fifty years. Nowadays, a large body of information concerning the total cross-sections – both at fixed energies and in a wide energy range – has been accumulated. Therefore, in order to reproduce the energy dependence of the total cross-section in the reactor-spectrum energy range, there is no necessity to cite the data of all works available in the literature; for this purpose, the data of works [10–13], which are most comprehensive and regular, will suffice. The data in those works were obtained with a high resolution; therefore, not all the corresponding points were plotted in Fig. 1, but mainly those obtained by averaging the experimental data falling within the energy interval of about 0.25 MeV. Such data are more suitable to be compared with one another and with the results of theoretical calculations. The figure demonstrates that the averaged data of the cited works are in mutual agreement – to the accuracy of experimental errors – within the common energy interval and, hence, reliably illustrate the energy dependence in a wide energy range. It should be noted that the experimental data, which were obtained with a high energy resolution, testify to the existence of a considerable resonant structure in the energy dependence of the total cross-section in the energy range below 5 MeV. The resonance structure of the total cross-sections for aluminum nuclei will manifest itself in the cross-sections of elastic and inelastic neutron scatterings as well, in the form of an appreciable dispersion of experimental cross-section values.

2.2. Cross-sections of elastic neutron scattering by ^{27}Al nuclei

In a number of experimental works devoted to studying the angular distributions of elastic neutron scattering, the energy range below 2.0 MeV [10, 14–16] was examined most completely. The comparative analysis of

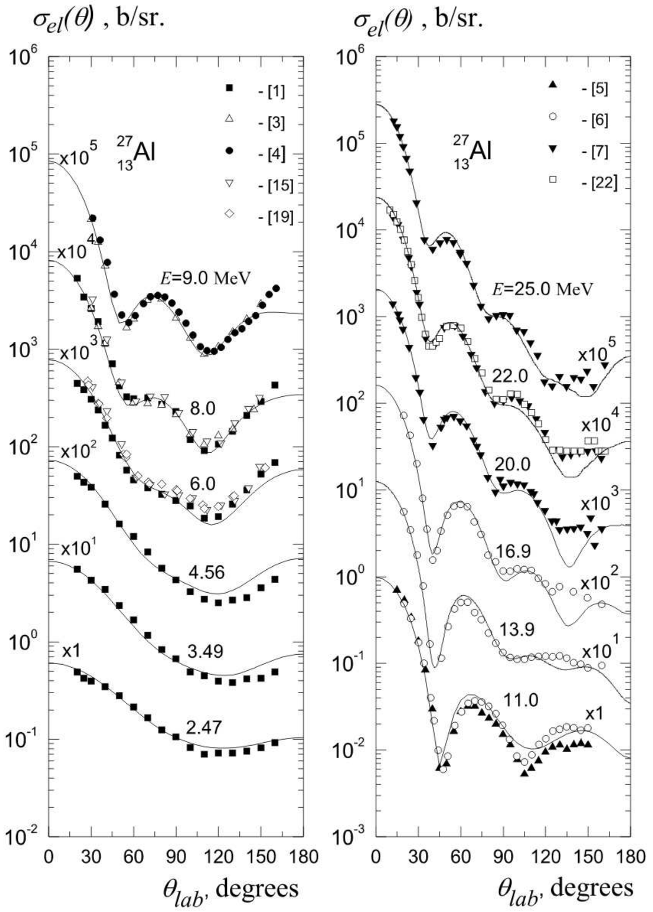

Fig. 3. Differential cross-sections of elastic neutron scattering by ^{27}Al nuclei in the energy range 2.47–25.0 MeV. Symbols denote experimental data of works [1,3–7,15,19,22], curves correspond to the results of calculations in the framework of the SOM and the SM with parameters (1) and (2)

