REFLECTION AND DAMPING OF BULK SPIN WAVES IN THE UNIAXIAL MULTILAYER FERROMAGNETIC STRUCTURE

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The processes of propagation and damping of spin waves in magnetic materials with periodically modulated parameters of uniaxial magnetic anisotropy, exchange interaction, and saturation magnetization have been studied theoretically. The influence of damping on the coefficient of bulk spin wave reflection from the uniaxial ferromagnetic multilayer structure has been studied. The dependences of the reflection amplitude on a wave frequency, thickness of one of the layers, and damping parameter have been investigated.

1. Introduction

In view of the peculiar properties characteristic of multilayers and exciting prospects of their practical applications, the investigation of multilayer magnetic materials has attracted a great interest of researchers in recent decades. On the one hand, this interest is brought about by the necessity of both the transition to more compact electronic devices and the operation in the high-frequency range. On the other hand, it is caused by a specific character of the wave propagation in multilayer structures. Works [1–3] were aimed at the investigation of the spectrum of bulk spin waves and their reflection from an infinite uniaxial magnetic mulilayer structure under the assumption about ideal exchange boundary conditions without regard for a damping. More recent papers devoted to the problem of the spin wave propagation in multilayer systems have tended to turn from the studies of idealized structures to the models which are much closer to real magnetics. For this reason, it is important to account for the effect of the spin wave damping on the processes which occur in the multilayer structures [4, 5]. The present paper considers the influence of a weak damping on the coefficient of spin wave reflection from a uniaxial multilayer ferromagnet.

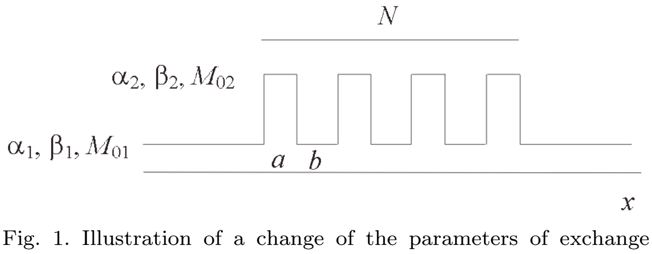

Fig. 1. Illustration of a change of the parameters of exchange interaction α, uniaxial anisotropy β, and saturation magnetization M_0 along the multilayer structure

2. Statement of the Problem

Consider a system which consists of three parts, whose contact planes are parallel to the yz plane. The first and the third (along the x direction) components are the homogeneous uniaxial semiinfinite ferromagnets. The slice between them is an N-layered ferromagnet with the modulated constants of exchange interaction α, uniaxial magnetic anisotropy β, and saturation magnetization M_0. The layers are parallel to the yz plane, and their thicknesses are equal to a and b, respectively. The quantities α, β, η, and M_0 take the values α_1, β_1, η_1, M_{01} and α_2, β_2, η_2, M_{02} for the corresponding layers, as shown in Fig. 1. An easy axis is parallel to the direction of the external homogeneous magnetic field H_0 and the z axis. Let us calculate the coefficient of spin wave reflection from this structure.

3. Main Equations

We utilize the spin density formalism [6], according to which the magnetization can be presented in the form

$$\mathbf{M}_j(\mathbf{r},t) = M_{0j}\Psi_j^+(\mathbf{r},t)\,\sigma\Psi_j(\mathbf{r},t), \quad j = 1,2, \tag{1}$$

where Ψ_j are the quasiclassical wave functions which play the role of order parameter for the spin density, $\mathbf{r}$ is the radius-vector of the Cartesian coordinate system, t is time, and σ are the Pauli matrices.

Lagrange's equations for Ψ_j have the form

$$i\hbar\frac{\partial\Psi_j(\mathbf{r},t)}{\partial t}=-\mu_0\mathbf{H}_{ej}(\mathbf{r},t)\,\sigma\Psi_j(\mathbf{r},t)+$$

$$+\frac{\eta_j\hbar}{2}\frac{\partial(\Psi_j^+(\mathbf{r},t)\,\sigma\Psi_j(\mathbf{r},t))}{\partial t}\sigma\Psi_j(\mathbf{r},t), \qquad (2)$$

