

---

## POLARIZATION EFFECTS AT EMISSION AND ABSORPTION OF LIGHT BY HOT ELECTRONS IN MULTIVALLEY SEMICONDUCTORS

P.M. TOMCHUK, V.M. BONDAR

UDC 538  
©2008Institute of Physics, Nat. Acad. of Sci. of Ukraine  
(46, Nauky Ave., Kyiv 03680, Ukraine; e-mail: ptomchuk@iop.kiev.ua)

---

The angular dependences of spontaneous light emission by hot electrons in multivalley semiconductors have been studied theoretically and experimentally, making use of *n*-Ge as an example. It has been demonstrated that the change of the scattering mechanism caused by the growth of electron temperature can bring about a modification of the behavior of the scattering angular dependence. The angular dependence of the emission in the case where the heating field is applied along the symmetry axis of the crystal – for *n*-Ge, it is the axis (1,0,0) – has been observed experimentally for the first time, and the corresponding theory has been developed. For the case where electrons have identical concentration and temperature in every valley, the angular dependence of emission was shown to be associated with the symmetry violation of the distribution function of electrons over the energy (in the theory, this means going beyond the scope of the traditional diffusion approximation).

---

### 1. Introduction

Free charge carriers in semiconductors can both absorb and emit light. If a semiconductor is irradiated with an external electromagnetic flux, the absorption process dominates, while in the absence of external irradiation, but provided that the semiconductor contains hot carriers, dominating is the process of light emission. In the thermodynamic equilibrium, the amount of energy absorbed by carriers for a unit time interval is equal to the amount of energy emitted by them. These processes are well studied, and their main regularities are described in available monographies. Nevertheless, in multivalley semiconductors of the *n*-Ge and *n*-Si types, light absorption and emission by free electrons are characterized by their specific features which have not been ultimately elucidated. These features stem from the strong anisotropy of the law of electron energy dispersion, electron filling of several equivalent minima (valleys) in the conduction band, and, at last, scattering anisotropy. The character of scattering governs the mechanisms of light absorption and emission, because, for the conservation laws of energy and momentum to be simultaneously valid at these processes, another –

besides an electron and a photon, – a “third” body is required, the role of which can be played by phonons, impurities, or defects.

The features of light absorption and emission by electrons in multivalley semiconductors were already examined by us earlier in works [1, 2]. In work [1], a more attention was paid to studying the role of anisotropic scattering of electrons by acoustic vibrations of the lattice, while, in work [2], to studying the role of impurity scattering by charged centers. It has been shown that, despite the cubic symmetry of multivalley semiconductors of the *n*-Ge and *n*-Si types, the polarization dependences can be observed in the phenomena of light absorption and emission under definite conditions. These polarization dependences have been researched both theoretically [1, 2] and experimentally [1, 3]. Nevertheless, some of those features in polarization dependences remained unclear. In particular, the experiment demonstrates (we will discuss this issue below) that the coefficient which characterizes the polarization emission dependence can change its sign as the temperature of hot electrons grows. Another unrevealed feature of emission is that, in rather strong electric fields, the polarization dependences arise, even if the carrier-heating field is directed symmetrically with respect to the valleys. Therefore, there appears a necessity to continue the study of those objects.

The theoretical consideration of the phenomena of light absorption and emission by free carriers was carried out in the framework of various methods. Concerning the analysis of anisotropic scattering, we believe that the method proposed by us [1, 2], which is based on the collision integral and takes the influence of electromagnetic waves on the processes of electron scattering into account, is most convenient and effective for this purpose.

The advantage of this method consists in its ability to consider the quantum-mechanical – if the energy of an electron is lower than the energy of a light quantum – and the opposite classical case on the equal footing.

Moreover, in addition to the absorption, the emission of free carriers induced by the electromagnetic wave can be easily determined as well. Hence, one can easily determine the spontaneous emission of hot electrons. At last, the general expression for the emission (absorption) in this approach includes integrals over the momenta of all electrons and phonons (impurities). This enables one to arbitrarily change the order of integration over the angles, which – if the scattering is anisotropic – strongly simplifies calculations and makes them less cumbersome.

## 2. Formulation of the Problem. Acoustic Scattering

In this section, we start from the integral of collisions between electrons and acoustic vibrations of the lattice, which already takes into account the influence of the electromagnetic wave on the collision event. It is known that, in semiconductors of the  $n$ -Ge and  $n$ -Si types, the dispersion law for electron energy  $\varepsilon_{\vec{p}}$  looks like

$$\varepsilon_{\vec{p}} = \sum_{j=1}^3 \frac{p_j^2}{2m_j} = \frac{p_x^2 + p_y^2}{2m_{\perp}} + \frac{p_z^2}{2m_{\parallel}}, \quad (1)$$

where  $p_j$  are the components of the momentum, and  $m_{\perp}$  and  $m_{\parallel}$  are the transverse and the longitudinal effective masses of an electron, respectively. To obtain the required collision integral, it is necessary – while deriving the kinetic equation – to use the wave functions  $\Psi_{\vec{p}}$  of an electron in the field of an electromagnetic wave rather than the basic wave functions of a free electron:

$$\Psi_{\vec{p}} = \frac{1}{\sqrt{V}} \exp\left(\frac{i}{\hbar} \vec{p} \vec{r}\right) \times \exp\left\{-\frac{i}{\hbar} \int_0^t dt' \sum_{j=1}^3 \frac{1}{2m_j} \left(p_j - \frac{e_0}{c} A_j(t')\right)^2\right\}. \quad (2)$$

Here,  $V$  is the volume,  $e_0$  the charge,  $c$  the speed of light,  $\vec{A}$  the vector-potential of the electromagnetic wave which is adopted in the form

$$\vec{A} = \vec{A}^{(0)} \cos \omega t, \quad (3)$$

$\omega$  the electromagnetic wave frequency, and  $t$  the time. Details on the procedure of derivation of the collision integral for the case of the impurity scattering, provided the dispersion law (1) and taking the influence of light on the collision event into account, can be found, e.g., in work [2]. For the first time, such a collision integral for the standard dispersion law was obtained in work [4].

Hence, the electron-phonon collision integral, which makes allowance for the influence of an electromagnetic wave on a scattering event, looks like

$$If = \sum_{s=1}^3 \sum_{\vec{q}} \sum_{l=-\infty}^{\infty} W^s(\vec{q}) J_l^2\left(\frac{e_0 \gamma}{m_{\perp} \omega c}\right) \times \left\{ \left[ f(\vec{p} + \hbar \vec{q}) (N_{\vec{q}}^{(s)} + 1) - f(\vec{p}) N_{\vec{q}}^{(s)} \right] \times \right. \\ \times \delta \left[ \varepsilon_{\vec{p} + \hbar \vec{q}} - \varepsilon_{\vec{p}} - \hbar \omega_{\vec{q}}^{(s)} - l \hbar \omega \right] + \\ \left. + \left[ f(\vec{p}) - \hbar \vec{q} N_{\vec{q}}^{(s)} - f(\vec{p}) (N_{\vec{q}}^{(s)} + 1) \right] \times \right. \\ \left. \times \delta \left[ \varepsilon_{\vec{p} - \hbar \vec{q}} - \varepsilon_{\vec{p}} + \hbar \omega_{\vec{q}}^{(s)} - l \hbar \omega \right] \right\}. \quad (4)$$

Here,  $f(\vec{p})$  is the distribution function of electrons over the momentum  $\vec{p}$ ,  $N_{\vec{q}}^{(s)}$  the distribution function of phonons belonging to the  $s$ -th branch over the momentum  $\hbar \vec{q}$ , and  $J_l\left(\frac{e_0 \gamma}{m_{\perp} \omega c}\right)$  the Bessel function of the  $l$ -order, which takes the influence of the wave on the collision event into account. In the argument of the Bessel function, the quantity

$$\gamma = \vec{A}^{(0)} \vec{q} + \left(\frac{m_{\parallel}}{m_{\perp}} - 1\right) \left(\vec{A}^{(0)} \vec{l}_0\right) \left(\vec{q} \vec{l}_0\right), \quad (5)$$

where  $\vec{l}_0$  is the unit vector which fixes the orientation of the rotation axis of the effective-mass ellipsoid.