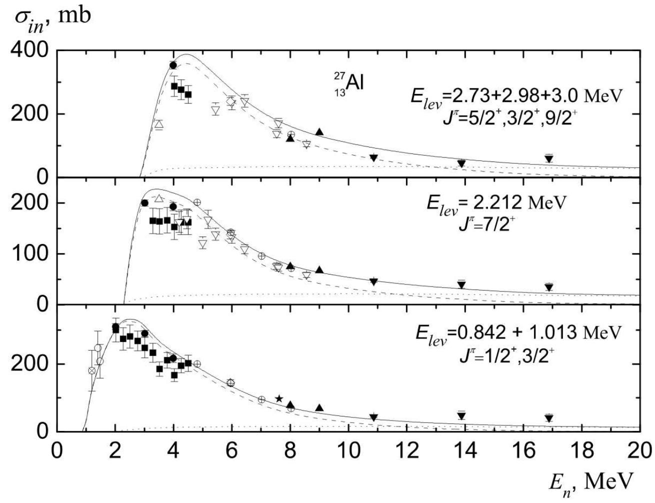

Fig. 4. Energy dependences of the cross-sections of inelastic neutron scattering with the excitation of the first levels of ^{27}Al nucleus. Points denote the experimental data: ▲ [3], ▼ [6], ⋆ [8], ○ [10], ● [18], ⋄ [19], ⊕ [20], ▽ [21], ■ [25], △ [26], and ⊗ [27]. Curves correspond to the results of cross-section calculations in the framework of the SOM and the ECM (solid), the SM (dashed), and the ECM (dotted curves)

the experimental material reported in those works revealed an essential discrepancy between the data obtained by different authors, this discrepancy being originated from the difference in the experimental energy resolution. In the range of neutron energy from 2 to 8 MeV, experimental data obtained in the earlier [1, 15–19] and later [8, 20, 21] works agree in general with one another. At energies higher that 8 MeV, the experimental data measured at different laboratories [3, 5–7, 22, 23] are also in good agreement.

This agreement is illustrated in Figs. 2 and 3, where the energy dependences of the differential and integral elastic scattering cross-sections are depicted. From the figures, one can see that the angular distributions of elastically scattered neutrons demonstrate a smooth variation of scattering anisotropy with the variation of neutron energy; the integral cross-sections of elastic scattering also smoothly change with the variation of this parameter. It should be noted that most of the data concerning the elastic scattering in this energy range were obtained by appying the high-precision time-of-flight method. Therefore, they were used by us further as the reference experimental data for the determination of the optical model parameters.

2.3. Cross-sections of inelastic fast neutron scattering by ^{27}**Al** *nuclei*

Experimental researches of the energy and angular dependences of the cross-sections of fast neutron inelastic scattering by aluminum nuclei were started almost simultaneously with the experiments on measuring $\sigma_{\rm el}(\theta, E_n)$. Nevertheless, the experimental database for $\sigma_{\rm in}(E)$ for aluminum nuclei is not so complete and accurate as that for $\sigma_{\rm el}(\theta, E_n)$. For today, the cross-sections of inelastic neutron scattering with the excitation of six lowest levels of the ^{27}Al nucleus have been studied in the energy range lower than 17 MeV. We used the data on the differential cross-sections of inelastic scattering with the excitation of individual levels or their sums taken from works [3, 19, 20] in order to calculate the integral ones. These data, as well as the data on the integral cross-section of inelastic scattering from works [6, 8, 10, 18, 21, 25–27], are exhibited in Fig. 4. From this figure, one can see that measurements were systematically made only in the energy range below 5 MeV, and the largest dispersion of data was observed just in this interval. In work [10], the inelastic neutron scattering cross-section with excitation of 1/2- and 3/2-levels of the ^{27}Al nucleus was obtained with a high resolution. Those data were not included into the figure; instead, the points corresponding to data averaged within the interval of about 200 keV were plotted. The figure testifies that the inelastic scattering cross-sections have not been studied enough at energies above 8 MeV; moreover, there is only a single work [6] devoted to studying the energy range above 10 MeV.

Besides the data on the energy dependence of inelastic scattering cross-sections, the literature database contains those on the differential cross-sections of inelastic scattering with the excitation of aluminum levels [3, 6, 18–20]. The shape of experimental angular distributions of inelastically scattered neutrons measured at energies higher than 5 MeV testifies that both mechanisms of scattering – through a compound nucleus and the direct one – take place at such energies.