where μ_0 is the Bohr magneton, $\hbar$ is the Planck constant, η_j is the damping parameter in the Gilbert form. $\mathbf{H}_{ej}=-\frac{\partial w_j}{\partial\mathbf{M}_j}+\frac{\partial}{\partial x_k}\frac{\partial w_j}{\partial(\partial\mathbf{M}_j/\partial x_k)}$, w_j is the energy density. It is noteworthy that, in the exchange approximation provided that $L >> l = a+b$, where L is the characteristic length of a material, the energy density for each of the homogeneous layers can be written as

$$w_j=\frac{\alpha_j}{2}\left(\frac{\partial m_j}{\partial x_k}\right)^2+\frac{\beta_j}{2}\left(m_{jx}^2+m_{jy}^2\right)-H_0M_{jz},\quad j=1,2. \qquad (3)$$

Here, the facts are taken into account that, in the basic state, the magnetization is parallel to $\mathbf{e}_z$, $M_j^2(\mathbf{r},t)=\text{const}$, and $\mathbf{M}_j(\mathbf{r},t)=M_{0j}\mathbf{e}_z+\mathbf{m}_j(\mathbf{r},t)$, where $\mathbf{m}_j(\mathbf{r},t)$ is a small correction term. Then, by using the linear perturbation theory, a solution of (2) can be written as

$$\Psi_j(\mathbf{r},t)=\exp(i\mu_0H_0t/\hbar)\begin{pmatrix}1\\ \chi_j(\mathbf{r},t)\end{pmatrix}, \qquad (4)$$

where $\chi_j(\mathbf{r},t)$ is a small addition which characterizes a deviation of the magnetization from that of the basic state. By linearizing Eq. (2) with regard for (4) and carrying out the Fourier transformation with respect to the time and coordinates, we obtain

$$\left(\alpha_j\frac{d^2}{dx^2}+\Omega_j-\alpha_jk_\perp^2-\beta_j-\tilde{H}_{0j}+i\eta_j\Omega_j\right)\chi_{j\omega,k_\perp}(x)=0, \qquad (5)$$

where $\tilde{H}_{0j}=H_0/M_{0j}$, $\Omega_j=\omega\hbar/2\mu_0M_{0j}$, ω is the frequency, $\mathbf{k}_\perp=(0,k_y,k_z)$, $j=1,2$, and $\chi_{j\omega,k_\perp}$ is the Fourier transform of the function $\chi_j(\mathbf{r},t)$.

By analogy with [7], the amplitude of the spin wave reflection from the multilayer structure consisting of N layers can be presented in the form

$$R_N=R\frac{1-\exp(2iqlN)}{1-R^2\exp(2iqlN)}, \qquad (6)$$

where R is the amplitude of the reflection from the semiinfinite multilayer structure ($N=\infty$),

$$R=\frac{\sqrt{(\rho+1)^2-\tau^2}-\sqrt{(\rho-1)^2-\tau^2}}{\sqrt{(\rho+1)^2-\tau^2}+\sqrt{(\rho-1)^2-\tau^2}}, \qquad (7)$$

q is the Bloch quasiwave vector which is determined from the equation

$$\exp(iqlN)=\frac{\sqrt{(\tau+1)^2-\rho^2}+\sqrt{(\tau-1)^2-\rho^2}}{\sqrt{(\tau+1)^2-\rho^2}-\sqrt{(\tau-1)^2-\rho^2}}, \qquad (8)$$

$l=a+b$ is the period of the structure; ρ and τ are the complex amplitudes of reflection and transmission, respectively, for the symmetric (relative to its center) unit period.

From these expressions, using the known methods of quantum mechanics with regard for formula (5), one can find the reflection and transmission amplitudes for a single period.