In Eq. (4),  $W^{(s)}$  is the probability for an electron to be scattered by an acoustic phonon of the  $s$ -th branch. The explicit form of this quantity will be discussed below.

Leaving very low temperatures beyond the scope of the present consideration, we may assume that the acoustic quantum energy  $\hbar \omega_{\vec{q}}^{(s)} \ll T$ , where the temperature of the lattice  $T$  is expressed in power units. Then,

$$N_{\vec{q}}^{(s)} \approx T / (\hbar \omega_{\vec{q}}^{(s)}) \gg 1.$$

In Eq. (4), in all the terms but the summand, where the number of light quanta is equal to zero ( $l = 0$ ), the energy of acoustic quanta can be put equal to zero. Then, making the substitution  $\vec{q} \rightarrow -\vec{q}$  in the second

term of Eq. (4) and taking into account that  $W^{(s)}(\vec{q}) = W^{(s)}(-\vec{q})$ , we obtain from this equation that

$$If = I^{(0)}f + \sum_{\vec{q}} \sum_{s=1}^3 \frac{2T}{\hbar\omega_{\vec{q}}^{(s)}} W^{(s)}(\vec{q}) \times$$

$$\times \sum_{l=-\infty}^{\infty} J_l^2\left(\frac{e_0\gamma}{m_{\perp}\omega c}\right) [f(\vec{p} + \hbar\vec{q}) - f(\vec{p})] \times$$

$$\times \delta[\varepsilon_{\vec{p}+\hbar\vec{q}} - \varepsilon_{\vec{p}} - l\hbar\omega]. \quad (6)$$

In this expression,  $I^{(0)}$  denotes the value of the integral  $I$  in Eq. 4) at  $l = 0$ . This term makes allowance for the inelasticity of acoustic scattering. The prime in the sum over  $l$  means that the term with  $l = 0$  is omitted.

Changing in Eq. (6) from the sum over the index  $\vec{q}$  to the integral over the variable  $\vec{p}' = \vec{p} + \hbar\vec{q}$  brings us to the result

$$If = I^{(0)}f + \int d\vec{p}' \sum_{l=-\infty}^{\infty} W_a(\vec{q}) J_l^2\left(\frac{e_0\gamma}{m_{\perp}\omega c}\right) \times$$

$$\times [f(\vec{p}') - f(\vec{p})] \delta[\varepsilon_{\vec{p}'} - \varepsilon_{\vec{p}} - l\hbar\omega]. \quad (7)$$

Here, we introduced the notation

$$W_a(\vec{q}) = \frac{V}{(2\pi\hbar)^3} \sum_{s=1}^3 \frac{2T}{\hbar\omega_{\vec{q}}^{(s)}} W^{(s)}(\vec{q}). \quad (8)$$

The explicit expression for the probability of the electron scattering by all three branches of acoustic vibrations can be obtained in the deformation potential approximation [5, page 162]. The calculation gives

$$W_a(\vec{q}) = \frac{T}{4\pi^2\hbar^4\rho} \left\{ \frac{1}{s_{\parallel}^2} \left[ \sum_d + \sum_u \left( \frac{\vec{l}_0\vec{q}}{q} \right)^2 \right]^2 + \right.$$

$$\left. + \frac{\sum_u}{s_{\perp}^2} \left[ 1 - \left( \frac{\vec{l}_0\vec{q}}{q} \right)^2 \right] \left( \frac{\vec{l}_0\vec{q}}{q} \right)^2 \right\}. \quad (9)$$

In this expression,  $\rho$  is the density;  $s_{\parallel}$  and  $s_{\perp}$  are the speeds of longitudinal and transverse sound, respectively;  $\sum_d$  and  $\sum_u$  are the constants of deformation potential; and  $\vec{l}_0$  is the unit vector that fixes the valley orientation (the rotation axis of the mass ellipsoid).

Now, let us write down the expression for the energy exchanged by electrons and acoustic phonons in a unit time in the presence of an electromagnetic wave:

$$P = \int d\vec{p}' \varepsilon_{\vec{p}'} \hat{I} f(\vec{p}') = P_0 + \hbar\omega \sum_{l=-\infty}^{\infty} l \int d\vec{p}' f(\vec{p}') \times$$

$$\times \int d\vec{p}' W_a(\vec{q}) J_l^2\left(\frac{e_0\gamma}{m_{\perp}\omega c}\right) \delta[\varepsilon_{\vec{p}'} - \varepsilon_{\vec{p}} - l\hbar\omega]. \quad (10)$$

Here, the term  $P_0$  is associated with the first term in Eq. (6) and characterizes the energy that the electron gas (if it is hot) transfers to acoustic vibrations of the lattice for a unit time.

In what follows, we confine ourselves to the consideration of single-quantum processes only, i.e., we consider only those terms in sum (10) which correspond to the values  $l = \pm 1$  only. Moreover, if extremely strong fields are not examined, the argument of the Bessel function is considerably smaller than unity, and, consequently, we may confine the consideration to the first term of the expansion. Taking all those approximations into account, we rewrite Eq. (10) as follows:

$$P \approx P_0 \pm \hbar\omega \int d\vec{p}' f(\vec{p}') \times$$

$$\times \int d\vec{p}' W_a(\vec{q}) \left( \frac{e_0\gamma}{2m_{\perp}\omega c} \right)^2 \delta[\varepsilon_{\vec{p}'} - \varepsilon_{\vec{p}} \mp \hbar\omega]. \quad (11)$$

The argument of the  $\delta$ -function in Eq. (11) demonstrates that the energy of the electron after scattering is equal to  $\varepsilon_{\vec{p}'} = \varepsilon_{\vec{p}} \pm \hbar\omega$ , i.e. expression (11) describes both the single-quantum radiation absorption and the single-quantum emission.

To move further, we should specify the momentum-distribution function of electrons  $f(\vec{p})$ . Hence, we consider multivalley semiconductors of the  $n$ -Ge and  $n$ -Si types, in which electrons occupy lower (equivalent) valleys. The distribution function for the electrons in the  $i$ -th valley is supposed to be Maxwellian-like, but with its own electron concentration  $n_i$  and own electron temperature  $T_i$ :

$$f^{(i)}(\vec{p}) = \frac{n_i}{(2\pi T_i)^{3/2} m_{\perp} \sqrt{m_{\parallel}}} e^{-\varepsilon_{\vec{p}}/T_i}. \quad (12)$$