The analysis of the whole scope of experimental data concerning the cross-sections of interaction between fast neutrons and aluminum nuclei allows a conclusion to be drawn that the body of experimental data on the total cross-sections and the cross-sections of elastic scattering is mainly sufficient – by its volume and quality – to be used as a basis for the determination of optical potential parameters and to carry out the research of scattering mechanisms in a wide energy range.

3. Theoretical Analysis of Experimental Data

The theoretical analysis of experimental data on the cross-sections of interaction between fast neutrons and ^{27}Al nuclei was fulfilled in the framework of the optical-statistical approach based on the SOM, the coupled-channel method (CCM), the ECM, and the statistical model (SM). In the framework of this approach, one can give a reliable consideration to the direct and compound mechanisms of scattering. To calculate the cross-sections by using those models, the individual set of optical potential parameters for the aluminum nucleus should be determined, because the averaged parameters of the SOM obtained from the description of cross-sections of even-even nuclei with medium atomic weights do not adequately describe experimental data for this odd nucleus.

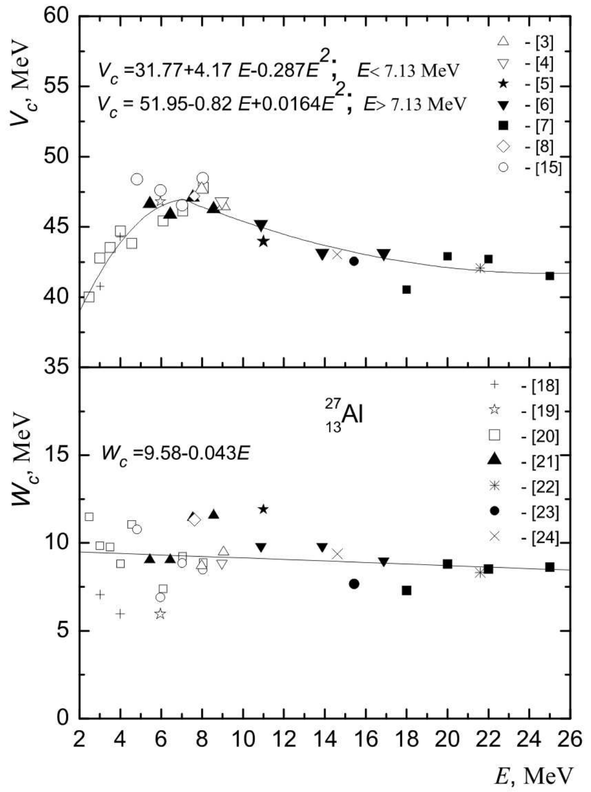

Fig. 5. Energy dependences of the optical potential parameters V_c and W_c for the aluminum nucleus determined by analyzing the experimental differential cross-sections of elastic neutron scattering obtained in works [3, 6–8, 15, 18–24]

The individual set of the SOM parameters was obtained making use of the ABAREX computer code [28]. In particular, such values for the parameters V_c and W_c were sought, at which the χ^2-quantity became minimal while fitting the calculated dependences $\sigma_{\rm el}(\theta)$ to the corresponding experimental ones at various energies. The other SOM parameters remained constant at this stage of calculations; namely, they were taken equal to those in work [29]:

$$V_{s0} = 7.5 \text{ MeV}, \quad a_V = a_W = 0.65 \text{ fm},$$

$$b = 0.47 \text{ fm}, \quad r_V = r_W = r_{s0} = 1.25 \text{ fm}. \qquad (1)$$

The values of the parameters determined (Fig. 5) in such a way were used to find the energy dependences for the quantities V_c and W_c. As a result, the following dependences were obtained:

$$V_c(E_n < 7.13) = 31.77 + 4.17E_n - 0.287E_n^2,$$

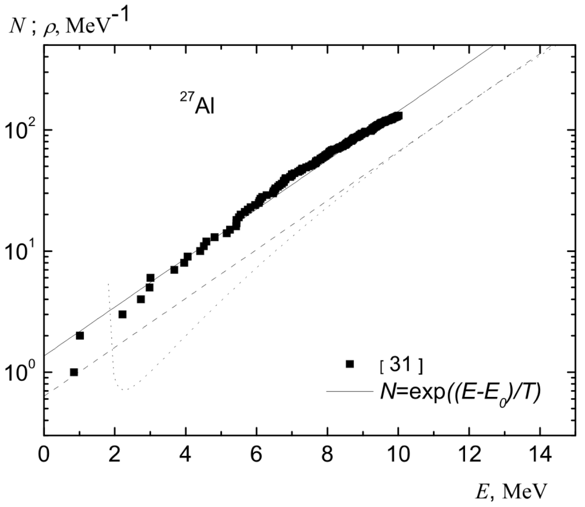

Fig. 6. Dependence of the number of levels N in the ^{27}Al nucleus up to the energy E versus E (symbols) and its exponential approximation with the parameters T = 2.15 MeV and E_0 = −0.66 MeV (solid curve). The density of levels ρ calculated by the N/T-formula (dashed line) and the formula for Fermi gas (dotted curve)

$$V_c(E_n > 7.13) = 51.95 - 0.82E_n + 0.0164E_n^2,$$

$$W_c = 9.58 - 0.043E_n. \quad (2)$$

Here, all the parameters V_c, W_c, and E_n are expressed in MeV. In what follows, the systematic calculations of the total cross-sections and the cross-sections of fast neutron scattering by aluminum nuclei were carried out making use of the optical potential parameters in forms (1) and (2).

The compound components of the elastic and inelastic neutron scattering cross-sections were calculated in the framework of the Hauser–Feshbach–Moldauer model [30] with the help of the ABAREX computer code. The characteristics of discrete excited states of the aluminum nucleus were taken from work [31]. In the case of neutron energies above 7.4 MeV, computations of the compound component of elastic and inelastic scatterings were carried out in the approximation that considers the channels of both the discrete and continuous spectra as the competitive output channels of inelastic scattering. The level density was calculated by the Gilbert–Cameron formula [32] with the following parameter values: $E_0 = -0.66$ MeV, $T = 2.15$ MeV, and $\sigma = 2.0$. The values for E_0 and T were obtained in this work by applying the method similar to that used in work [32] and taking into account 130 levels (the authors of work [32] considered the first 30 levels) up to an energy of about 10 MeV. These values, together with the value $a = 3.45$ MeV^{-1} for the parameter describing the energy dependence of the level density in the formula for Fermi gas [32], provide an adequate description for the level density in the ^{27}Al nucleus (see Fig. 6).

The direct component of the inelastic neutron scattering with the excitation of the lowest levels in the aluminum nucleus was calculated in the framework of the ECM. In this model, the ground state of ^{27}Al can be considered as a proton hole in the $1d_{5/2}$ shell weakly coupled with the ground state of the ^{28}Si core nucleus. The lowest excited states of the ^{27}Al nucleus with spins ranging from $1/2^+$ to $9/2^+$ are formed by the coupling of this proton hole with states 2^+ and 4^+ in the ^{28}Si nucleus at energies 1.779 and 4.618 MeV, respectively. The cross-section of inelastic scattering with the excitation of ^{28}Si's 2^+-state is distributed among the quintet of ^{27}Al levels according to the formula

$$\frac{d\sigma}{d\Omega}(J^\pi, {}^{27}\mathrm{Al}) = \frac{2J+1}{30}\frac{d\sigma}{d\Omega}(2^+, {}^{28}\mathrm{Si}), \quad (3)$$