4. Boundary Conditions

Equation (5) describes the magnetization dynamics in the exchange approximation. Its solution should be continuous and have a continuous derivative $\frac{\partial\chi_{\omega,k_\perp}(x)}{\partial x}$. The boundary conditions will have the form (indices ω and $k_\perp$ are omitted):

$$\chi_1(x_0-0)=\chi_2(x_0+0),$$

$$\alpha_1\chi_1'(x_0-0)=\alpha_2\chi_2'(x_0+0). \qquad (9)$$

5. Reflection and Transmission Amplitudes for a Single Period

We represent the incident, reflected, and transmitted waves in (5) as $\chi_I=\exp\left(\left[ik_1^+-k_1^-\right]x\right)$, $\chi_\rho=\rho\exp\left(-\left[ik_1^+-k_1^-\right]x\right)$, and $\chi_\tau=\tau\exp\left(\left[ik_1^+-k_1^-\right]x\right)$, respectively. Substituting these expressions into (9) with regard for the expression $\chi_{\text{layer}}=C_1\exp\left(\left[ik_2^+-k_2^-\right]x\right)+C_2\exp\left(-\left[ik_2^+-k_2^-\right]x\right)$ which describes a wave in the intermediate layer, we find the expressions for the amplitudes of the spin wave reflection and transmission for each of the boundaries of the single period:

$$\rho=\exp\left(ik_1^+b\right)\cdot\exp\left(-k_1^-b\right)\frac{E_-F_-}{GE_+-F_+E_-},$$

$$\tau=\exp\left(-ik_1^+a\right)\cdot\exp\left(k_1^-a\right)\frac{2G}{GE_+-F_+E_-}, \qquad (10)$$

where

$$G = 2\alpha_1\alpha_2\left(ik_1^+ - k_1^-\right)\left(ik_2^+ - k_2^-\right),$$

$$F_\pm = \alpha_1^2\left(ik_1^+ - k_1^-\right)^2 \pm \alpha_2^2\left(ik_2^+ - k_2^-\right)^2$$

$$E_\pm = e^{\left(ik_2^+ - k_2^-\right)a} \pm e^{-\left(ik_2^+ - k_2^-\right)a},$$

$$k_j^\pm = \sqrt{\left(\pm m_j + \sqrt{m_j^2 + n_j^2}\right)/2},$$

$$m_j = \left(\Omega_j - \beta_j - \alpha_j k_\perp^2 - \tilde{H}_{0j}\right)/\alpha_j, \quad n_j = \eta_j\Omega_j/\alpha_j.$$

6. Reflection and Transmission Amplitudes for a Multilayer Structure

With the use of expressions (10), Eq. (7) can be rewritten as

$$R = \frac{\sqrt{A_+A_-} - \sqrt{B_+B_-}}{\sqrt{A_+A_-} + \sqrt{B_+B_-}}, \tag{11}$$

where

$$A_\pm = q_+\left[p_-\cos(u_2a) + ip_+\sin(u_2a)\right] +$$

$$+2g_+\left[p_+\sin(u_2a) - ip_-\cos(u_2a)\right] -$$

$$-2\alpha_1\alpha_2\left(\breve{y}\left[p_+\cos(u_2a) + ip_-\sin(u_2a)\right] +\right.$$

$$\left.+\widehat{y}\left[p_-\sin(u_2a) - ip_+\cos(u_2a)\right]\right) -$$

$$-\exp(-\nu_1 b)\left[q_-p_-\cos(u_2a) + 2g_-p_+\sin(u_2a)\right] \times$$

$$\times\left[\cos(u_1b) + i\sin(u_1b)\right] + \exp(-\nu_1 b)\left[q_-p_+\sin(u_2a) +\right.$$

$$\left.+2g_-p_-\cos(u_2a)\right]\cdot\left[\sin(u_1b) - i\cos(u_1b)\right] \mp$$

$$\mp 4\alpha_1\alpha_2\exp(-\nu_1 b)\left\{\left[\breve{y} - i\widehat{y}\right]\cos(u_1a) -\right.$$

$$\left.-\left[\widehat{y} - i\breve{y}\right]\sin(u_1a)\right\},$$

$$B_\pm = -q_+\left[p_-\cos(u_2a) + ip_+\sin(u_2a)\right] - 2g_+\times$$

$$\times\left[p_+\sin(u_2a) - ip_-\cos(u_2a)\right] - 2\alpha_1\alpha_2\left(\breve{y}\left[p_+\cos(u_2a) +\right.\right.$$

$$\left.\left.+ip_-\sin(u_2a)\right] + \widehat{y}\left[p_-\sin(u_2a) - ip_+\cos(u_2a)\right]\right) -$$

$$-\exp(-\nu_1 b)\left[q_-p_-\cos(u_2a) + 2g_-p_+\sin(u_2a)\right] \times$$

$$\times\left[\cos(u_1b) + i\sin(u_1b)\right] + \exp(-\nu_1 b)\left[q_-p_+\sin(u_2a) +\right.$$