The concentration  $n_i$  and the temperature  $T_i$  depend on a number of external factors (pressure, electric field, irradiation) and should be determined from the corresponding balance equations for the concentration

and the energy. In our theory, they are considered as the known parameters. If the function  $f(\vec{p})$  in Eq. (11) denotes function (12) and the quantity  $\gamma$  is determined from Eq. (5), then expression (11) describes the contribution of charge carriers from the  $i$ -th valley to the processes of light absorption and emission. Below, the quantities associated with the  $i$ -th valley are designated by the subscript  $i$ . Substituting Eq. (12) into Eq. (11) and changing to the deformed coordinate system, where the energy does not depend on angles (see details in works [1, 2]), the integrals can be easily calculated, and, as a result, we obtain

$$\Delta P_i(+)=\frac{2e_0^2}{3\sqrt{\pi}}\left\{\frac{A_{\perp}^{(0)2}}{m_{\perp}\tau_{\perp}^{(0)}}+\frac{A_{\parallel}^{(0)2}}{m_{\parallel}\tau_{\parallel}^{(0)}}\right\}\frac{n_iT_i^{3/2}}{T^{1/2}c^2\hbar\omega}\times$$

$$\times\left\{a_i^3e^{a_i}\frac{d}{da_i}\left(\frac{K_1(a_i)}{a_i}\right)\right\}_{a_i=\hbar\omega/T_i}, \quad (13)$$

$$\Delta P_i(-)=-\exp\left(-\frac{\hbar\omega}{T_i}\right)\Delta P_i(+).$$

In expression (13), we took into account only the second term in Eq. (11), which describes single-quantum transitions (the term including  $P_0$  is not of interest for us). The quantities  $\Delta P_i(+)$  and  $\Delta P_i(-)$  describe the contributions of the  $i$ -th valley to absorption and emission, respectively.

In Eq. (13),  $K_1(a_i)$  is the so-called Bessel function of an imaginary argument, which has the asymptote

$$K_1(a_i)=\begin{cases} \frac{1}{a_i}, & \text{при } a_i \rightarrow 0, \\ \sqrt{\frac{\pi}{2a_i}} e^{-a_i}, & \text{при } a_i \rightarrow \infty. \end{cases} \quad (14)$$

In addition, two new physical quantities,  $\tau_{\parallel}^{(0)}$  and  $\tau_{\perp}^{(0)}$ , were introduced in Eq. (13); these are the so-called ‘‘longitudinal’’ and ‘‘transverse’’, respectively, components of the relaxation time. Their expressions in terms of the parameters  $m_{\parallel}$ ,  $m_{\perp}$ ,  $\Sigma_d$ , and  $\Sigma_u$  are rather cumbersome (see, e.g., [5, page 166]). Nevertheless, their relationships with the corresponding components of the acoustic mobility tensor for cold (not heated) electrons are simple, namely,

$$\mu_{\parallel}^{(a)}=\frac{4}{3\sqrt{\pi}}\frac{e\tau_{\parallel}^{(0)}}{m_{\parallel}}, \quad \mu_{\perp}^{(a)}=\frac{4}{3\sqrt{\pi}}\frac{e\tau_{\perp}^{(0)}}{m_{\perp}}. \quad (15)$$

In formulas (13), the quantity  $A_{\parallel}^{(0)}$  means the component of the  $\vec{A}^{(0)}$  vector directed along the rotation axis

of the  $i$ -th mass ellipsoid, and  $A_{\perp}^{(0)}$  the component perpendicular to this axis, i.e.

$$(A_{\parallel}^{(0)})^2=(\vec{A}^{(0)}\vec{l}_0)^2, \quad (A_{\perp}^{(0)})^2=(A^{(0)})^2-(\vec{A}^{(0)}\vec{l}_0)^2. \quad (16)$$

In Introduction, we mention that, depending on the situation, the process of absorption or emission of light by free charge carriers can dominate. Therefore, before speaking about the polarization dependences of emission by free carriers in multivalley semiconductors, let us write down the absorption coefficient. It equals

$$K=\frac{\sum_{(i)}\{P_i(+)+P_i(-)\}}{\Pi}=\frac{\sum_{(i)}\{1-e^{-\hbar\omega/T_i}\}P_i(+)}{\Pi}. \quad (17)$$

Here,  $\Pi$  is the electromagnetic flux that – in the case of absorption – falls onto the semiconductor:

$$\Pi=\frac{\chi_0^{1/2}}{8\pi}\frac{\omega^2}{c}(A^{(0)})^2, \quad (18)$$

$\chi_0$  being the static dielectric constant.

From Eqs. (13) and (16)–(18), we obtain

$$K=\frac{16\sqrt{\pi}}{3}\frac{e_0^2}{c\hbar}\sum_i\frac{n_iT_i^{3/2}}{\omega T^{1/2}}\times$$

$$\times\left\{\frac{1}{m_{\perp}\tau_{\perp}^{(0)}}+\left[\frac{1}{m_{\parallel}\tau_{\parallel}^{(0)}}-\frac{1}{m_{\perp}\tau_{\perp}^{(0)}}\right](\vec{g}_0\vec{l}_0)^2\right\}\times$$

$$\times\left(1-e^{-\hbar\omega/T_i}\right)\left\{a_i e^{a_i}\frac{d}{da_i}\left(\frac{K_1(a_i)}{a_i}\right)\right\}_{a_i=\hbar\omega/2T_i}, \quad (19)$$

where  $\vec{g}_0$  is a unit vector which sets the polarization, i.e.

$$\vec{A}^{(0)}=\vec{g}_0A^{(0)}.$$

### 3. Polarization Dependences of Spontaneous Emission by Hot Electrons. Acoustic Scattering

In this section, we discuss the spontaneous emission by electrons. We consider the electron gas to be heated by means of an electric field applied to the semiconductor. Therefore, both the electron concentration  $n_i$  in the  $i$ -th valley and the corresponding electron temperature  $T_i$  in it differ from the relevant equilibrium values of those

parameters. According to formulas (13), the quantity  $P_i(-)$  is responsible for the emission of hot electrons from that valley induced by the wave field. To derive the expression describing the spontaneous emission, we will act in the following way.

First, we normalize the vector-potential of the wave, so that the volume  $V$  should contain  $N_{pb}$  photons, i.e. we assume that

$$\frac{1}{V} N_{pb} \hbar \omega = \frac{\bar{E}^2}{4\pi} \equiv \frac{1}{8\pi} \left(\frac{\omega}{c}\right)^2 \left(A^{(0)}\right)^2. \quad (20)$$

Whence, we obtain

$$A^{(0)} = 2c \left(\frac{2\pi\hbar}{V\omega} N_{pb}\right)^{1/2}. \quad (21)$$

We now substitute expression (21) into Eq. (13), put  $N_{pb} = 1$  there, and multiply the expression obtained for  $P_i(-)$  by the final state density of the electromagnetic field in the unit interval of frequencies and the solid angle  $d\Omega$  to obtain

$$d\rho(\omega) = \frac{V}{(2\pi c)^3} \omega^2 d\Omega. \quad (22)$$

As a result, the expression for the energy, which is radiated by electrons from all valleys in a unit time, in a unit volume, and into the solid angle  $d\Omega$ , looks like

$$\begin{aligned} W^{(-)} = & -\frac{2e_0^2}{3\pi^{5/2}c^3T^{1/2}} \sum_{(i)} n_i T_i^{3/2} \left\{ \frac{1}{m_{\perp}\tau_{\perp}^{(0)}} + \right. \\ & \left. + \left( \frac{1}{m_{\parallel}\tau_{\parallel}^{(0)}} - \frac{1}{m_{\perp}\tau_{\perp}^{(0)}} \right) (\vec{l}_0\vec{g}_0)^2 \right\} \times \\ & \times \left( a_i^3 e^{-a_i} \frac{d}{da_i} \left( \frac{K_1(a_i)}{a_i} \right) \right)_{a_i=\hbar\omega/2T_i} d\Omega. \end{aligned} \quad (23)$$

Formula (23) describes both the cases of classic (if  $a_i \ll 1$ ) and quantum-mechanical (if  $a_i \gg 1$ ) emission. In particular, making use of asymptotics (14), Eq. (23) in the classical case yields

$$\begin{aligned} W^{(-)} = & \frac{4e_0^2}{3\pi^{5/2}c^3T^{1/2}} \times \\ & \times \sum_{(i)} n_i T_i^{3/2} \left\{ \frac{\sin^2 \varphi_i}{m_{\perp}\tau_{\perp}^{(0)}} + \frac{\cos^2 \varphi_i}{m_{\parallel}\tau_{\parallel}^{(0)}} \right\} d\Omega. \end{aligned} \quad (24)$$

Here,  $\cos^2 \varphi_i = (\vec{l}_0\vec{g}_0)^2$ , so that  $\sin^2 \varphi_i = 1 - (\vec{l}_0\vec{g}_0)^2$ .