where J^π is the spin and the parity of the excited state of the quintet in ^{27}Al. The cross-section of direct excitation of the ^{28}Si's collective level 2^+ was calculated in the framework of the CCM (the ECIS94 computer code [33]) and the rotational model with the deformation parameters $\beta_2 = -0.36$ and $\beta_4 = 0.2$ [8]. The calculations of the direct neutron scattering by the ^{28}Si nucleus were carried out making use of the averaged optical model parameters (1) and (2), except for W_c, the magnitude of which was reduced by 20%. The calculated cross-section of direct excitation of the ^{28}Si's level 2^+ smoothly increases from the threshold, reaches its maximum of about 100 mb at an energy of 10 MeV, and smoothly decreases down to 82 mb at an energy of 20 MeV. These values of cross-sections are distributed among the members of the studied level quintet of the ^{27}Al nucleus according to the proportion 0.067 : 0.133 : 0.267 : 0.2 : 0.333. A correction due to the effect of mixing of the ground and excited states with $J^\pi = 5/2^+$ was introduced into the cross-sections of direct inelastic excitation of quintet's levels. The correction brought about a reduction of the cross-sections of direct inelastic scattering by a factor $(1-A^2)$ in the case $J^\pi \neq 5/2^+$ and by a factor $(1-2A^2)^2$ in the case $J^\pi = 5/2^+$, where A is the state mixing amplitude. The value $A = 0.436$ [6] used by us is in satisfactory agreement with the value $A = 0.417$ calculated by us in the framework of the ECM. The importance of the introduction of this correction is evidenced by the fact that the cross-section of direct excitation of the whole quintet of ^{27}Al states decreases by a factor of 0.733 as compared with that for state 2^+ in the ^{28}Si nucleus, while the cross-section of the $J^\pi = 5/2^+$-state excitation decreases by a factor of 0.384.

The results of our calculations dealing with the total cross-sections, the differential and integral cross-sections of elastic scattering, and the integral cross-sections of inelastic scattering of fast neutrons by ^{27}Al nuclei are compared with experimental ones in Figs. 1 to 4. Figure 1 demonstrates that the calculated total cross-sections agree well with the experimental ones in a wide energy range, excluding the interval of energies lower than 2.5 MeV, for which the optical potential parameters were obtained by their extrapolation to the low-energy region making use of relations (2). The results of calculations of the integral and differential cross-sections of elastic scattering are presented in Figs. 2 and 3 by solid curves. It is evident from the figures that the calculated cross-sections agree well with the experimental ones in a wide energy range and that the compound component of elastic scattering makes an appreciable contribution even at neutron energies up to 12 MeV. The results of calculations of the inelastic neutron scattering cross-sections with the excitation of six lowest levels in the ^{27}Al nucleus are exhibited in Fig. 4, from which one can see that the theoretical calculations of cross-sections, which take the compound and direct mechanisms of scattering into consideration, describe well the experimental data, except for the sum of cross-sections of the first two levels at energies above 14 MeV, where the calculated cross-section values are smaller than the experimental ones. In those energy intervals, for which no experimental data are available, the results of calculations can be regarded as the recommended values for the cross-sections of inelastic neutron scattering.

To summarize, it is worth noting that the adequate description of the cross-sections of neutron scattering by aluminum nuclei in the wide energy range allows reliable conclusions concerning the scattering mechanisms to be done. It follows from the analysis that the compound mechanism gives an appreciable contribution to the elastic scattering cross-sections even at neutron energies close to 12 MeV, and it dominates in the excitation of the multiplet of levels at energies up to 11 MeV. The direct mechanism of elastic scattering dominates over the compound one within the whole energy range under investigation. The components of direct inelastic neutron scattering with the excitation of the studied multiplet levels of the ^{27}Al nucleus dominate only at energies above 12 MeV.

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ВПЛИВ ЗМІШУВАННЯ СТАНІВ МУЛЬТИПЛЕТІВ У ЯДРІ ^{27}Al НА ПЕРЕРІЗИ РОЗСІЯННЯ НЕЙТРОНІВ

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Р е з ю м е

Проведено порівняльний аналіз експериментальних даних з повних перерізів та перерізів розсіяння нейтронів ядрами ^{27}Al в діапазоні енергій від 0,2 до 25 МеВ і досліджено застосовність оптико-статистичного підходу та моделі збудженого остова до опису цих перерізів. Результати теоретичного аналізу експериментальних даних використано для вивчення у досліджуваному діапазоні енергій внеску прямого механізму і механізму розсіяння через складене ядро в пружне та непружне розсіяння нейтронів ядрами ^{27}Al.