$$\left.+2g_-p_-\cos(u_2a)\right]\cdot\left[\sin(u_1b) - i\cos(u_1b)\right] \mp 4\alpha_1\alpha_2\times$$

$$\times\exp(-\nu_1 b)\left\{\left[\breve{y} - i\widehat{y}\right]\cos(u_1a) - \left[\widehat{y} - i\breve{y}\right]\sin(u_1a)\right\},$$

$$p_\pm = \frac{1 \pm \exp(2\nu_2 a)}{\exp(2\nu_2 a)}, \quad q_\pm = \alpha_2^2\left[\nu_2^2 - u_2^2\right] \pm \alpha_1^2\left[\nu_1^2 - u_1^2\right],$$

$$g_\pm = \alpha_1^2 u_1\nu_1 \pm \alpha_2^2 u_2\nu_2, \quad \breve{y} = \nu_1\nu_2 - u_1u_2,$$

$$\widehat{y} = u_1\nu_2 + u_2\nu_1, \quad u_j = k_j^+, \quad \nu_j = k_j^-.$$

As was noted above, the reflection amplitude for the multilayer structure consisting of N layers is determined by expression (6).

7. Discussion of Results

All the calculations will be carried out under the assumption that the Gilbert damping parameters are the same for both layers, $\eta_1 = \eta_2 = \eta$, and the value of the external permanent magnetic field is fixed and equals H_0. Figure 2 shows the plots of the frequency dependences of the reflection amplitude from the multilayer structure calculated in the cases where η takes two values, 0 and 0.003, as it is typical of ferrite-garnets. Figure 3 depicts the dependences of the

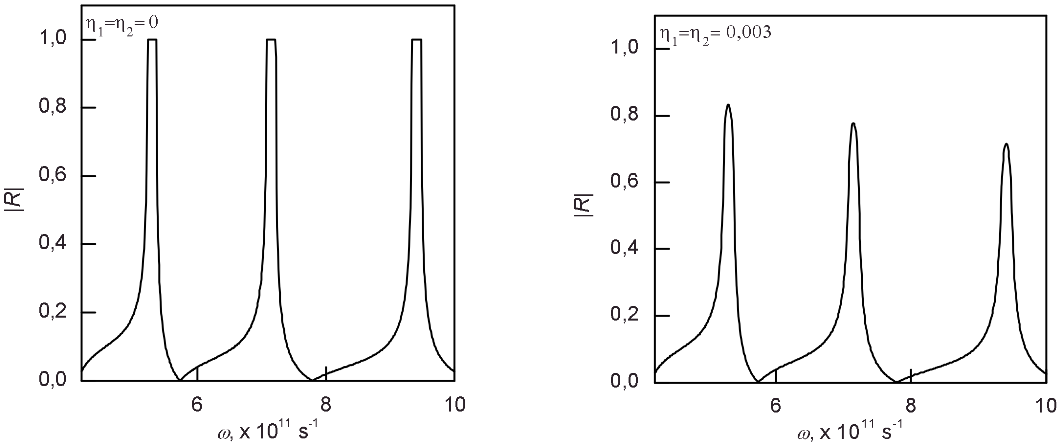

Fig. 2. Frequency dependences of the amplitude of reflection from the multilayer structure calculated for two values of damping parameters. The plots are referred to the case of a normal wave incidence on the periodic structure with the following parameters: $\alpha_1 = \alpha_2 = 2 \times 10^{-11} m^2$, $\beta_1 = 20$, $\beta_2 = 100$, $H_0 = 1000$ Oe, $M_{01} = 90$ Gs, $M_{02} = 95$ Gs, a=3.8 μm, b=0.2μm