Similarly, in the quantum-mechanical case, we have

$$\begin{aligned} W^{(-)} = & \frac{e_0^2(\hbar\omega)^3}{6\pi^2 c^3 T^{1/2}} \times \\ & \times \sum_{(i)} n_i e^{-\hbar\omega/T_i} \left\{ \frac{\sin^2 \varphi_i}{m_{\perp}\tau_{\perp}^{(0)}} + \frac{\cos^2 \varphi_i}{m_{\parallel}\tau_{\parallel}^{(0)}} \right\} d\Omega. \end{aligned} \quad (25)$$

The polarization dependence of the emission is characterized by the angles  $\varphi_i$  between the valley orientation (the rotation axis of the mass ellipsoid) and the polarization of the emitted wave. To decipher these angular dependences, it is necessary to find the distributions of the parameters  $n_i$  and  $T_i$  over the valleys. Therefore, let us consider a specific case where the electron-heating electric field is directed along the (1,1,1)-axis.

As was mentioned earlier, isoenergetic surfaces around every minimum look like ellipsoids of revolution (1). The rotation axes of these ellipsoids in  $n$ -Ge are given by the unit vectors

$$\begin{aligned} \vec{l}_1 = & \frac{1}{\sqrt{3}}(1, 1, 1), \quad \vec{l}_2 = \frac{1}{\sqrt{3}}(-1, 1, 1), \\ \vec{l}_3 = & \frac{1}{\sqrt{3}}(1, -1, 1), \quad \vec{l}_4 = \frac{1}{\sqrt{3}}(-1, -1, 1). \end{aligned} \quad (26)$$

Up to now, we used the notation  $\vec{l}_0$  to designate the unit vector which determined the rotation axis of an arbitrary  $i$ -th ellipsoid. Below, we use the notation  $\vec{l}_k$  ( $k = 1, 2, 3, 4$ ) to fix the direction of specific valleys in  $n$ -Ge.

The energy supplied by a constant electric field  $\vec{F}$  in a unit time to the electrons of the  $k$ -th valley is equal to

$$W_D = n_k \left\{ \mu_{\perp} F^2 + (\mu_{\parallel} - \mu_{\perp})(\vec{l}_k\vec{F})^2 \right\}. \quad (27)$$

Here,  $\mu_{\parallel}$  is the longitudinal and  $\mu_{\perp}$  the transverse component of the mobility tensor. From Eq. (27), it is clear that, provided the field  $\vec{F}$  is oriented along the (1,1,1)-direction, the energies released per one electron in valleys 2, 3, and 4 are identical to each other, being higher than that released in valley 1. So, the valley represented by the unit vector  $\vec{l}_1$  is "cold", while the valleys characterized by the unit vectors  $\vec{l}_2, \vec{l}_3,$  and  $\vec{l}_4$  are "hot". In this situation, we have two sets of parameters:  $(n_1, T_1)$  and  $(n_2 = n_3 = n_4, T_2 = T_3 = T_4)$ .

But before analyzing the situation where the parameters for different valleys can be different, let us consider the case with identical concentrations and temperatures in all valleys. This situation takes place, if the field is directed along the direction (1,0,0). In this case, if one takes advantage of formula (24) and, while carrying out summation over  $i$ , takes unit vectors (26) instead of the unit vector  $\vec{l}_0$ , the following formula is obtained:

$$W^{(-)} \equiv W \{n_e, T_e\} = \frac{16}{9} \frac{e_0^2 n_e T_e^{3/2}}{\pi^{5/2} c^3 T^{1/2}} \times \left\{ \frac{2}{m_{\perp} \tau_{\perp}^{(0)}} + \frac{1}{m_{II} \tau_{II}^{(0)}} \right\} d\Omega. \quad (28)$$

Here, the fact that  $(n_k, T_k) = (n_e, T_e)$  at  $k = 1, 2, 3$ , and 4 was taken into account.

From Eq. (28), one can see that, if the concentration of electrons and their temperatures in all the valleys are identical, the dependence of the emission on the polarization disappears. This conclusion is in agreement with the symmetry of the problem and remains valid as far as the influence of the electric field on the symmetry of the distribution function of electrons over the energy can be neglected. (In this connection, see the consideration presented below.)

Now, let us come back to the case where the electric field is directed along the unit vector  $\vec{l}_1 = \frac{1}{\sqrt{3}}(1, 1, 1)$ . Then, the values of the parameters  $n_k$  and  $T_k$  are identical for valleys 2, 3, and 4, but differ from the relevant magnitudes for  $n_1$  and  $T_1$ . If formula (24) is written down in the form

$$W^{(-)} = \sum_{k=1}^4 W^{(-)}(n_k, T_k, \varphi_k), \quad (29)$$

then, in the case concerned, we can expand it as follows:

$$W^{(-)} = \sum_{k=1}^4 W^{(-)}(n_2, T_2, \varphi_k) + W^{(-)}(n_1, T_1, \varphi_1) - W^{(-)}(n_2, T_2, \varphi_1). \quad (30)$$

That is, to the contributions made by three valleys ( $k = 2, 3$ , and 4) with identical parameters  $\{n_k, T_k\} = \{n_2, T_2\}$ , we added and subtracted the contribution of the first valley with the same parameters. Then, the sum in Eq. (30) coincides with expression (28), where

the substitution  $\{n_k, T_k\} \leftrightarrow \{n_e, T_e\}$  is made, and does not depend on the polarization.

The dependence of the emission on the polarization is governed by two last terms in Eq. (30):

$$W^{(-)}(n_1, T_1, \varphi_1) - W^{(-)}(n_2, T_2, \varphi_1) = \frac{4e_0^2}{3\pi^{5/2} c^3 T^{1/2}} (n_1 T_1^{3/2} - n_2 T_2^{3/2}) \times \left\{ \frac{\sin^2 \varphi_1}{m_{\perp} \tau_{\perp}^{(0)}} + \frac{\cos^2 \varphi_1}{m_{\parallel} \tau_{\parallel}^{(0)}} \right\} d\Omega. \quad (31)$$

Expressing  $\sin^2 \varphi_1$  and  $\cos^2 \varphi_1$  in terms of  $\cos 2\varphi_1$  and substituting expression (31) into Eq. (30), we obtain

$$W^{(-)} = W \{n_2, T_2\} + \frac{4e_0^2}{3\pi^{5/2} c^3 T^{1/2}} \times (n_1 T_1^{3/2} - n_2 T_2^{3/2}) \left\{ \frac{1}{m_{\perp} \tau_{\perp}^{(0)}} + \frac{1}{m_{\parallel} \tau_{\parallel}^{(0)}} \right\} d\Omega + \frac{4e_0^2}{3\pi^{5/2} c^3 T^{1/2}} (n_1 T_1^{3/2} - n_2 T_2^{3/2}) \times \left\{ \frac{1}{m_{\parallel} \tau_{\parallel}^{(0)}} - \frac{1}{m_{\perp} \tau_{\perp}^{(0)}} \right\} \cos 2\varphi_1 d\Omega. \quad (32)$$

We are interested in the sign of the coefficient of  $\cos 2\varphi_1$ . The matter is that, in the experiment with high-resistance specimens, the growth of the electric field can be accompanied by the change of the sign of this coefficient. The growth of the field increases the electron temperature, and the growth of the latter can result in the change of the scattering mechanism. Therefore, we want to elucidate whether and, if yes, how the sign of this coefficient depends on the scattering mechanism. In this section, we deal with acoustic scattering. For the corresponding scenario,

$$\frac{1}{m_{\parallel} \tau_{\parallel}^{(0)}} - \frac{1}{m_{\perp} \tau_{\perp}^{(0)}} \approx -\frac{1}{m_{\perp} \tau_{\perp}^{(0)}} < 0. \quad (33)$$

Therefore, the sign of the coefficient of  $\cos 2\varphi_1$  is determined by the sign of the expression  $n_1 T_1^{3/2} - n_2 T_2^{3/2}$ . Since valley 1 is "cold" and valleys 2, 3, and 4 are "hot", we have  $T_2 > T_1$ . The question concerning the

relationship between the parameters  $n_1$  and  $n_2$  remains unanswered.

The parameters  $n_k$  and  $T_k$  must be determined from the concentration and energy balance equations, respectively. But, it should be bear in mind that the very form of these equations is based on the assumption about the Maxwellian distribution function of hot electrons in every valley. This assumption, in its turn, admits that the electron-electron scattering dominates over the electron-phonon one. The intensity of  $e - e$  scattering is known to be proportional to the squared electron concentration, while the corresponding cross-section is reciprocal to the fourth degree of relative electron-electron speed. Therefore, the situation can be realized in high-resistance specimens (i.e. with a low concentration of electrons), when the  $e - e$  interaction “Maxwellizes” the distribution function in the range of medium energies ( $\varepsilon_{\vec{p}} \sim T_e$ ), but plays an auxiliary role in the formation of the shape of the high-energy “tail” of the distribution function [5]. At the same time, since the intervalley repopulation of electrons (from hotter valleys to colder ones) is governed just by the “tail” of the distribution function, this repopulation in high-resistance specimens can be neglected, so that the electron concentrations in all valleys can be considered approximately equal to one another. In this case, taking inequalities (33) and  $T_2 > T_1$  into account, we see that the sign of the coefficient of  $\cos 2\varphi_1$  is positive. We will analyze this issue in more details after having considered, in the next section, the Coulomb mechanism of scattering.

#### 4. Spontaneous Emission by Hot Electrons at Impurity (Coulomb) Scattering

In this section, we consider optical transitions in a system of free electrons, provided that they are scattered by charged impurities (ions). The electron-ion interaction potential looks like

$$U(\vec{r}) = \frac{e_0^2}{\chi_0 r} \exp(-r/r_D), \tag{34}$$

where  $r_D$  is the Debye radius. The expression for the integral of electron-ion collision in the presence of an electromagnetic wave and provided the dispersion law (1) was derived by us in work [2]. For electrons from the  $i$ -th valley, the collision integral can be written down in the form

$$\hat{I}f^{(i)} = \frac{4e_0^4}{\chi_0^2} N_D \sum_{l=-\infty}^{\infty} \int d\vec{p} \frac{f^{(i)}(\vec{p}') - f^{(i)}(\vec{p})}{\{(\vec{p} - \vec{p}')^2 + (\hbar/r_D)^2\}^2} \times$$

$$\times J_l^2 \left( \frac{e_0}{c\hbar\omega} \sum_{\alpha=1}^3 A_{\alpha}^{(0)} \frac{p_{\alpha} - p'_{\alpha}}{m_{\alpha}} \right) \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} - \hbar\omega), \tag{35}$$

where  $N_D$  is the concentration of ionized impurities. Using the collision integral (35) and applying the same approximation as that in the derivation of Eq. (11), we obtain

$$P = \pm \hbar\omega \frac{4e_0^4 N_D}{\chi_0^2} \int d\vec{p} f^{(i)}(\vec{p}) \times \int d\vec{p}' \frac{\left( \frac{e_0\gamma}{2m_{\perp}\omega c} \right)^2 \delta\{\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}'} \pm \hbar\omega\}}{\{(\vec{p} - \vec{p}')^2 + (\hbar/r_D)^2\}^2}. \tag{36}$$

Impurity scattering is elastic. Therefore,  $P_0 = 0$  (see Eq. (10)), and  $\Delta P_i$  coincides with  $P_i$ . The parameter  $\gamma$  in Eq. (36) stands for the quantity given by formula (5) which concerns the  $i$ -th valley.

After substituting formula (12) into Eq. (36) and passing to the deformed coordinate system, where the electron energy is independent of angles, all the integrals over the remaining angles can be calculated (see work [2]), and the following result is obtained:

$$P_i(+)= \frac{e_0^6 N_D n_i}{4\chi_0^2 c^2 \hbar\omega} \left( \frac{2\pi m_{\parallel}}{T_i} \right)^{1/2} \frac{A^{(0)2}}{(m_{\parallel} - m_{\perp})^2} \times \int_0^{\infty} \frac{dx e^{-x} \{\Psi(q_{\max}^+) + \Psi(q_{\min}^+)\}}{\sqrt{x(x + \hbar\omega/T_i)}}, \tag{37}$$

Here, the notations

$$\Psi(q) = B_1(q) + (i_0 \vec{g}_0)^2 \left[ -B_1(q) + \frac{2m_{\perp}}{m_{\parallel}} B_2(q) \right],$$

$$q_{\max}^+ = \frac{(2m_{\perp} T_i)^{1/2}}{\hbar} \left[ x^{1/2} + (x + \hbar\omega/T_i)^{1/2} \right],$$

$$q_{\min}^+ = \frac{(2m_{\perp} T_i)^{1/2}}{\hbar} \left[ -x^{1/2} + (x + \hbar\omega/T_i)^{1/2} \right], \tag{38}$$

$$B_1 = \frac{1}{b^2} + \frac{1 - b^2}{b^3} \operatorname{arctg} \frac{1}{b},$$

$$B_2 = -\frac{1}{1 + b^2} + \frac{1}{b} \operatorname{arctg} \frac{1}{b},$$

$$b^2 = \frac{m_{\perp}}{m_{\parallel} - m_{\perp}} \left( 1 + \frac{1}{(qr_D)^2} \right)$$

were introduced. The quantities  $P_i(-)$  and  $P_i(+)$  are interconnected by an equation which is similar to Eq. (13).

In the general case, expression (37) is still cumbersome. The matter is that, in the classical case ( $\hbar\omega \ll T_i$ ), one has to take the charge screening into account (otherwise, integral (37) diverges), whereas such a procedure is not necessary in the quantum-mechanical case. Both cases include integral (37); and, in both limit cases, integral (37) is calculable to the end.