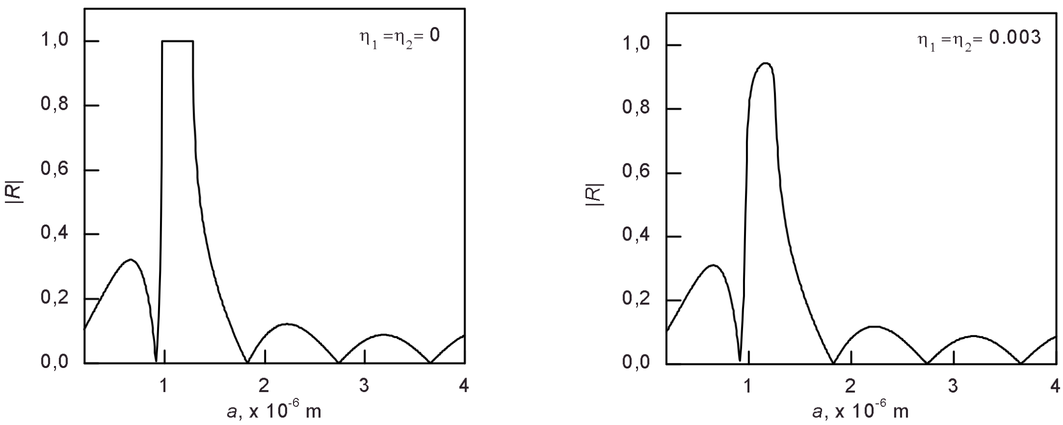

Fig. 3. Dependence of the reflection amplitude on a layer thickness for $\omega = 5.8 \times 10^{11}$ s^{-1} and $L = a + b = 4$ μm. All other parameters are the same as in Fig. 2

reflection amplitude $|R|$ on a thickness a of one of the layers, provided that the values of the structure period l and wave frequency ω are fixed. It is seen that the amplitude of a reflected wave strongly depends on the frequency and the ratio between the layer thicknesses. It should be stressed that a profound characteristic of both the figures is the presence of periodically repeated points which correspond to a complete transmission of a wave through the multilayer magnet. It is noteworthy that the idealized case $\eta = 0$ corresponds to such parameters of the actual structure where η is small enough, so that it can be neglected.

It should also be noted that, as seen from Fig. 4, an increase in η results in the enhancement of the total damping, which, in turn, leads to a decrease in the reflection amplitude.

8. Conclusions

In summary, the reflectivity of the multilayer structure under study displays the periodic dependence on the frequency and the ratio between the thicknesses of layers. What is more, the account for the damping essentially affects the features of the spin wave reflection from the multilayer structure and qualitatively changes the characteristic parts of the curves which illustrate the dependences of the reflection amplitude on the frequency and the layer thickness. As for the future prospects of

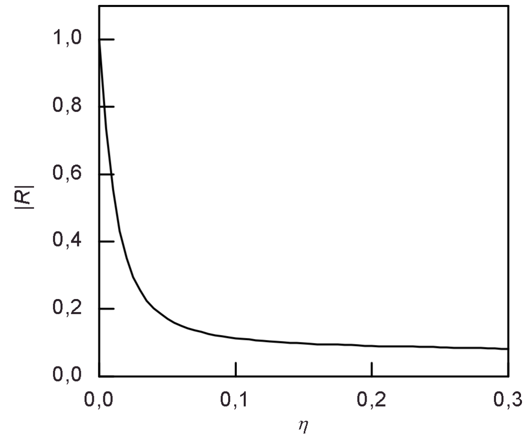

Fig. 4. Dependence of the reflection amplitude on a value of the Gilbert damping parameter for the structure with the same parameters as used for the plots in Fig. 2

developing the problems outlined in this paper, it is of interest to combine the obtained results with the data on the field dependence of the coefficient of spin wave reflection from the multilayer structures under study, which will make it possible to create a complex approach applicable to the design of spin wave devices.

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ВІДБИТТЯ ТА ЗАГАСАННЯ ОБ'ЄМНИХ СПІНОВИХ ХВИЛЬ У ОДНОВІСНІЙ БАГАТОШАРОВІЙ ФЕРОМАГНІТНІЙ СТРУКТУРІ

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Р е з ю м е

Теоретично вивчено процес поширення спінових хвиль у магнітних матеріалах з періодично модульованими параметрами одновісної магнітної анізотропії, обмінної взаємодії та намагніченості насичення. Досліджено вплив слабкого загасання на коефіцієнт відбиття спінових хвиль від одновісної багатошарової феромагнітної структури, а також залежності коефіцієнта відбиття від частоти хвилі, товщини одного з шарів та від параметра загасання.