In particular, let us consider the classical case for the impurity scattering. By using formulas (37) and (17), we obtain the coefficient of absorption by free carriers in the following form:

$$K = \frac{3\pi^{3/2}}{2} \frac{e_0^2}{\chi_0^{1/2} c\omega^2} \sum_{i=1}^4 n_i \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(T_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(T_i)} \right\}. \quad (39)$$

Here,  $\tau_{\perp}(T_i)$  and  $\tau_{\parallel}(T_i)$  are the ‘‘transverse’’ and ‘‘longitudinal’’, respectively, components of the relaxation time at the impurity scattering:

$$\frac{1}{\tau_{\perp}(T_i)} = \frac{8}{3} \frac{e_0^4 (2m_{\parallel})^{1/2}}{\chi_0^2 m_{\perp} T_i^{3/2}} \times$$

$$N_D \frac{b_0}{2} \left[ b_0 + (1 - b_0^2) \operatorname{arctg} \frac{1}{b_0} \right] \ln (C_1 x_{\min})^{-1},$$

$$\frac{1}{\tau_{\parallel}(T_i)} = \frac{8}{3} \frac{e_0^4 (2m_{\parallel})^{1/2}}{\chi_0^2 m_{\parallel} T_i^{3/2}} \times$$

$$\times N_D b_0 \left[ -b_0 + (1 + b_0^2) \operatorname{arctg} \frac{1}{b_0} \right] \ln (C_1 x_{\min})^{-1}. \quad (40)$$

Here,  $b_0^2 = m_{\perp}/(m_{\parallel} - m_{\perp})$ ,  $\ln C_1 = 0.577\dots$  is the Euler constant, and

$$x_{\min} = \frac{1}{8} \frac{\hbar^2}{m_{\perp} T_i r_D}.$$

Unfortunately, in work [2], the expressions for  $\tau_{\perp}(T_i)$  and  $\tau_{\parallel}(T_i)$  were confused.

In the same classical case within the same calculation scheme as in the derivation of formula (24), we obtain

that the energy of spontaneous emission by hot electrons on the impurity (Coulomb) scattering is as follows:

$$W^{(-)} = \frac{3e_0^2}{16\pi^{3/2}c^3} \sum_{i=1}^4 n_i T_i \left\{ \frac{\sin^2 \varphi_i}{m_{\perp} \tau_{\perp}(T_i)} + \frac{\cos^2 \varphi_i}{m_{\parallel} \tau_{\parallel}(T_i)} \right\} d\varphi. \quad (41)$$

If the electron-heating electric field is directed along the unit vector  $\vec{l}_1 = \frac{1}{\sqrt{3}}(1, 1, 1)$ , the dependence of the emission on the polarization, similarly to Eq. (31) in the case of the impurity scattering, reads

$$\begin{aligned} & W^{(-)}(n_1, T_1, \varphi_1) - W^{(-)}(n_2, T_2, \varphi_1) = \\ & = \frac{3e_0^2}{16\pi^{3/2}c^3} \left\{ \left[ \frac{n_1 T_1}{m_{\perp} \tau_{\perp}(T_1)} - \frac{n_2 T_2}{m_{\perp} \tau_{\perp}(T_2)} \right] \times \right. \\ & \left. \times \sin^2 \varphi_1 + \left[ \frac{n_1 T_1}{m_{\parallel} \tau_{\parallel}(T_1)} - \frac{n_2 T_2}{m_{\parallel} \tau_{\parallel}(T_2)} \right] \cos^2 \varphi_1 \right\} d\varphi. \quad (42) \end{aligned}$$

Let the repopulation of valleys be inessential ( $n_1 \approx n_2$ ) owing to the reasons discussed above. Then, having expressed  $\sin^2 \varphi_1$  and  $\cos^2 \varphi_1$  in terms of  $\cos 2\varphi_1$  and taking into account that – according to Eq. (40) –  $m_{\perp} \tau_{\perp} \ll m_{\parallel} \tau_{\parallel}$ , we see that the coefficient of  $\cos 2\varphi_1$  is negative. This means that the change of the scattering mechanism along with the temperature variation (e.g., a transition from the impurity scattering to the acoustic one) can induce the change of the sign of the coefficient of  $\cos 2\varphi_1$ . Different signs of this coefficient at the impurity and acoustic scatterings are associated with the fact that the impurity scattering intensity falls down, while that of the acoustic scattering grows up with increase in the electron temperature.

## 5. Polarization Dependences Connected With the Distortion of the Electron Distribution Function

As the experiment shows, if the fields applied to  $n$ -Ge are strong, then, even provided that the electron-heating electric field is oriented in the symmetric – with respect to the valleys – direction  $\vec{l}_0 = \frac{1}{\sqrt{3}}(1, 0, 0)$ , the polarization dependence of the emission by hot electrons is observed (see a more detailed discussion below). In this case, the origin of symmetry violation can be only the electric field itself. Since neither the scattering anisotropy nor the anisotropy of the dispersion law play any important role in this situation, we consider the influence of the electric field on the polarization dependence of the emission



by hot electrons in the framework of a simpler model. Namely, we consider a mono-valley model with the dispersion law

$$\varepsilon_p = p^2/2m \tag{43}$$

and the isotropic acoustic scattering, where – instead of Eq. (9) – the equation

$$W_a = \frac{T}{4\pi^2\hbar^4\rho s_{\parallel}^2} \Sigma_d = \text{const} \tag{44}$$

is now valid. In this model, if a strong electric field is applied, the expression for the electron distribution function  $f(\vec{p})$  should be searched in the form

$$f(\vec{p}) = f_0(\varepsilon_p) + f_1(\varepsilon_p)\mathcal{P}_1(\cos\theta) + f_2(\varepsilon_p)\mathcal{P}_2(\cos\theta) + \dots, \tag{45}$$

where  $P_n(\cos\theta)$  are Legendre polynomials, and  $\theta$  is the angle between the field  $\vec{F}$  and the momentum  $\vec{p}$  direction. It is worth noting that, in the general case – i.e. if the dispersion law (1) and the scattering mechanism (9) were relevant, – we would have, instead of expression (45), the expansion

$$f(\vec{p}) = \sum_{lm} f_{lm}(\varepsilon_p) \Upsilon_{lm}(\theta, \varphi), \tag{46}$$

where  $\Upsilon_{lm}(\theta, \varphi)$  are spherical functions. However, such a generalization makes the calculations excessively cumbersome but does not change, in fact, the result of our estimation of the influence of the distribution function distortion by the field on the polarization dependences of the emission by hot electrons. In the single-quantum approximation, the energy absorbed and emitted by electrons in a unit time interval is given by expression (11), which is valid for an arbitrary distribution function.

Earlier, we adopted the Maxwell function (12) as the distribution function with the dispersion law (1). In so doing, we did not consider the influence of the field  $\vec{F}$  on the distribution function symmetry (although the field did affect the electron temperature). In our present model, the violation of the distribution function symmetry in expansion (45) is characterized by the expressions  $f_1(\varepsilon_p)\mathcal{P}_1(\cos\theta)$ ,  $f_2(\varepsilon_p)\mathcal{P}_2(\cos\theta)$ , and so on. The term proportional to  $f_1(\varepsilon_p)$ , owing to its oddness, gives no contribution to expression (11). We should note that expansion (45) is actually an expansion in the dimensionless parameter  $e_0F\tau/p$ , i.e. in the ratio between the field-induced momentum gain of the electron within the mean free time and the electron

momentum itself [5]. Therefore, for  $f_2(\varepsilon_p)$ , according to the results of work [5], we obtain

$$f_2(\varepsilon_p) \approx \frac{2}{3} (e_0F)^2 \tau(\rho)p \frac{d}{d\rho} \left( \frac{\tau(\rho)}{p} \frac{df_0}{d\rho} \right), \tag{47}$$

where the notation

$$\frac{1}{\tau(\rho)} = \frac{1}{\pi} \frac{mT}{\rho s_{\parallel}^2 \hbar^4} p \equiv \frac{1}{\tau^{(0)}} \left( \frac{\varepsilon}{T} \right)^{1/2}$$

was introduced.

Substituting expansion (45) into Eq. (11) and integrating over the angles, we obtain

$$\begin{aligned} \Delta P(-) = & \frac{8\pi^2}{3} \frac{e_0^2(A^{(0)})^2 W_a}{m\hbar\omega c^2} \times \\ & \times \int_{\sqrt{2m\hbar\omega}}^{\infty} d\rho p^2 \sqrt{p^2 - 2m\hbar\omega} \left\{ (p^2 - 2m\hbar\omega) f_0(\varepsilon_p) + \right. \\ & + \frac{1}{5} \mathcal{P}_2(\cos\theta_0) \frac{(eF\tau^{(0)})^2}{mT_e^2} T \left[ -a^3 e^a \frac{d}{da} \left( \frac{K_1(a)}{a} \right) + \right. \\ & \left. \left. + \frac{1}{2} (1 + 4a) a e^a K_1(a) \right] \right\}_{a=\hbar\omega/2T_e}. \end{aligned} \tag{48}$$

Here, the quantities  $\Delta P(-)$  and  $\Delta P(+)$  are connected by relationship (13), and  $\theta_0$  is the angle between the polarization unit vector  $\vec{g}_0$  and the field  $\vec{F}$ .

Now, in Eq. (48), we substitute  $f_0(\varepsilon_p)$  by the Maxwell function with the effective electron temperature  $T_e$  and  $f_2(\varepsilon_p)$  by expression (47). After the integration, we have

$$\begin{aligned} \Delta P(-) = & \frac{2}{3\sqrt{\pi}} \frac{e_0^2(A^{(0)})^2 n T_e^{3/2}}{m\tau^{(0)}\hbar\omega c^2 T^{1/2}} \times \\ & \times e^{-\hbar\omega/T_e} \left\{ -a^3 e^a \frac{d}{da} \left( \frac{K_1(a)}{a} \right) + \frac{2}{15} \mathcal{P}(\cos\theta_0) \times \right. \\ & \times \frac{(eF\tau^{(0)})^2}{mT_e^2} T \left[ -a^3 e^a \frac{d}{da} \left( \frac{K_1(a)}{a} \right) + \right. \\ & \left. \left. + \frac{1}{2} (1 + 4a) a e^a K_1(a) \right] \right\}_{a=\hbar\omega/2T_e}. \end{aligned} \tag{49}$$

It is of interest to compare the first term in Eq. (49) with expression (13). Putting  $m_{\perp}\tau_{\perp}^{(0)} = m_{\parallel}\tau_{\parallel}^{(0)}$ ,  $T_i = T_e$ , and  $n_i = n$ , we get a complete coincidence. Formula (13) determines the contribution of a single  $i$ -th valley to the absorption and emission processes. Much more interesting for a comparison with expression (13) is not to reduce the model to the isotropic case but to take the identical contributions from all valleys into consideration, which occurs if the field  $\vec{F}$  is oriented along the (1,0,0) direction. In this case, the axes  $T_i$  are equal to each other (and to  $T_e$ ), and all  $n_i$  are equal to each other (and to  $n/4$ , where  $n$  is the total concentration over all valleys). Doing such a comparison, we see that the first term in Eq. (49) coincides with the expression for an anisotropic multivalley case, provided the formal substitution

$$\frac{1}{m\tau^{(0)}} \rightarrow \frac{1}{3} \left( \frac{2}{m_{\perp}\tau_{\perp}^{(0)}} + \frac{1}{m_{\parallel}\tau_{\parallel}^{(0)}} \right). \quad (50)$$

The matter is about the orientation of the heating field in the symmetric direction.

Knowing expressions for the quantities  $P(\pm)$  in the model, where the function  $f(\vec{p})$  is given by expansion (45), it is easy to find the energy of spontaneous emission by hot electrons. Similarly to the procedure of deriving Eq. (23) from Eq. (13), Eq. (49) yields

$$\begin{aligned} W^{(-)} &= \frac{2e_0 n T_e^{3/2}}{3\pi^{5/2} c^2 T^{1/2}} \frac{1}{m\tau^{(0)}} \times \\ &\times \left\{ -a^3 e^{-a} \frac{d}{da} \left( \frac{K_1(a)}{a} \right) + \frac{2}{15} \mathcal{P}_2(\cos \theta_0) \times \right. \\ &\times \left. \frac{(e_0 F \tau^{(0)})^2}{m T_e^2} T \left[ -a^3 e^{-a} \frac{d}{da} \left( \frac{K_1(a)}{a} \right) + \right. \right. \\ &\left. \left. \frac{1 + 4a}{2} a e^{-a} K_1(a) \right] \right\}_{a=\hbar\omega/2T_e} d\Omega. \quad (51) \end{aligned}$$

In the multivalley case, provided that the field  $\vec{F}$  is oriented in the direction (1,0,0), expression (23) is transformed – by formally applying substitution (50) – into the first summand in Eq. (51). This summand does not contain the polarization dependence. Such a dependence appears, as is evident, if the dependence  $f_2(\varepsilon_{\rho})$  is made allowance for in Eq. (45), i.e. if going

beyond the frame of the diffusion approximation. In other words, the polarization dependence of the emission by hot electrons, provided that the field is oriented in the symmetric direction, is associated with the symmetry violation of the electron distribution function over the energy (the even part of the distribution function!).

In the classical case (if  $a \ll 1$ ), expression (51) becomes simpler:

$$\begin{aligned} W^{(-)} &= \frac{4e_0^2 n T_e^{3/2}}{3\pi^{5/2} c^2 T^{1/2}} \frac{1}{m\tau^{(0)}} \times \\ &\times \left\{ 1 + \frac{(e_0 F \tau^{(0)})^2}{6m T_e^2} T \mathcal{P}_2(\cos \theta_0) \right\} d\Omega. \quad (52) \end{aligned}$$

By the order of magnitude, the second term in the braces is equal to the squared ratio between the drift and thermal speeds.

## 6. Discussion of Results. Comparison with Experimental Data

We have demonstrated in the previous sections that, in the  $n$ -Ge case, the change of the sign is possible for the coefficient that characterizes the angular dependence of the polarization, if the electron-heating field is oriented along the (1,1,1)-axis. This change is caused by the change of the scattering mechanism, which, in its turn, is caused by the increase of the electron temperature with the growing field. Figure 1 illustrates such a sign change observed experimentally. The measurement technique was described in work [3]. The curves in this figure correspond to specimens with a charge carrier concentration of  $2.5 \times 10^{15} \text{ cm}^{-3}$ .

Figure 1 depicts the dependences of the emission intensity by hot electrons on the polarization rotation angle at various values of the heating field (10, 20, 80, and 140 V). One can see that the growth of the field gives rise to the modification of the curve character. In particular, the angles, which correspond to observable minima at weak fields, become characterized by the maximal intensity at higher ones.

It is worth noting that the mechanism of scattering can change due to the variation of the lattice temperature as well. In this case, the change of the sign of the coefficient mentioned above is also possible. This

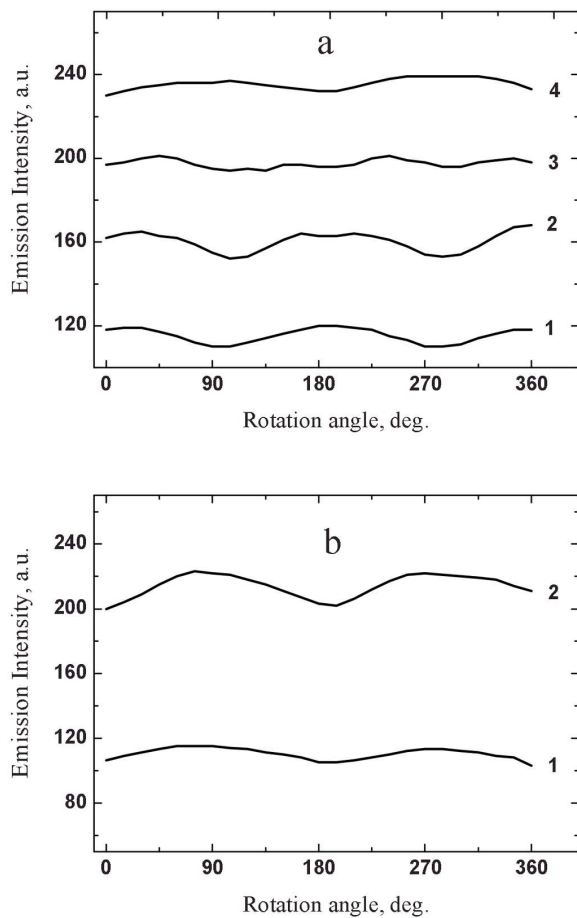


Fig. 1. Polarization angular dependences of the emission by hot charge carriers in an  $n$ -Ge specimen (the crystallographic direction  $\langle 111 \rangle$  and the carrier concentration  $n \approx 2.5 \times 10^{15} \text{ cm}^{-3}$ ) for various values of the heating electric field: (panel *a*) 10 (1), 20 (2), 40 (3), and 75 V/cm (4); (panel *b*) 80 (1) and 140 V/cm (2)

opportunity is illustrated in Fig. 2. In this figure, exhibited are the angular dependences of the emission at various temperatures. To illustrate the angular dependences, the curves are somewhat shifted vertically with respect to one another, although the intensities of the emission are approximately identical at different temperatures. The curves correspond to specimens with a carrier concentration of  $2 \times 10^{14} \text{ cm}^{-3}$ . The character of modifications of the angular dependences is the same as in Fig. 1. An opportunity for the angular dependence of the emission to change its profile with the modification of the scattering mechanism was pointed out in work [3].

Rather unexpected was the appearance of polarization dependence for the heating field oriented along the  $(1,0,0)$ -axis. Provided such a field orientation, all valleys

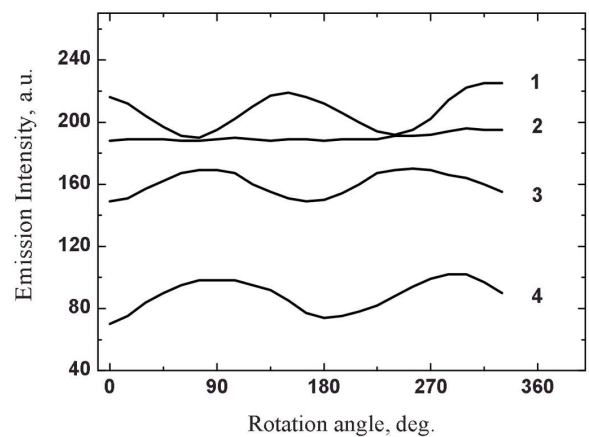


Fig. 2. Polarization angular dependences of the emission by hot charge carriers in an  $n$ -Ge specimen (the crystallographic direction  $\langle 111 \rangle$ ,  $n \approx 6 \times 10^{14} \text{ cm}^{-3}$ , the applied electric field  $E = 140 \text{ V/cm}$ ) for various temperatures: 6.6 (1), 7.7 (2), 14 (3), and 76 K (4)

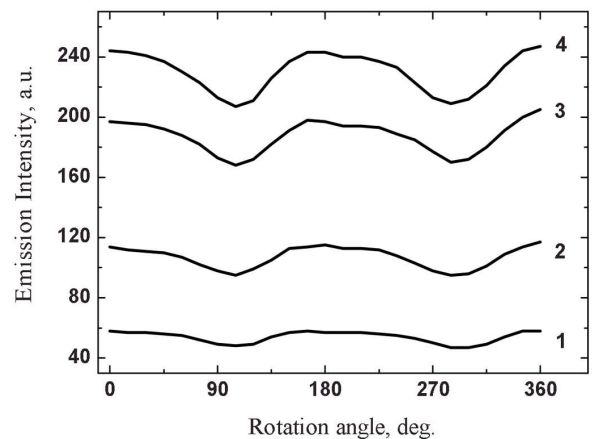


Fig. 3. Polarization angular dependences of the emission by hot charge carriers in an  $n$ -Ge specimen (the symmetric crystallographic direction  $\langle 100 \rangle$ ,  $n \approx 1.5 \times 10^{14} \text{ cm}^{-3}$ ) for various values of the heating electric field: 7 (1), 10 (2), 13 (3), and 15 V/cm (4)

are characterized by identical values of the concentration and the temperature of electrons. Figure 3 illustrates the corresponding polarization dependence obtained experimentally. Specimens with a charge carrier concentration of  $1.5 \times 10^{14} \text{ cm}^{-3}$  were used. The figure exhibits the angular dependence of the emission at various heating fields applied in the symmetric direction. The specimen temperature was 5 K. The curves evidently reveal the angular dependence of the emission, although all the parameters of valleys (the temperature and the concentration of electrons) are

identical. Such an unusual behavior can be associated with going beyond the traditional approach which is reduced to the so-called diffusion approximation.

Note that a combined influence on the character of the angular dependences – both the change of scattering mechanisms and the violation of conditions of the diffusion approximation – can take place at low temperatures and relatively strong fields.

Thus, we have shown that the emergence of the polarization dependence, if the heating field is oriented in the symmetric direction, is caused by a distortion of the distribution function of electrons over the energy (going beyond the range of validity of the diffusion approximation).

The authors express their gratitude to O.G. Sarbey for his permanent interest to this study and for fruitful discussions.

1. V.M. Bondar, O.G. Sarbey, and P.M. Tomchuk, *Fiz. Tverd. Tela* **9**, 1540 (2002).
2. P.M. Tomchuk, *Ukr. Fiz. Zh.* **49**, 681 (2004).
3. V.M. Bondar and N.F. Chernomorets, *Ukr. Fiz. Zh.* **48**, 51 (2003).
4. E.M. Epshtein, *Fiz. Tekh. Poluprovodn.* **11**, 2732 (1974).

5. I.M. Dykman and P.M. Tomchuk, *Transfer Phenomena and Fluctuations in Semiconductors* (Naukova Dumka, Kyiv, 1981) (in Russian).

Received Received 22.10.07.

Translated from Ukrainian by O.I. Voitenko

#### ПОЛЯРИЗАЦІЙНІ ЕФЕКТИ В ВИПРОМІНЮВАННІ І ПОГЛИНАННІ СВІТЛА ГАРЯЧИМИ ЕЛЕКТРОНАМИ У БАГАТОДОЛИННИХ НАПІВПРОВІДНИКАХ

*П.М. Томчук, В.М. Бондар*

Р е з ю м е

Теоретично і експериментально на прикладі *n*-Ge досліджено кутові залежності спонтанного випромінювання гарячих електронів в багатодолинних напівпровідниках. Показано, що зміна механізму розсіяння, зумовлена ростом електронної температури, може приводити до зміни характеру кутової залежності розсіяння. Вперше експериментально спостережено кутову залежність випромінювання у випадку прикладання гріючого електроні поля вздовж осі симетрії кристала (для *n*-Ge це вісь (1,0,0)). Побудована теорія такої залежності. Показано, що кутова залежність випромінювання у випадку, коли електрони всіх долин мають однакову концентрацію і температуру, пов'язана з порушенням симетрії функції розподілу електронів за енергіями (в теорії це означає вихід за рамки традиційного дифузійного наближення).